The Current I through Surface S

Given that we know volume current density $\mathbf{J}(\bar{r})$ throughout some volume, we can find the **total current** through **any** arbitrary **surface S** as:

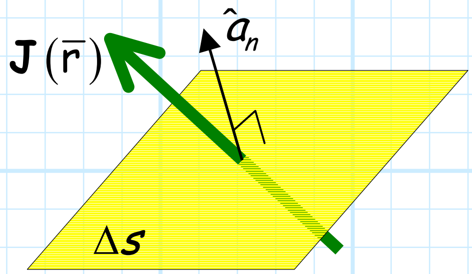
$$I = \iint_S \mathbf{J}(\bar{r}_s) \cdot \overline{ds} \qquad [Amps]$$

This integral is in the form of the **surface integral** we studied in Section 2-5.

Note the **integrand** has units of **current** (amps):

$$\mathbf{J}(\bar{r}_s) \cdot \overline{ds} = J_n(\bar{r}_s) \left|\overline{ds}\right| \qquad \left[\left(\frac{\text{Amps}}{\text{m}^2} \right) \left(\text{m}^2 \right) = \text{Amps} \right]$$

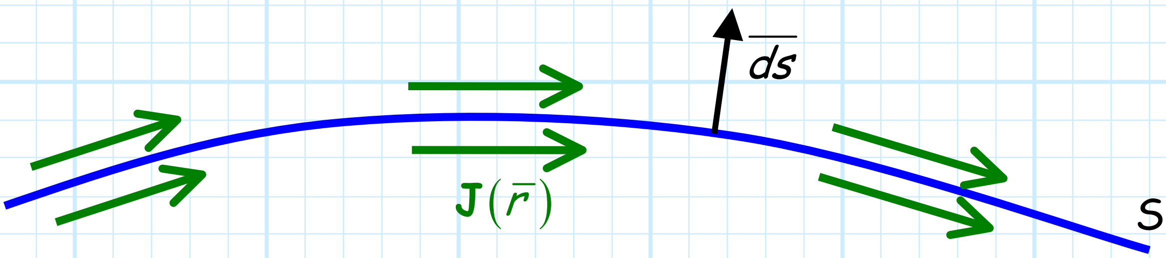
Physically, the value $\Delta I = \mathbf{J}(\bar{r}) \cdot \overline{ds}$ is the current flowing **through** the tiny differential surface Δs, located at point $\bar{r}$ on surface S.



* Therefore if we **add** up (i.e., integrate) the current flowing through **each** and every differential surface element Δs that makes up surface S, we determine the **total** current I flowing **through** surface S.

* Note the **sign** of current I is determined by the **direction** of differential surface vector $\overline{ds}$. For **example**, if I is **positive**, then the current is flowing **through** the surface in the direction of $\overline{ds}$.

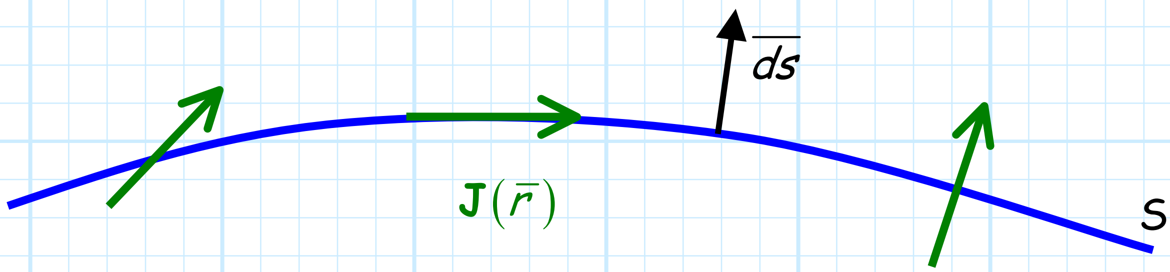
* So, consider the case where $\mathbf{J}(\bar{r})$ describes current that is flowing **tangential** to **every point** on surface S. In other words, the current density has no **normal** component on the surface S !



As a result, we find that $\mathbf{J}(\bar{r}) \cdot \overline{ds} = 0$ at **every** point on the surface, and therefore the surface **integral** results in $I = 0$.

This of course is **physically** the correct answer! Current is flowing **along** the surface, but none is flowing **through** it.

To get a **non-zero** amount of total current, the current density must have a **normal** component at **some** points on the surface.



For the case above, $I \neq 0$.

Q: *We know that if* $\mathbf{J}(\bar{\mathrm{r}}) \cdot \overline{ds} = 0$ *at all points on a surface, then the current flowing through the surface is zero (I=0).*

Is the ***converse*** *true? That is,* ***if*** *the total current through a surface is* ***zero****, does that mean that the current density is* ***tangential*** *to the surface at* ***all*** *points?*

A: