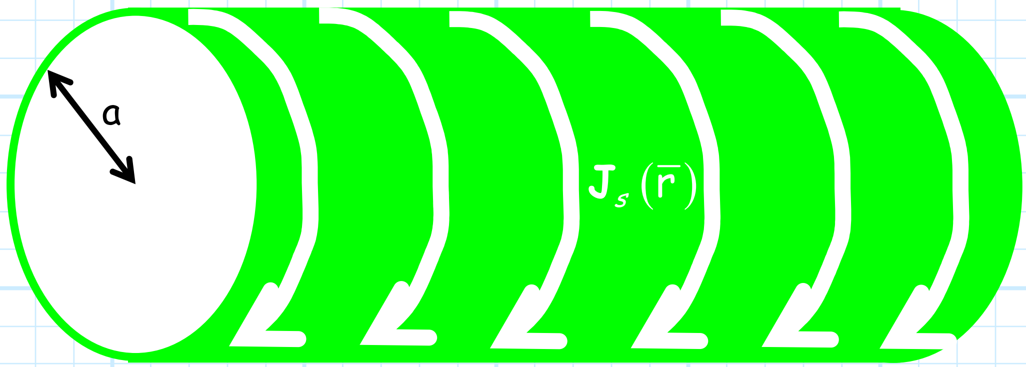
Solenoids

An important structure in electrical and computer engineering is the **solenoid**.

A solenoid is a **tube of current**. However, it is different from the hollow cylinder example, in that the current flows **around** the tube, rather than down the tube:



Aligning the center of the tube with the *z*-axis, we can express the **current density** as:

$$\mathbf{J}_s(\bar{r}) = \begin{cases} 0 & \rho < a \\ J_s\ \hat{a}_\phi & \rho = a \\ 0 & \rho > a \end{cases} \qquad \left[\frac{Amps}{m}\right]$$

where a is the **radius** of the solenoid, and J_s is the **surface** current density in Amps/meter.

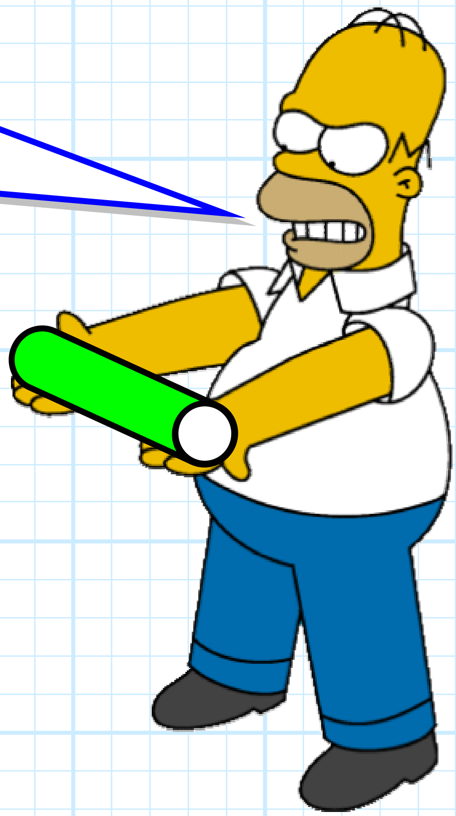
We can use **Ampere's Law** to find the magnetic flux density resulting from this structure. The result is:

$$\mathbf{B}(\bar{r}) = \begin{cases} \mu_0 J_s \, \hat{a}_z & \rho < a \\ 0 & \rho > a \end{cases}$$

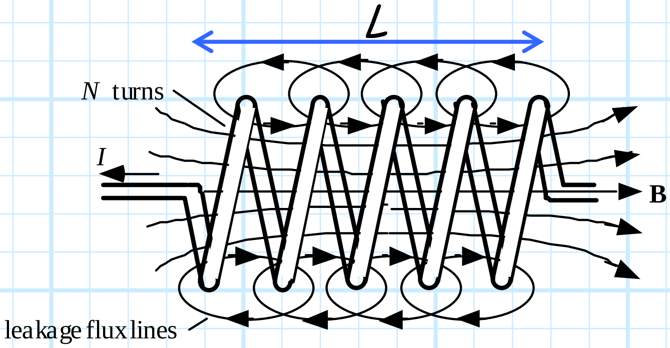
Note the direction of the magnetic flux density is in the direction $\hat{a}_z$--it points **down** the center of the solenoid.

Note also that the magnitude $|\mathbf{B}(\bar{r})|$ is **independent** of solenoid radius a!

Q: *Yeah right! How are we supposed to get current to flow **around** this tube? I don't see how this is even possible.*



A: We can easily make a solenoid by forming a **wire spiral** around a cylinder.



The surface current density J_s of this solenoid is **approximately** equal to:

$$J_s = \frac{N\ I}{L} = N_\ell\ I$$

where $N_\ell = N/L$ is the number of turns/unit length. Inserting this result into our expression for magnetic flux density, we find the magnetic flux density **inside** a solenoid:

$$\mathbf{B}(\bar{r}) = \mu_0 \frac{N\ I}{L}\ \hat{a}_z$$
$$= \mu_0 N_\ell\ I\ \hat{a}_z$$