Field Equations in Magnetic Materials

Now that we have defined a magnetic field $\mathbf{H}(\bar{r})$ and material permeability $\mu(\bar{r})$, we can write the magnetostatic (point form) equations for fields in **magnetic material**.

$$\nabla x \mathbf{H}(\bar{r}) = \mathbf{J}(\bar{r})$$

$$\nabla \cdot \mathbf{B}(\bar{r}) = 0$$

$$\mathbf{B}(\bar{r}) = \mu(\bar{r})\mathbf{H}(\bar{r})$$

We likewise can express these equations in **integral form** as:

$$\oint_C \mathbf{H}(\bar{r}) \cdot \overline{d\ell} = I_{enc}$$

$$\oiint_S \mathbf{B}(\bar{r}) \cdot \overline{ds} = 0$$

$$\mathbf{B}(\bar{r}) = \mu(\bar{r})\mathbf{H}(\bar{r})$$

First, note the **new** form of **Ampere's Law**:

$$\oint_C \mathbf{H}(\bar{r}) \cdot \overline{d\ell} = I_{enc}$$

Where I_{enc} is the conduction current only (i.e., it does not include magnetization current!).

Again, note the **analogies** to the new form of **Gauss's Law** we derived for electrostatics:

$$\iint_S \mathbf{D}(\bar{r}) \cdot \overline{ds} = Q_{enc}$$

where Q_{enc} is the free-charge enclosed by surface S.

Perhaps the most important result of expressing magnetostatic fields in terms of material **permeability** $\mu(\bar{r})$ is that we **do not** have to **rederive** any of the results from Chapter 7!

In Chapter 7, the "material" we were concerned with was **free space**. The permeability of free space is by definition, $\mu(\bar{r}) = \mu_0$.

If the material is **not** free space, then we simply **change** the results of Chapter 7 to reflect the **correct value** of **permeability** $\mu(\bar{r})$.

For example, we found that the Biot-Savart Law becomes,:

$$\mathbf{B}(\bar{\mathbf{r}}) = \frac{\mu I}{4\pi} \oint_C \frac{\overline{d\ell'} \times (\bar{\mathbf{r}} - \bar{\mathbf{r}}')}{|\bar{\mathbf{r}} - \bar{\mathbf{r}}'|^3}$$

magnetic vector potential is,:

$$\mathbf{A}(\bar{\mathbf{r}}) = \frac{\mu}{4\pi} \iiint_V \frac{\mathbf{J}(\bar{\mathbf{r}}')}{|\bar{\mathbf{r}} - \bar{\mathbf{r}}'|} dv'$$

or the magnetic flux produced by a infinite line current is:

$$\mathbf{B}(\bar{\mathbf{r}}) = \frac{\mu I}{2\pi \rho} \hat{a}_\phi$$