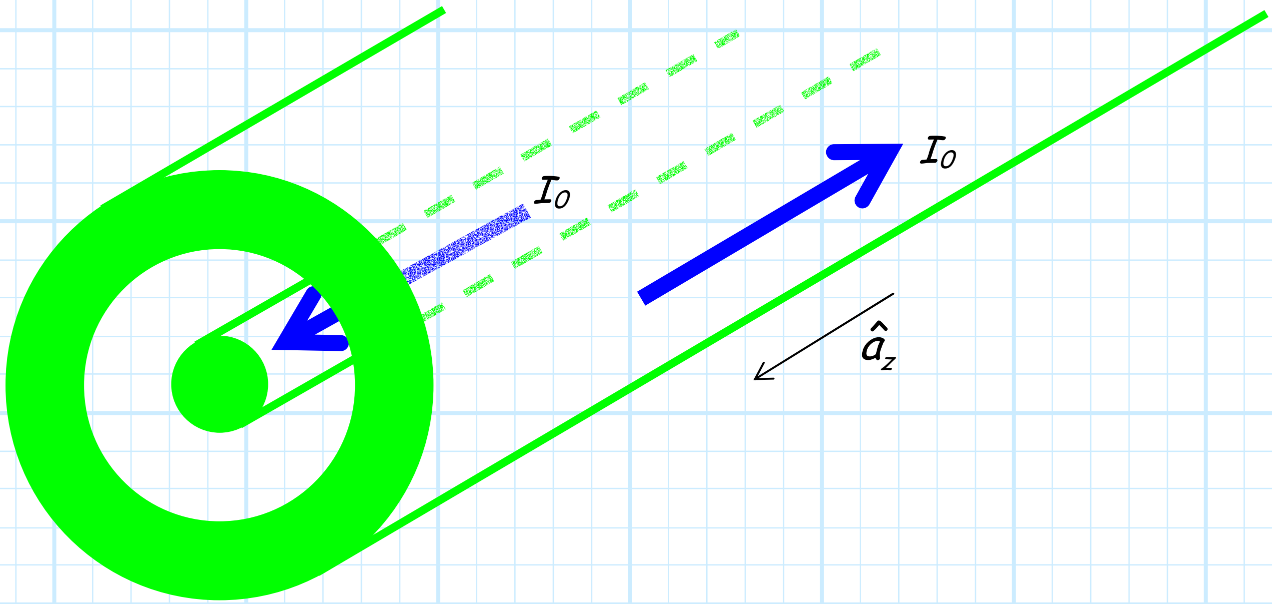
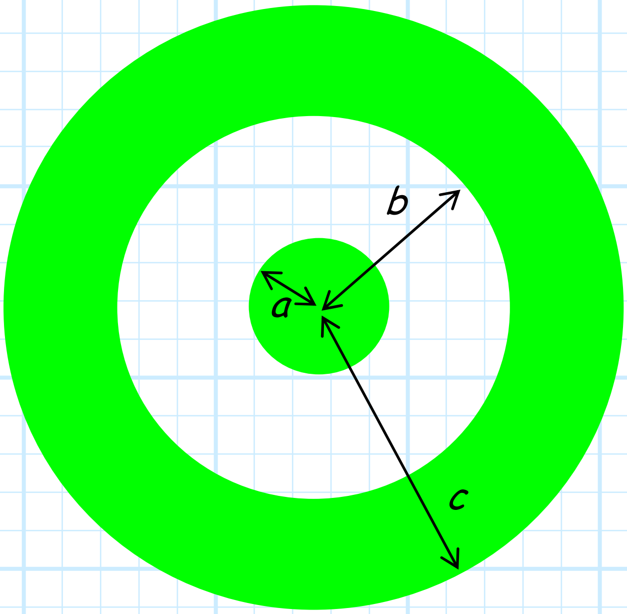
Example: The B-Field of Coaxial Transmission Line

Consider now a **coaxial cable**, with inner radius a:



The **outer** surface of the **inner** conductor has radius a, the **inner** surface of the **outer** conductor has radius b, and the **outer** radius of the **outer** conductor has radius c.



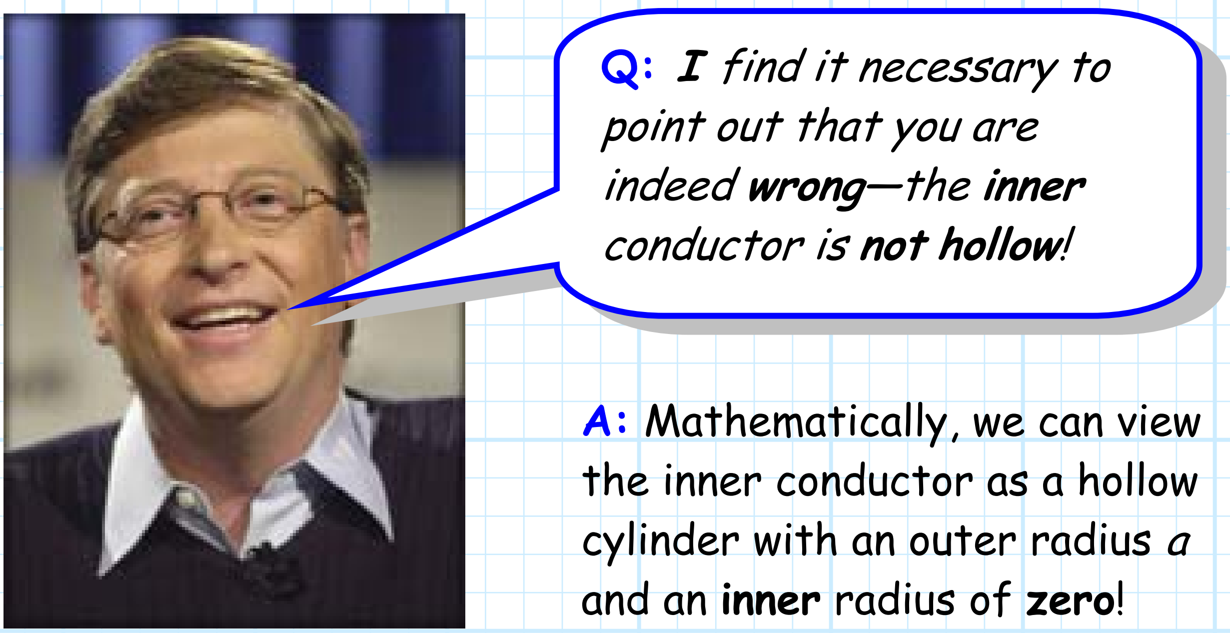
Typically, the **current** flowing on the **inner** conductor is **equal but opposite** that flowing in the **outer** conductor. Thus, if current I_0 is flowing in the **inner** conductor in the direction $\hat{a}_z$, then current I_0 will be flowing in the **outer** conductor in the opposite (i.e., $-\hat{a}_z$) direction.

Q: *Hey! If there is current, a magnetic flux density must be created. What is the vector field* $\mathbf{B}(\bar{r})$*?*

A: We've already determined this (sort of) !

Recall we found the magnetic flux density produced by a hollow cylinder—we can use this to determine the magnetic flux density in a coaxial transmission line.

A coaxial cable can be viewed as **two** hollow cylinders!



A: Mathematically, we can view the inner conductor as a hollow cylinder with an outer radius *a* and an **inner** radius of **zero**!

Thus, we can use the results of the previous handout to conclude that the magnetic flux density produced by the current flowing in the inner conductor is:

$$\mathbf{B}_{inner}(\bar{\mathbf{r}}) = \begin{cases} \dfrac{I_0\ \mu_0}{2\pi\rho}\left(\dfrac{\rho^2 - 0^2}{a^2 - 0^2}\right)\hat{a}_\phi = \dfrac{I_0\ \mu_0}{2\pi a^2}\rho\ \hat{a}_\phi & \rho < a \\ \\ \dfrac{I_0\ \mu_0}{2\pi\rho}\hat{a}_\phi & \rho > a \end{cases} \qquad \left[\frac{Webers}{m^2}\right]$$

Likewise, we can use the same result to determine the magnetic flux density of the current flowing in the outer conductor:

$$\mathbf{B}_{outer}(\bar{\mathbf{r}}) = \begin{cases} 0 & \rho < b \\ \\ \dfrac{-I_0\ \mu_0}{2\pi\rho}\left(\dfrac{\rho^2 - b^2}{c^2 - b^2}\right)\hat{a}_\phi & b < \rho < c \\ \\ \dfrac{-I_0\ \mu_0}{2\pi\rho}\hat{a}_\phi & \rho > c \end{cases} \qquad \left[\frac{Webers}{m^2}\right]$$

Note the **minus sign** is due to direction of the current $(-\hat{a}_z)$ in the outer conductor.

We can now apply **superposition** to determine the total magnetic flux density in a coaxial transmission line! Specifically:

$$\text{if} \qquad \mathbf{J}(\bar{r}) = \mathbf{J}_{inner}(\bar{r}) + \mathbf{J}_{outer}(\bar{r})$$

$$\text{then} \qquad \mathbf{B}(\bar{r}) = \mathbf{B}_{inner}(\bar{r}) + \mathbf{B}_{outer}(\bar{r})$$

Note due to the **piecewise** nature of these solutions, we must evaluate this sum for **4** distinct regions:

1) $\rho < a$ (in the inner conductor)

2) $a < \rho < b$ (in the region between the conductors)

3) $b < \rho < c$ (in the outer conductor)

4) $\rho > c$ (outside the coaxial cable)

$\underline{\rho < a}$

$$\begin{aligned} \mathbf{B}(\bar{r}) &= \mathbf{B}_{inner}(\bar{r}) + \mathbf{B}_{outer}(\bar{r}) \\ &= \frac{I_0 \, \mu_0}{2\pi a^2} \rho \, \hat{a}_\phi + 0 \\ &= \frac{I_0 \, \mu_0}{2\pi a^2} \rho \, \hat{a}_\phi \end{aligned}$$

$a < \rho < b$

$$
\begin{aligned}
\mathbf{B}(\bar{\mathbf{r}}) &= \mathbf{B}_{inner}(\bar{\mathbf{r}}) + \mathbf{B}_{outer}(\bar{\mathbf{r}}) \\
&= \frac{I_0\ \mu_0}{2\pi\ \rho}\,\hat{a}_\phi + 0 \\
&= \frac{I_0\ \mu_0}{2\pi\ \rho}\,\hat{a}_\phi
\end{aligned}
$$

$b < \rho < c$

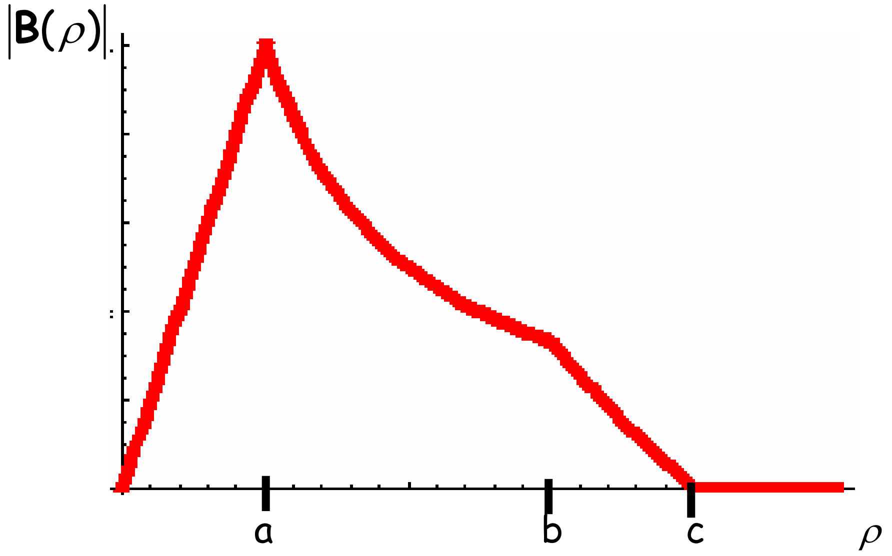
$$
\begin{aligned}
\mathbf{B}(\bar{\mathbf{r}}) &= \mathbf{B}_{inner}(\bar{\mathbf{r}}) + \mathbf{B}_{outer}(\bar{\mathbf{r}}) \\
&= \frac{I_0\ \mu_0}{2\pi\ \rho}\,\hat{a}_\phi + \frac{-I_0\ \mu_0}{2\pi\rho}\left(\frac{\rho^2 - b^2}{c^2 - b^2}\right)\hat{a}_\phi \\
&= \frac{I_0\ \mu_0}{2\pi\ \rho}\left(\frac{c^2 - \rho^2}{c^2 - b^2}\right)\hat{a}_\phi
\end{aligned}
$$

$\rho > c$

$$
\begin{aligned}
\mathbf{B}(\bar{\mathbf{r}}) &= \mathbf{B}_{inner}(\bar{\mathbf{r}}) + \mathbf{B}_{outer}(\bar{\mathbf{r}}) \\
&= \frac{I_0\ \mu_0}{2\pi\ \rho}\,\hat{a}_\phi + \frac{-I_0\ \mu_0}{2\pi\rho}\,\hat{a}_\phi \\
&= 0
\end{aligned}
$$

Summarizing, we find the total magnetic flux density to be:

$$\mathbf{B}(\bar{\mathbf{r}}) = \begin{cases} \dfrac{I_0\ \mu_0}{2\pi a^2}\rho\ \hat{a}_\phi & \rho < a \\[2ex] \dfrac{I_0\ \mu_0}{2\pi\ \rho}\ \hat{a}_\phi & a < \rho < b \\[2ex] \dfrac{I_0\ \mu_0}{2\pi\ \rho}\left(\dfrac{c^2-\rho^2}{c^2-b^2}\right)\hat{a}_\phi & b < \rho < c \\[2ex] 0 & \rho > c \end{cases} \qquad \left[\frac{Webers}{m^2}\right]$$



The magnetic flux density **and** the electric field **outside** of a coaxial transmission line are **zero**!