Faraday's Law of Induction

Say instead of a static magnetic flux density, we consider a **time-varying** B field (i.e., $B(\overline{r}, t)$). Recall that one of **Maxwell's** equations is:

$$\nabla \mathbf{x} \mathbf{E}(\bar{r}) = -\frac{\partial \mathbf{B}(\bar{r}, t)}{\partial t}$$

Yikes! The curl of the electric field is therefore **not zero** if the magnetic flux density is **time-varying**!

If the magnetic flux density is changing with time, the electric field will **not be conservative**!

Q: What the heck does this equation mean ?!?

A: Integrate both sides over some surface S:

$$\iint_{S} \nabla \mathsf{x} \mathsf{E}(\bar{r}) \cdot \overline{ds} = -\frac{\partial}{\partial t} \iint_{S} \mathsf{B}(\bar{r}, t) \cdot \overline{ds}$$

Applying Stoke's Theorem, we get:

$$\oint_{\mathcal{C}} \mathbf{E}(\bar{r}) \cdot \overline{d\ell} = -\frac{\partial}{\partial t} \iint_{\mathcal{S}} \mathbf{B}(\bar{r}, t) \cdot \overline{ds}$$

where C is the contour that surrounds the boundary of S.

Note that $\oint \mathbf{E}(\overline{\mathbf{r}}) \cdot \overline{\mathbf{d}\ell} \neq \mathbf{0}$.

This equation is called Faraday's Law of Induction.

- Q: Again, what does this mean?
- A: It means that a time varying magnetic flux density $\mathbf{B}(\overline{r},t)$ can **induce** an electric field (and thus an electric potential difference)!

Faraday's Law describes the behavior of such devices such as generators, inductors, and transformers !



Michael Faraday (1791-1867), an English chemist and physicist, is shown here in an early daguerreotype holding a bar of glass he used in his 1845 experiments on the effects of a magnetic field on polarized light. Faraday is considered by many scientists to be the greatest experimentalist ever! (from "Famous Physicists and Astronomers" www.phy.hr/~dpaar/fizicari/index.html)