**Stokes' Theorem**

Consider a vector field $\mathbf{B}(\mathbf{r})$ where:

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$$

Say we wish to integrate this vector field over an open surface $S$:

$$\int \int \int_{S} \mathbf{B}(\mathbf{r}) \cdot d\mathbf{s} = \int \int \int_{S} \nabla \times \mathbf{A}(\mathbf{r}) \cdot d\mathbf{s}$$

We can likewise evaluate this integral using **Stokes' Theorem**:

$$\int \int \nabla \times \mathbf{A}(\mathbf{r}) \cdot d\mathbf{s} = \oint \mathbf{A}(\mathbf{r}) \cdot d\ell$$

In this case, the contour $C$ is a closed contour that surrounds surface $S$. The direction of $C$ is defined by $d\mathbf{s}$ and the right-hand rule. In other words $C$ rotates counterclockwise around $d\mathbf{s}$. E.G.,
* Stokes' Theorem allows us to evaluate the surface integral of a curl as simply a contour integral!

* Stokes' Theorem states that the summation (i.e., integration) of the circulation at every point on a surface is simply the total “circulation” around the closed contour surrounding the surface.

In other words, if the vector field is rotating counterclockwise around some point in the volume, it must simultaneously be rotating clockwise around adjacent points within the volume—the net effect is therefore zero!

Thus, the only values that make any difference in the surface integral is the rotation of the vector field around points that lie on the surrounding contour (i.e., the very edge of the surface S). These vectors are likewise rotating in the opposite direction around adjacent points—but these points do not lie on the surface (thus, they are not included in the integration). The net effect is therefore non-zero!
Note that if \( S \) is a closed surface, then there is no contour \( C \) that exists! In other words:

\[
\oint \nabla \times \mathbf{A}(\mathbf{r}) \cdot d\mathbf{s} = \int_{0}^{s} \mathbf{A}(\mathbf{r}) \cdot d\mathbf{l} = 0
\]

Therefore, integrating the curl of any vector field over a closed surface always equals zero.