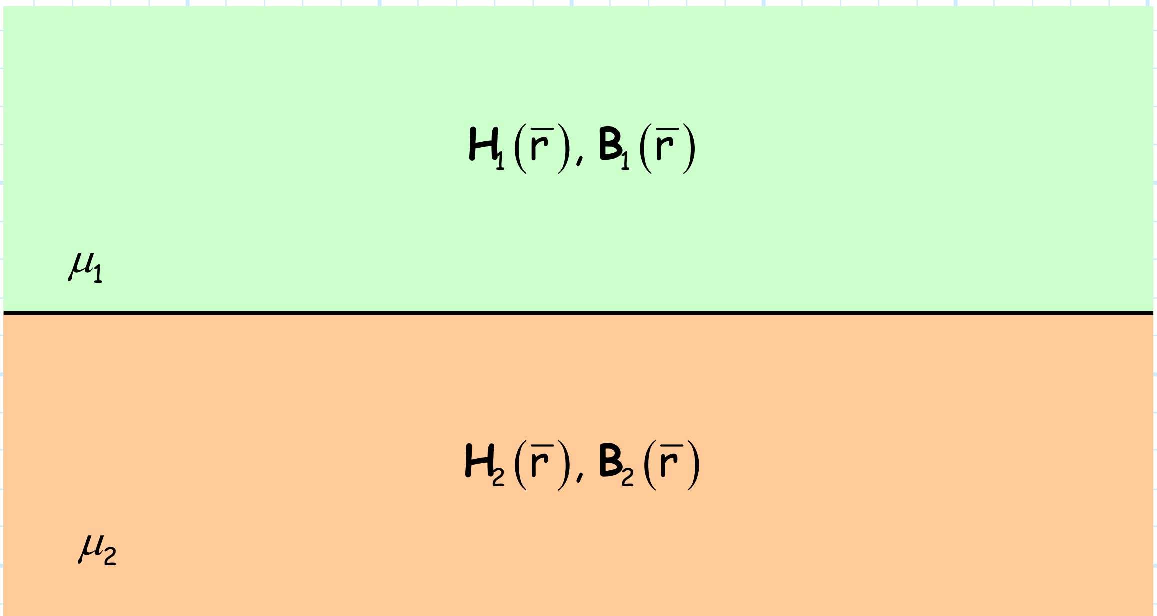
Magnetic Boundary Conditions

Consider the **interface** between two different materials with dissimilar permeabilities:

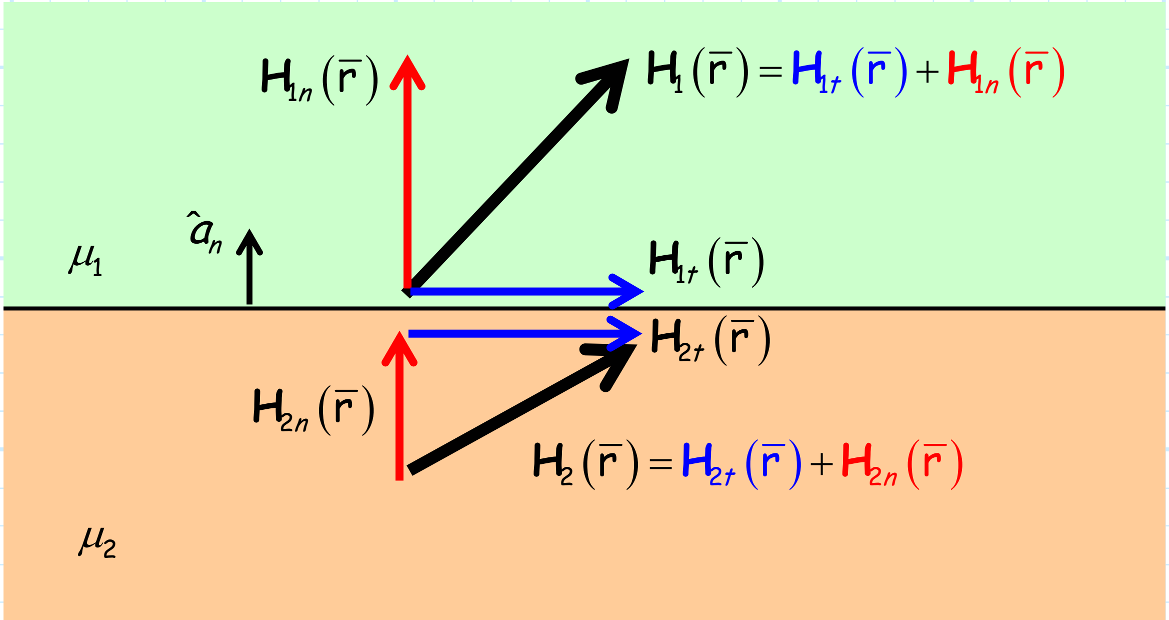


Say that a magnetic field and a magnetic flux density is present in **both** regions.

Q: *How are the fields in dielectric **region 1** (i.e., $\mathbf{H}_1(\bar{r}), \mathbf{B}_1(\bar{r})$) related to the fields in **region 2** (i.e., $\mathbf{H}_2(\bar{r}), \mathbf{B}_2(\bar{r})$)?*

A: They must satisfy the **magnetic boundary conditions** !

First, let's write the fields **at the interface** in terms of their **normal** (e.g., $\mathbf{H}_n(\bar{r})$) and **tangential** (e.g., $\mathbf{H}_t(\bar{r})$) vector components:



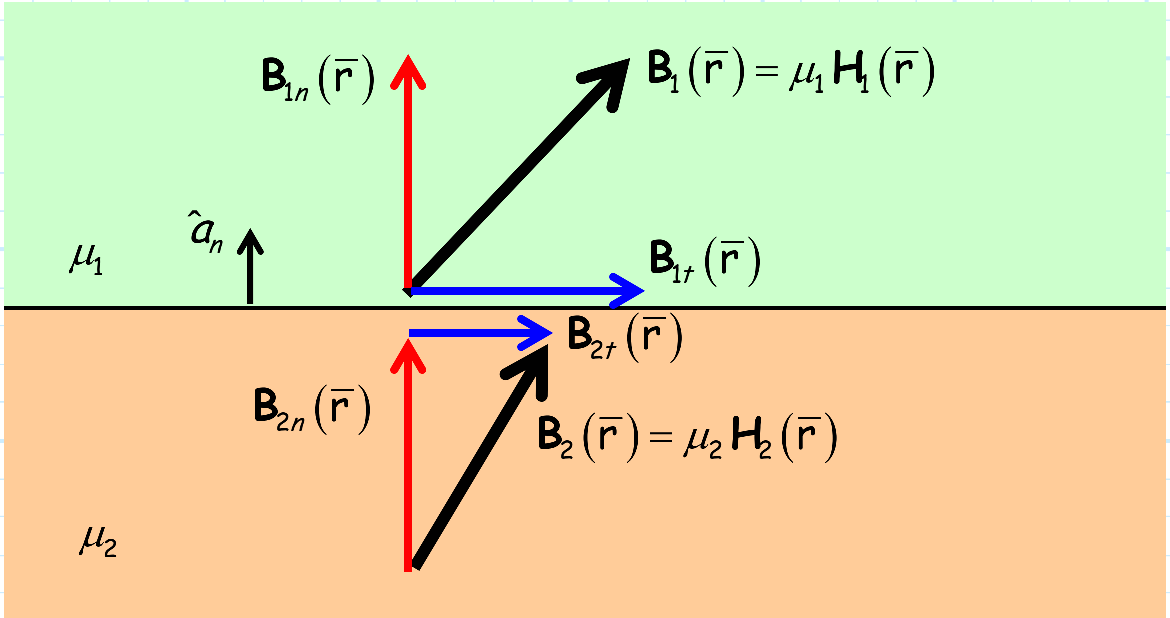
Our first boundary condition states that the **tangential** component of the magnetic field is **continuous** across a boundary. In other words:

$$\mathbf{H}_{1t}(\bar{r}_b) = \mathbf{H}_{2t}(\bar{r}_b)$$

where $\bar{r}_b$ denotes to **any** point along the interface (e.g., material boundary).

The **tangential** component of the magnetic field on **one** side of the material boundary is **equal** to the tangential component on the **other** side !

We can likewise consider the **magnetic flux densities** on the material interface in terms of their **normal** and **tangential** components:



The second magnetic boundary condition states that the **normal** vector component of the **magnetic flux density** is **continuous** across the material boundary. In other words:

$$\mathbf{B}_{1n}(\bar{r}_b) = \mathbf{B}_{2n}(\bar{r}_b)$$

where $\bar{r}_b$ denotes **any** point along the interface (i.e., the material boundary).

Since $\mathbf{B}(\bar{r}) = \mu\,\mathbf{H}(\bar{r})$, these boundary conditions can **likewise** be expressed as:

$$\mathbf{H}_{1t}(\bar{r}_b) = \mathbf{H}_{2t}(\bar{r}_b)$$

$$\frac{\mathbf{B}_{1t}(\bar{r}_b)}{\mu_1} = \frac{\mathbf{B}_{2t}(\bar{r}_b)}{\mu_2}$$

and as:

$$\mathbf{B}_{1n}(\bar{r}_b) = \mathbf{B}_{2n}(\bar{r}_b)$$

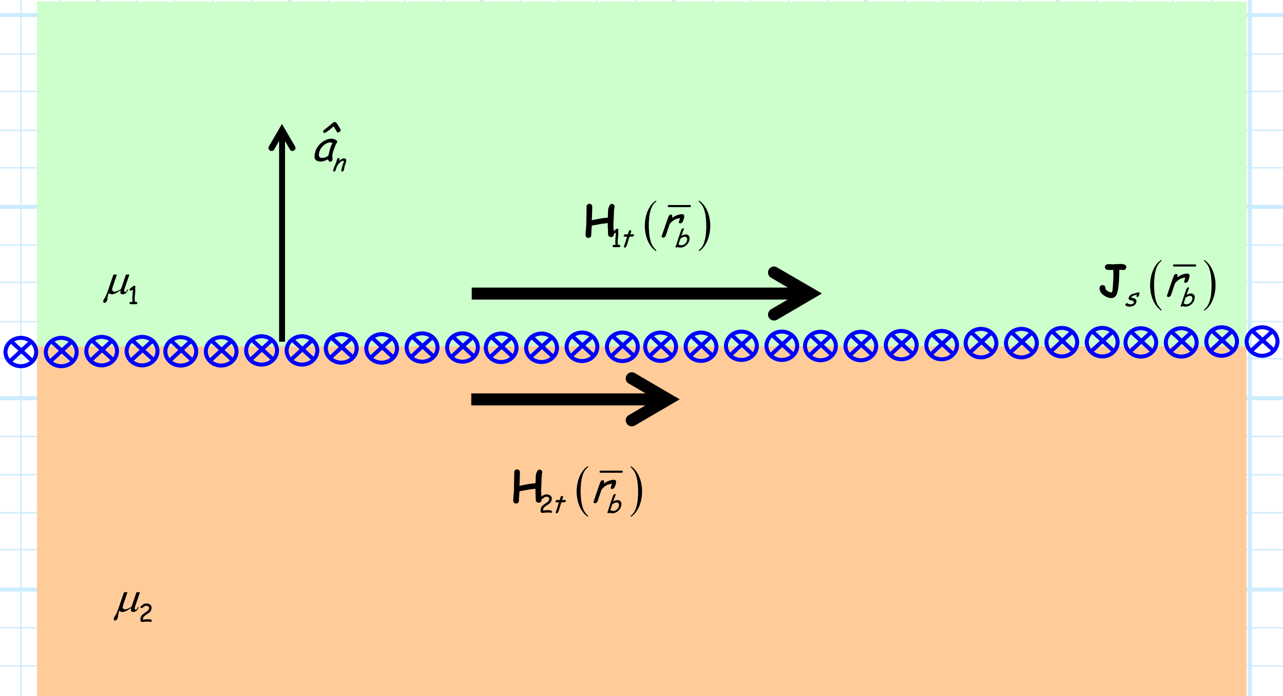
$$\mu_1\mathbf{H}_{1n}(\bar{r}_b) = \mu_2\mathbf{H}_{2n}(\bar{r}_b)$$

Note again the perfect **analogy** to the boundary conditions of **electrostatics**!

Finally, recall that if a layer of **free charge** were lying at a dielectric boundary, the boundary condition for electric flux density was **modified** such that:

$$\hat{a}_n \cdot \left[\mathbf{D}_1(\bar{r}_b) - \mathbf{D}_2(\bar{r}_b)\right] = \rho_s(\bar{r}_b)$$
$$D_{1n}(\bar{r}_b) - D_{2n}(\bar{r}_b) = \rho_s(\bar{r}_b)$$

There is an **analogous** problem in magnetostatics, wherein a **surface current** is flowing at the interface of two magnetic materials:



In this case the tangential components of the magnetic field will **not** be continuous!

Instead, they are related by the boundary condition:

$$\hat{a}_n \times \left(\mathbf{H}_1(\bar{r}_b) - \mathbf{H}_2(\bar{r}_b)\right) = \mathbf{J}_s(\bar{r}_b)$$

This expression means that:

1) $\mathbf{H}_{1t}(\bar{r}_b)$ and $\mathbf{H}_{2t}(\bar{r}_b)$ point in the **same** direction.

2) $\mathbf{H}_{1t}(\bar{r}_b)$ and $\mathbf{H}_{2t}(\bar{r}_b)$ are **orthogonal** to $\mathbf{J}_s(\bar{r}_b)$.

3) The difference between $\left|\mathbf{H}_{1t}(\bar{r}_b)\right|$ and $\left|\mathbf{H}_{2t}(\bar{r}_b)\right|$ is $\left|\mathbf{J}_s(\bar{r}_b)\right|$.

Recall that $\mathbf{H}(\bar{r})$ and $\mathbf{J}_s(\bar{r})$ have the same units—**Amperes/meter**!

Note for this case, the boundary condition for the magnetic flux density remains **unchanged**, i.e.:

$$\mathbf{B}_{1n}(\bar{r}_b) = \mathbf{B}_{2n}(\bar{r}_b)$$

regardless of $\mathbf{J}_s(\bar{r}_b)$.