Example: The Inductance of a Solenoid

Many **inductors** used in electronic circuits are simply **solenoids**. Let's determine the **inductance** of this structure!

First, we recall that inductance is the **ratio** of the **current** and the **flux linkages** that the current produces:

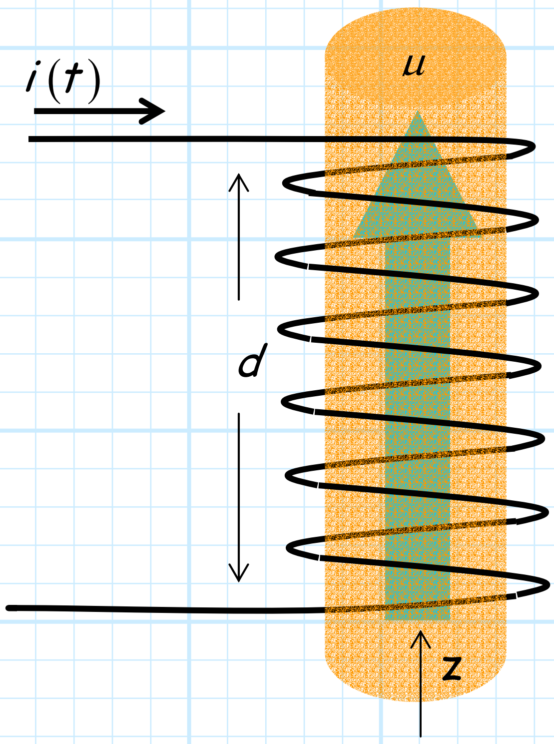
$$L \doteq \frac{\Lambda}{i} = \text{inductance} \quad \left[\frac{\text{Webers}}{\text{Amp}}\right]$$

The question then is, what is flux linkages Λ for a **solenoid**?

Recall that the magnetic flux density in the **interior** of a solenoid is:

$$\mathbf{B}(\bar{r}) \approx \frac{\mu N i}{d} \hat{a}_z$$

where N is the number of loops and d is the length of the solenoid.



The total **magnetic flux** flowing through the solenoid is therefore found by integrating across the **cross-section** of the solenoid:

$$\Phi = \iint_S \mathbf{B}(\bar{r}) \cdot \overline{ds}$$
$$= \frac{\mu N i}{d} S$$

where S is the cross-sectional area of the solenoid (e.g., $S = \pi a^2$ if solenoid is circular with radius a).

Recall the total **flux linkage** is just the **product** of the **magnetic flux** and the **number of loops**:

$$\Lambda = N\Phi$$
$$= \frac{\mu N^2 S}{d} i$$

Thus, we now find that the **inductance of a solenoid** is:

$$L = \frac{\Lambda}{i} = \frac{\mu N^2 S}{d}$$

Note if we wish to **increase** the inductance of this solenoid, we can either:

1) **Increase** the **permeability** μ of the core material.

2) **Increase** the number of **turns** N.

3) **Increase** the cross-sectional **area** S

4) **Decrease** the **length** d (while keeping N constant).

Note all of the derivations in this handout are derived from the solution to an **infinite** solenoid. As a result, they are **approximations**, but are typically accurate ones **provided** that:

$$d >> \sqrt{S}$$

In other words, provided that the inductor **length** is significantly **greater** than its **radius**.