The Gradient Operator in Coordinate Systems

For the Cartesian coordinate system, the Gradient of a scalar field is expressed as:

\[ \nabla g(\vec{r}) = \frac{\partial g(\vec{r})}{\partial x} \hat{a}_x + \frac{\partial g(\vec{r})}{\partial y} \hat{a}_y + \frac{\partial g(\vec{r})}{\partial z} \hat{a}_z \]

Now let’s consider the gradient operator in the other coordinate systems.

Q: Pfft! This is easy! The gradient operator in the spherical coordinate system is:

\[ \nabla g(\vec{r}) = \frac{\partial g(\vec{r})}{\partial r} \hat{a}_r + \frac{\partial g(\vec{r})}{\partial \theta} \hat{a}_\theta + \frac{\partial g(\vec{r})}{\partial \phi} \hat{a}_\phi \]

Right ??

A: NO!! The above equation is not correct!

Instead, we find that for spherical coordinates, the gradient is expressed as:
\[ \nabla g(\vec{r}) = \frac{\partial g(\vec{r})}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial g(\vec{r})}{\partial \theta} \hat{a}_\theta + \frac{1}{r \sin \theta} \frac{\partial g(\vec{r})}{\partial \phi} \hat{a}_\phi \]

And for the **cylindrical** coordinate system we likewise get:

\[ \nabla g(\vec{r}) = \frac{\partial g(\vec{r})}{\partial \rho} \hat{a}_\rho + \frac{1}{\rho} \frac{\partial g(\vec{r})}{\partial \phi} \hat{a}_\phi + \frac{\partial g(\vec{r})}{\partial z} \hat{a}_z \]