Solenoids

An important structure in electrical and computer engineering is the solenoid.

A solenoid is a tube of current. However, it is different from the hollow cylinder example, in that the current flows around the tube, rather than down the tube:

Aligning the center of the tube with the z-axis, we can express the current density as:

\[
J_s(\vec{r}) = \begin{cases} 
  0 & \rho < a \\
  J_s \hat{\phi} & \rho = a \\
  0 & \rho > a 
\end{cases}
\]

where \( a \) is the radius of the solenoid, and \( J_s \) is the surface current density in Amps/meter.
We can use **Ampere's Law** to find the magnetic flux density resulting from this structure. The result is:

\[
B(\vec{r}) = \begin{cases} 
\mu_0 J_s \hat{a}_z & \rho < a \\
0 & \rho > a
\end{cases}
\]

Note the direction of the magnetic flux density is in the direction \( \hat{a}_z \)--it points **down** the center of the solenoid.

Note also that the magnitude \( |B(\vec{r})| \) is **independent** of solenoid radius \( a \)!

**Q:** Yeah right! How are we supposed to get current to flow **around** this tube? I don’t see how this is even possible.

**A:** We can easily make a solenoid by forming a **wire spiral** around a cylinder.
The surface current density $\mathbf{J}_s$ of this solenoid is approximately equal to:

$$\mathbf{J}_s = \frac{N I}{L} = N_l I$$

where $N_l = N/L$ is the number of turns/unit length. Inserting this result into our expression for magnetic flux density, we find the magnetic flux density inside a solenoid:

$$\mathbf{B}(\mathbf{r}) = \mu_0 \frac{N I}{L} \hat{a}_z$$

$$= \mu_0 N_l I \hat{a}_z$$