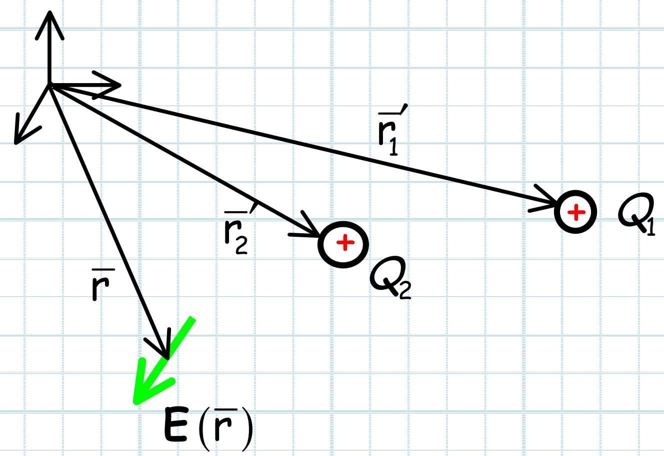
Coulomb's Law for Charge Density

Consider the case where there are **multiple** point charges present. What is the resulting **electrostatic field** ?



The electric field produced by the charges is simply the **vector** sum of the electric field produced by each (i.e., **superposition**!):

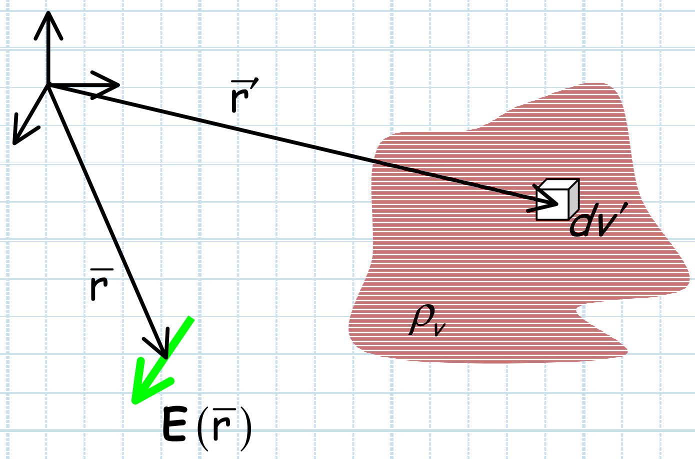
$$\mathbf{E}(\bar{r}) = \frac{Q_1}{4\pi\varepsilon_0}\frac{\bar{r}-\bar{r}_1'}{\left|\bar{r}-\bar{r}_1'\right|^3} + \frac{Q_2}{4\pi\varepsilon_0}\frac{\bar{r}-\bar{r}_2'}{\left|\bar{r}-\bar{r}_2'\right|^3}$$

Or, more generally, for N point charges:

$$\mathbf{E}(\bar{r}) = \sum_{n=1}^{N} \frac{Q_n}{4\pi\varepsilon_0}\frac{\bar{r}-\bar{r}_n'}{\left|\bar{r}-\bar{r}_n'\right|^3}$$

Consider now a volume V that is filled with a "cloud" of charge, described by **volume charge density** $\rho_v(\bar{r})$.

A very small differential volume *dv*, located at point $\bar{r}'$, will thus contain charge $dQ = \rho_v(\bar{r}')\,dv'$.



This differential charge produces an electric field at point $\bar{r}$ equal to :

$$\mathbf{dE}(\bar{r}) = \frac{\rho_v(\bar{r}')\,dv'}{4\pi\varepsilon_0}\,\frac{\bar{r}-\bar{r}'}{|\bar{r}-\bar{r}'|^3}$$

The **total** electric field at $\bar{r}$ (i.e., $\mathbf{E}(\bar{r})$) is the summation (i.e., **integration**) of all the electric field vectors produced by all the little differential charges *dQ* that make up the charge cloud:

$$\mathbf{E}(\bar{r}) = \iiint_V \frac{\rho_v(\bar{r}')}{4\pi\varepsilon_0}\,\frac{\bar{r}-\bar{r}'}{|\bar{r}-\bar{r}'|^3}\,dv'$$

Note: The variables of integration are the **primed** coordinates, representing the locations of the charges (i.e., **sources**).

Similarly, we can show that for **surface** charge:

$$\mathbf{E}(\bar{\mathbf{r}}) = \iint_S \frac{\rho_s(\bar{\mathbf{r}}')}{4\pi\varepsilon_0} \frac{\bar{\mathbf{r}}-\bar{\mathbf{r}}'}{\left|\bar{\mathbf{r}}-\bar{\mathbf{r}}'\right|^3} \, ds'$$

And for **line** charge:

$$\mathbf{E}(\bar{\mathbf{r}}) = \int_C \frac{\rho_\ell(\bar{\mathbf{r}}')}{4\pi\varepsilon_0} \frac{\bar{\mathbf{r}}-\bar{\mathbf{r}}'}{\left|\bar{\mathbf{r}}-\bar{\mathbf{r}}'\right|^3} \, d\ell'$$