Maxwell's Equations

Consider what we now know:

1) Law of Charge Conservation

2) Coulomb's Law of Force

3) Ampere's Law of Force

4) Lorentz Force Law

These are all valid laws, but they are **not complete**. That is, they do not completely describe the relationships between $\mathbf{J}(\bar{r})$, $\rho_v(\bar{r})$, $\mathbf{B}(\bar{r})$, and $\mathbf{E}(\bar{r})$.

In 1873, **James Clerk Maxwell** published a book on electromagnetics, which included a complete, unified theory.

This theory includes 4 equations relating $\mathbf{J}(\bar{r},t)$, $\rho_v(\bar{r},t)$, $\mathbf{B}(\bar{r},t)$, and $\mathbf{E}(\bar{r},t)$, called **Maxwell's Equations**.

$$\nabla \times \mathbf{E}(\bar{r},t) = -\frac{\partial \mathbf{B}(\bar{r},t)}{\partial t}$$

$$\nabla \cdot \mathbf{E}(\bar{r},t) = \frac{\rho_v(\bar{r},t)}{\varepsilon_0}$$

$$\nabla \times \mathbf{B}(\bar{r},t) = \mu_0 \mathbf{J}(\bar{r},t) + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}(\bar{r},t)}{\partial t}$$

$$\nabla \cdot \mathbf{B}(\bar{r},t) = 0$$

From **Helmholtz's Theorems**, we know that we must know **both** the **divergence** and the **curl** of a vector field in order to determine the vector field.

Note Maxwell's Equation does this for both the electric field $\mathbf{E}(\bar{r},t)$ and magnetic flux density $\mathbf{B}(\bar{r},t)$!

Q: *Is the magnetic flux density* $\mathbf{B}(\bar{r},t)$ *conservative, solenoidal, or neither?*

A:

* Since the divergence of the magnetic flux density is zero ($\nabla \cdot \mathbf{B}(\bar{r},t) = 0$), it is a **solenoidal** vector field.

* Thus, **all** the things that we learned about solenoidal fields are true for the magnetic flux density $\mathbf{B}(\bar{r},t)$.

* Likewise, the **sources** of this rotational field appear to be **current** (i.e., $\mu_0 \mathbf{J}(\bar{r},t)$), and/or a **time-varying electric field**:

$$\nabla \times \mathbf{B}(\bar{r},t) = \mu_0 \mathbf{J}(\bar{r},t) + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}(\bar{r},t)}{\partial t}$$

Note that **permittivity** ε_0 and **permeability** μ_0 of free space appear **also** in Maxwell's Equations !

Q: Is the electric field $\mathbf{E}(\bar{r},t)$ conservative, solenoidal, or neither?

A:

* Since **neither** the curl **nor** the divergence of the electric field is zero, the electric field is **neither** conservative **nor** solenoidal.

* Instead, it is apparent that the electric field has **both** a solenoidal and conservative vector component!

* The **source** of the **solenoidal** component of the electric field $\mathbf{E}(\bar{r},t)$ appears to be a **time-varying magnetic flux density**:

$$\nabla \times \mathbf{E}(\bar{r},t) = -\frac{\partial \mathbf{B}(\bar{r},t)}{\partial t}$$

* Whereas the **source** of the **conservative** component of $\mathbf{E}(\bar{r},t)$ appears to be **charge**:

$$\nabla \cdot \mathbf{E}(\bar{r},t) = \frac{\rho_v(\bar{r},t)}{\varepsilon_0}$$

Q: *But, what **else** do Maxwell's Equations **mean**?*

A: They mean that the rest of the semester will be **very** busy!