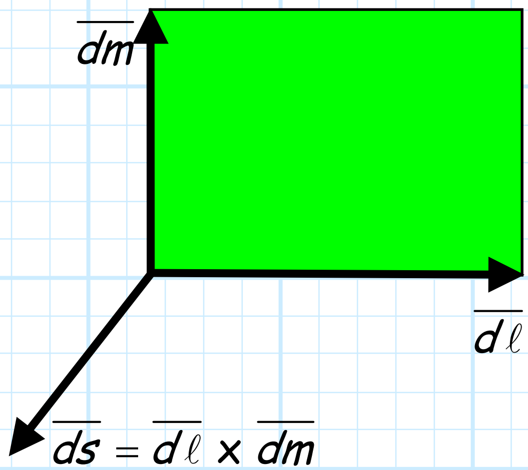
The Differential Surface Vector for Coordinate Systems

Given that $\overline{ds} = \overline{d\ell} \times \overline{dm}$, we can determine the differential surface vectors for each of the **three** coordinate systems.



Cartesian

$$\overline{ds_x} = \overline{dy} \times \overline{dz} = \hat{a}_x \, dy\, dz$$
$$\overline{ds_y} = \overline{dz} \times \overline{dx} = \hat{a}_y \, dx\, dz$$
$$\overline{ds_z} = \overline{dx} \times \overline{dy} = \hat{a}_z \, dx\, dy$$

We shall find that these differential surface vectors define a small patch of area on the surface of **flat plane**.

Cylindrical

$$\overline{ds_\rho} = \overline{d\phi} \times \overline{dz} = \hat{a}_\rho \ \rho \, d\phi \, dz$$
$$\overline{ds_\phi} = \overline{dz} \times \overline{dp} = \hat{a}_\phi \ d\rho \, dz$$
$$\overline{ds_z} = \overline{d\rho} \times \overline{d\phi} = \hat{a}_z \ \rho \, d\rho \, d\phi$$

We shall find that $\overline{ds_\rho}$ describes a small patch of area on the surface of a **cylinder**, $\overline{ds_\phi}$ describes a small patch of area on the surface of a **half-plane**, and $\overline{ds_z}$ again describes a small patch of area on the surface of a flat **plane**.

Spherical

$$\overline{ds_r} = \overline{d\theta} \times \overline{d\phi} = \hat{a}_r \ r^2 \sin\theta \ d\theta \, d\phi$$
$$\overline{ds_\theta} = \overline{d\phi} \times \overline{dr} = \hat{a}_\theta \ r \sin\theta \ dr \, d\phi$$
$$\overline{ds_\phi} = \overline{dr} \times \overline{d\theta} = \hat{a}_\phi \ r \ dr \, d\theta$$

We shall find that $\overline{ds_r}$ describes a small patch of area on the surface of a **sphere**, $\overline{ds_\theta}$ describes a small patch of area on the surface of a **cone**, and $\overline{ds_\phi}$ again describes a small patch of area on the surface of a **half plane**.