Coulomb's Law

Recall from Coulomb's Law of Force that a charge $Q_2$ located at point $\vec{r}_2$ applies a force $F_1$ on charge $Q_1$ (located at point $\vec{r}_1$):

$$F_1 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1Q_2}{R^2} \hat{a}_{21} = \frac{Q_1Q_2}{4\pi\varepsilon_0} \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3}$$

Likewise, from the Lorentz Force Law, we know that the force $F_1$ on a charge $Q_1$ located at point $\vec{r}_1$ is attributed to an electric field located at $\vec{r}_1$:

$$F_1 = Q_1 E(\vec{r}_1) \quad \Rightarrow \quad E(\vec{r}_1) = \frac{F_1}{Q_1}$$

Inserting Coulomb's Law of Force into this equation, we get the electric field at location $\vec{r}_1$, generated by charge $Q_2$ located at $\vec{r}_2$:

$$E(\vec{r}) = \frac{F_1}{Q_1} = \frac{Q_2}{4\pi\varepsilon_0} \frac{\hat{a}_{21}}{R^2}$$
In general, we can say the electric field $E(\vec{r})$ at location $\vec{r}$, generated by a charge $Q$ at point $\vec{r}'$, is:

$$E(\vec{r}) = \frac{Q}{4\pi\varepsilon_0} \frac{\hat{a}_r}{R^2} = \frac{Q}{4\pi\varepsilon_0} \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$

This is **Coulomb's Law** !! It describes the electric field $E(\vec{r})$ at location $\vec{r}$ that is created by a charge $Q$ at location $\vec{r}'$.

Note that:

$$\hat{a}_r = \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|}$$

Therefore, if the charge $Q$ is at the origin (i.e., $\vec{r}' = 0$), then:

$$\hat{a}_r = \frac{\vec{r}}{|\vec{r}|} = \hat{a}_r$$

Recall that the base vector $\hat{a}_r$ always points away from the origin. In other words, a charge located at the origin creates an electric field vector that points in the direction of base vector $\hat{a}_r$ (i.e., away from the origin) at all points $\vec{r}$!

Likewise, if the charge is at the origin, then:

$$R = |\vec{r}| = r$$
In other words, the magnitude of the electric field vector is proportional to \(1/r^2\). As a result, the magnitude of the electric field is dependent on its distance from the origin (i.e., distance from the charge). Therefore, if \(r'=0\):

\[
\mathbf{E}(\vec{r}) = \frac{Q}{4\pi \varepsilon_0} \frac{\hat{a}_r}{r^2} = \frac{Q}{4\pi \varepsilon_0} \frac{\vec{r}}{r^3}
\]

**Q:** What is the curl of \(\mathbf{E}(\vec{r})\)?

**A:** 

\[
\nabla \times \mathbf{E}(\vec{r}) = \nabla \times \frac{Q}{4\pi \varepsilon_0} \frac{\vec{r}}{r^3}
\]