The Gradient Operator in Other Coordinate Systems

Q: *Pfft! This is easy! The gradient operator in the spherical coordinate system is:*

$$\nabla g(\bar{r}) = \frac{\partial g(\bar{r})}{\partial r}\hat{a}_r + \frac{\partial g(\bar{r})}{\partial \theta}\hat{a}_\theta + \frac{\partial g(\bar{r})}{\partial \phi}\hat{a}_\phi$$

Right ??

A: **NO!!** The above equation is **not** correct!

Instead, we find that for **spherical** coordinates, the gradient is expressed as:

$$\boxed{\nabla g(\bar{r}) = \frac{\partial g(\bar{r})}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial g(\bar{r})}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial g(\bar{r})}{\partial \phi}\hat{a}_\phi}$$

And for the **cylindrical** coordinate system we get:

$$\boxed{\nabla g(\bar{r}) = \frac{\partial g(\bar{r})}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial g(\bar{r})}{\partial \phi}\hat{a}_\phi + \frac{\partial g(\bar{r})}{\partial z}\hat{a}_z}$$