## <u>The Curl of</u> <u>Conservative Fields</u>

Recall that every **conservative** field can be written as the gradient of some scalar field:

$$\boldsymbol{\mathcal{C}}(\overline{\mathbf{r}}) = \nabla \boldsymbol{\mathcal{G}}(\overline{\mathbf{r}})$$

Consider now the curl of a conservative field:

$$\nabla \mathbf{x} \boldsymbol{\mathcal{C}}(\bar{\mathbf{r}}) = \nabla \mathbf{x} \nabla \mathbf{g}(\bar{\mathbf{r}})$$

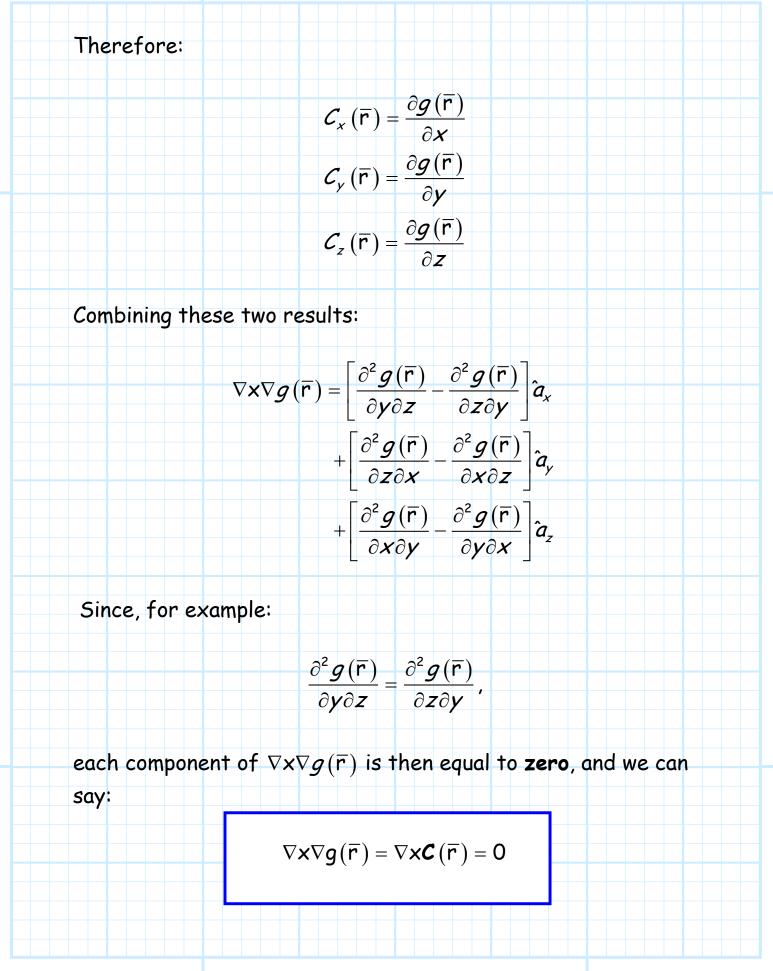
Recall that if  $C(\bar{r})$  is expressed using the **Cartesian** coordinate system, the curl of  $C(\bar{r})$  is:

$$\nabla \mathbf{x} \mathbf{C} \left( \overline{\mathbf{r}} \right) = \left[ \frac{\partial \mathcal{C}_z}{\partial \mathbf{y}} - \frac{\partial \mathcal{C}_y}{\partial \mathbf{z}} \right] \hat{\mathbf{a}}_x + \left[ \frac{\partial \mathcal{C}_x}{\partial \mathbf{z}} - \frac{\partial \mathcal{C}_z}{\partial \mathbf{x}} \right] \hat{\mathbf{a}}_y + \left[ \frac{\partial \mathcal{C}_y}{\partial \mathbf{x}} - \frac{\partial \mathcal{C}_x}{\partial \mathbf{y}} \right] \hat{\mathbf{a}}_z$$

Likewise, the gradient of  $g(\overline{r})$  is:

$$\nabla g(\bar{\mathbf{r}}) = \mathbf{C}(\bar{\mathbf{r}}) = \frac{\partial g(\bar{\mathbf{r}})}{\partial \mathbf{x}} \hat{a}_{\mathbf{x}} + \frac{\partial g(\bar{\mathbf{r}})}{\partial \mathbf{y}} \hat{a}_{\mathbf{y}} + \frac{\partial g(\bar{\mathbf{r}})}{\partial \mathbf{z}} \hat{a}_{\mathbf{z}}$$

Jim Stiles





The curl of every conservative field is equal to zero !

Likewise, we have determined that:

 $\nabla \mathbf{x} \nabla g(\overline{\mathbf{r}}) = \mathbf{0}$ 

for **all** scalar functions  $g(\overline{r})$ .

**Q:** Are there some **non**-conservative fields whose curl is also equal to zero?

A: NO! The curl of a conservative field, and only a conservative field, is equal to zero.

Thus, we have way to **test** whether some vector field  $A(\overline{r})$  is conservative: **evaluate its curl**!

1. If the result **equals zero**—the vector field **is** conservative.

 If the result is non-zero—the vector field is not conservative.

4/4

Let's again recap what we've learned about conservative fields:

- 1. The line integral of a conservative field is **path independent**.
- Every conservative field can be expressed as the gradient of some scalar field.
- The gradient of any and all scalar fields is a conservative field.
- The line integral of a conservative field around any closed contour is equal to zero.
- 5. The curl of every conservative field is equal to zero.
- 6. The curl of a vector field is zero only if it is conservative.