Orthogonal and Orthonormal Vector Sets

We often specify or relate a set of **scalar values** (e.g., x, y, z) using a set of **scalar equations**. For example, we might say:

$$x = y \quad \text{and} \quad z = x + 2$$

From which we can conclude a **third** expression:

$$z = y + 2$$

Say that we now add a **new** constraint to the first two:

$$x + y = 2$$

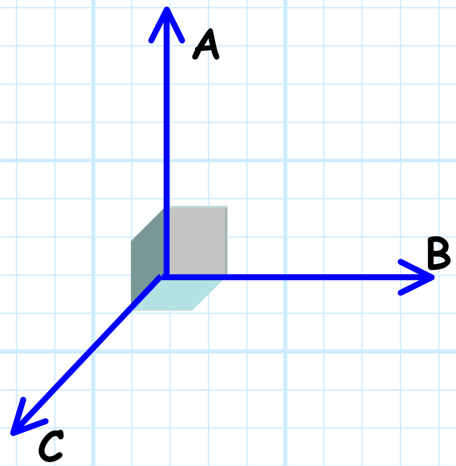
We can now **specifically** conclude that:

$$x = 1 \qquad y = 1 \qquad z = 3$$

Note we can likewise use **vector equations** to specify or relate a set of **vectors** (e.g., $\mathbf{A}, \mathbf{B}, \mathbf{C}$).

For example, consider a set of **three** vectors that are oriented such that they are **mutually orthogonal** !

In other words, each vector is **perpendicular** to each of the other two:



Note that we can **describe** this orthogonal relationship mathematically using **three simple equations**:

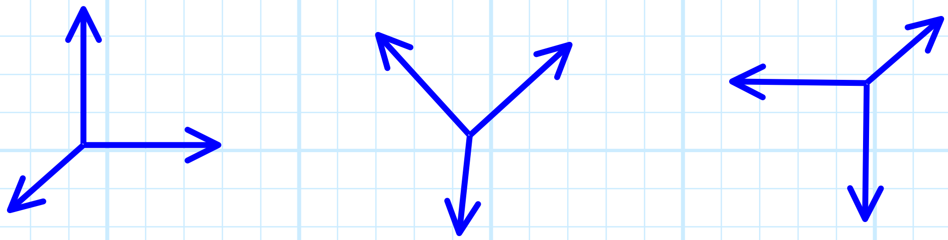
$$\mathbf{A} \cdot \mathbf{B} = 0$$
$$\mathbf{A} \cdot \mathbf{C} = 0$$
$$\mathbf{B} \cdot \mathbf{C} = 0$$

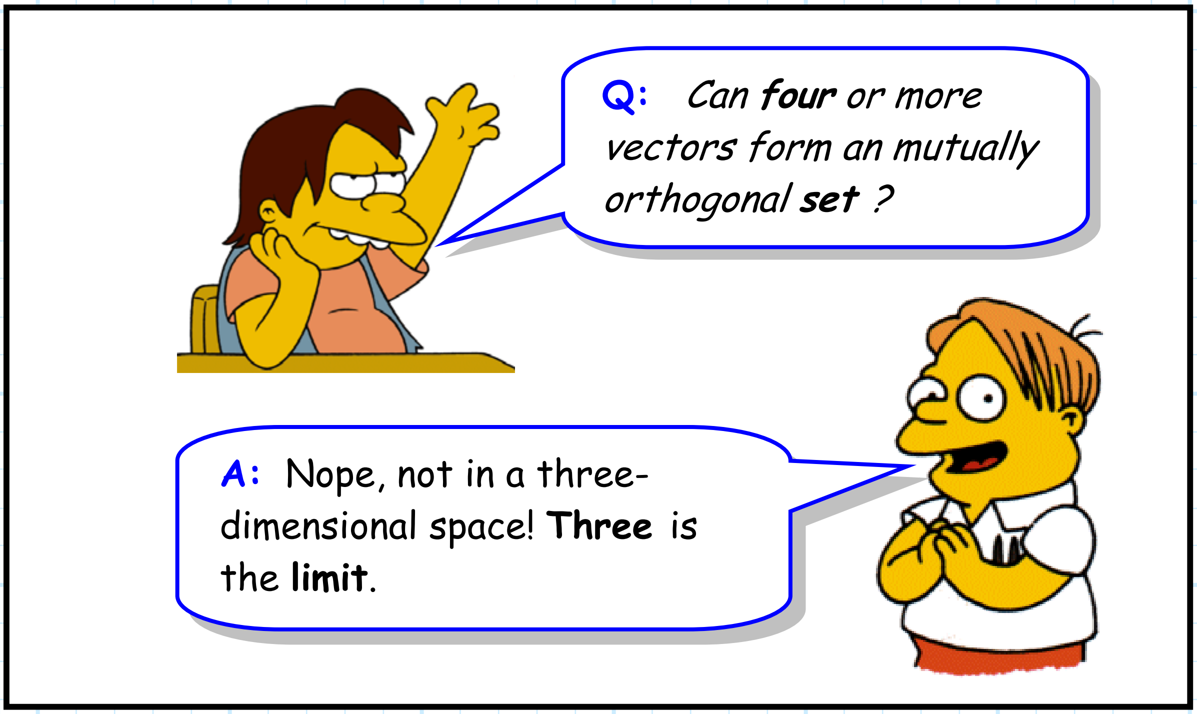
We can therefore **define** an orthogonal set of vectors using the **dot product**:

Three (non-zero) vectors **A**, **B** and **C** form an **orthogonal set** iff they satisfy $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{C} = \mathbf{C} \cdot \mathbf{A} = 0$

Note that there are an **infinite** number of mutually orthogonal vector **sets** that can be formed !

E.G:





Consider now a mutually orthogonal set of **unit vectors**. Such a set can be defined as **any** three vectors that satisfy these **six** equations:

$$\mathbf{A}\cdot\mathbf{B}=\mathbf{B}\cdot\mathbf{C}=\mathbf{C}\cdot\mathbf{A}=0 \qquad \text{(mutually orthognal)}$$

$$\mathbf{A}\cdot\mathbf{A}=\mathbf{B}\cdot\mathbf{B}=\mathbf{C}\cdot\mathbf{C}=1 \qquad \text{(unit magnitudes)}$$

A set of vectors that satisfy these equations are said to form an **orthonormal** set of vectors ! Therefore, an orthonormal set consists of **unit vectors** where:

$$\hat{a}_A \cdot \hat{a}_B = \hat{a}_B \cdot \hat{a}_C = \hat{a}_C \cdot \hat{a}_A = 0$$

Again, there are an **infinite** number of **orthonormal** vector sets, but each set consists of only **three** vectors.