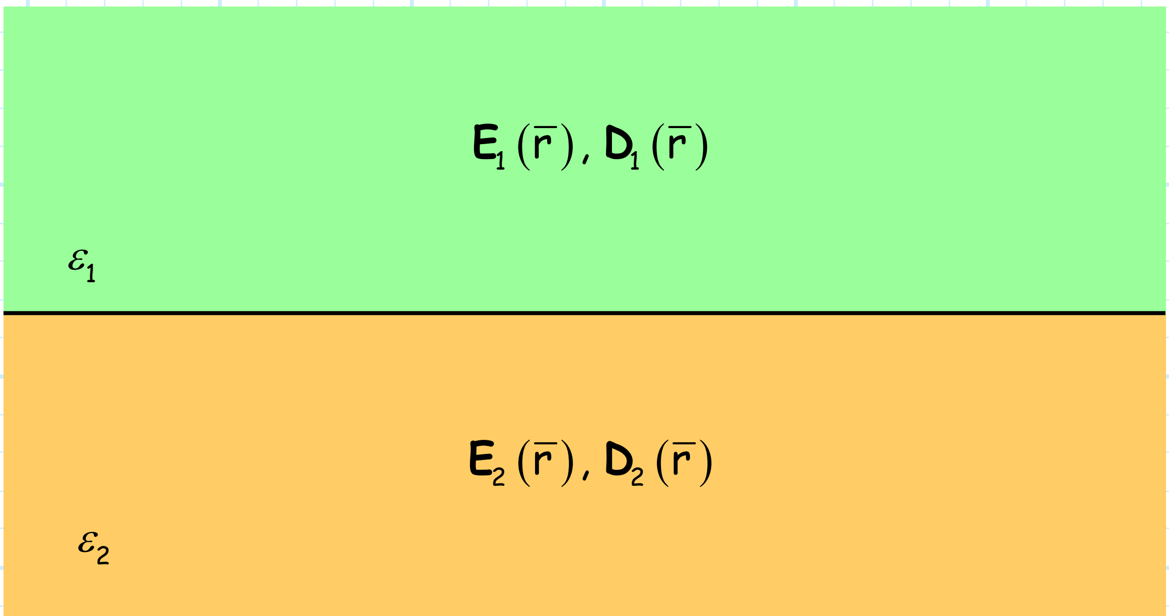
Dielectric Boundary Conditions

Consider the **interface** between two dissimilar **dielectric** regions:

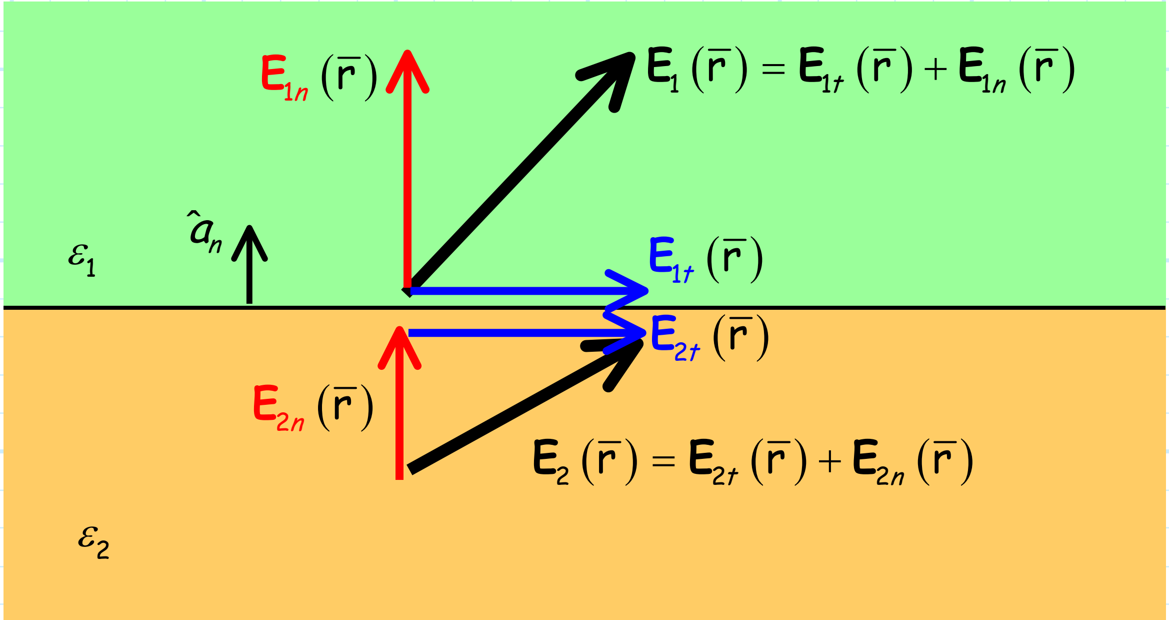


Say that an **electric field** is present in both regions, thus producing also an electric flux density ($\mathbf{D}(\bar{\mathbf{r}}) = \varepsilon \mathbf{E}(\bar{\mathbf{r}})$).

Q: *How are the fields in dielectric* ***region 1*** *(i.e., $\mathbf{E}_1(\bar{\mathbf{r}}), \mathbf{D}_1(\bar{\mathbf{r}})$) related to the fields in* ***region 2*** *(i.e., $\mathbf{E}_2(\bar{\mathbf{r}}), \mathbf{D}_2(\bar{\mathbf{r}})$)?*

A: They must satisfy the **dielectric boundary conditions** !

First, let's write the fields **at the dielectric interface** in terms of their **normal** ($\mathbf{E}_n(\bar{r})$) and **tangential** ($\mathbf{E}_t(\bar{r})$) vector components:



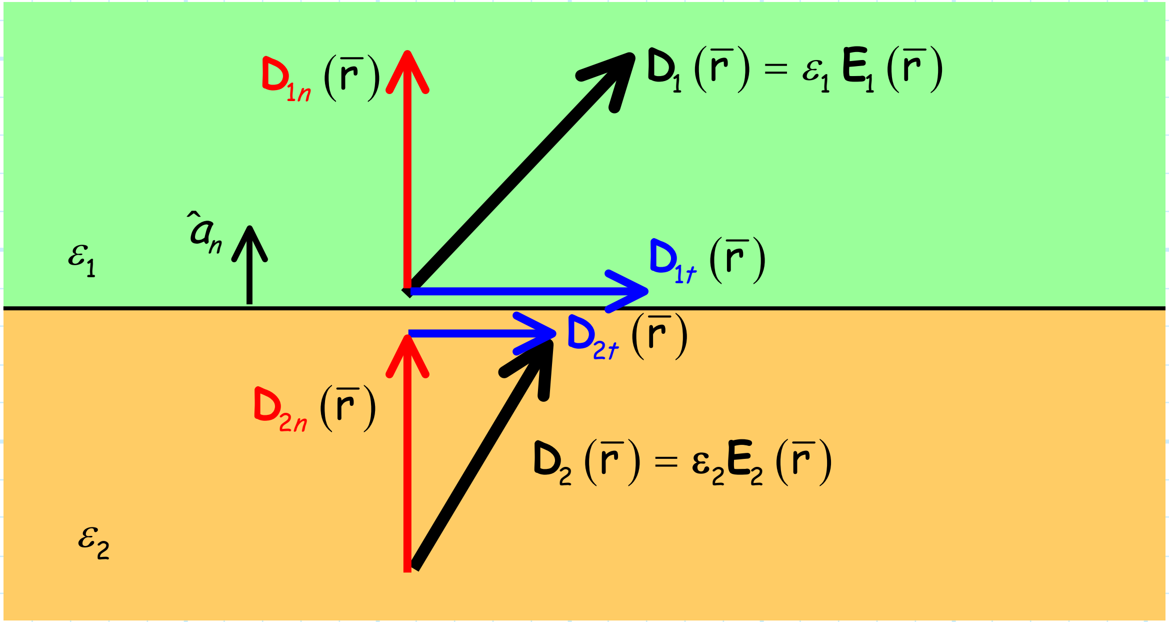
Our first boundary condition states that the **tangential** component of the electric field is **continuous** across a boundary. In other words:

$$\mathbf{E}_{1t}(\bar{r}_b) = \mathbf{E}_{2t}(\bar{r}_b)$$

where $\bar{r}_b$ denotes any point **on the boundary** (e.g., dielectric interface).

The **tangential** component of the electric field at **one** side of the dielectric boundary is **equal** to the tangential component at the **other** side !

We can likewise consider the **electric flux densities** on the dielectric interface in terms of their **normal** and **tangential** components:



The second dielectric boundary condition states that the **normal** vector component of the **electric flux density** is **continuous** across the dielectric boundary. In other words:

$$D_{1n}(\bar{r}_b) = D_{2n}(\bar{r}_b)$$

where $\bar{r}_b$ denotes any point **on the dielectric boundary** (i.e., dielectric interface).

Since $\mathbf{D}(\bar{r}) = \varepsilon\, \mathbf{E}(\bar{r})$, these boundary conditions can **likewise** be expressed as:

$$\mathbf{E}_{1t}(\bar{r}_b) = \mathbf{E}_{2t}(\bar{r}_b)$$

$$\frac{\mathbf{D}_{1t}(\bar{r}_b)}{\varepsilon_1} = \frac{\mathbf{D}_{2t}(\bar{r}_b)}{\varepsilon_2}$$

and as:

$$\mathbf{D}_{1n}(\bar{r}_b) = \mathbf{D}_{2n}(\bar{r}_b)$$

$$\varepsilon_1 \mathbf{E}_{1n}(\bar{r}_b) = \varepsilon_2 \mathbf{E}_{2n}(\bar{r}_b)$$

MAKE SURE YOU UNDERSTAND THIS:

These boundary conditions describe the relationships of the vector fields **at the dielectric interface** only (i.e., at points $\bar{r} = \bar{r}_b$)!!!! They say **nothing** about the value of the fields at points above or below the interface.