The Integral Form of Magnetostatics

Say we evaluate the **surface integral** of the point form of **Ampere's Law** over some arbitrary surface *S*.

$$\iint_S \nabla \times \mathbf{B}(\bar{\mathbf{r}}) \cdot \overline{ds} = \mu_0 \iint_S \mathbf{J}(\bar{\mathbf{r}}) \cdot \overline{ds}$$

Using **Stoke's Theorem**, we can write the **left** side of this equation as:

$$\iint_S \nabla \times \mathbf{B}(\bar{\mathbf{r}}) \cdot \overline{ds} = \oint_C \mathbf{B}(\bar{\mathbf{r}}) \cdot \overline{d\ell}$$

We also recognize that the **right** side of the equation is:

$$\mu_0 \iint_S \mathbf{J}(\bar{\mathbf{r}}) \cdot \overline{ds} = \mu_0 \mathrm{I}$$

where I is the **current** flowing through surface *S*.

Therefore, combing these two results, we find the integral form of **Ampere's Law** (Note the **direction** of I is defined by the **right-hand rule**):

$$\oint_C \mathbf{B}(\bar{\mathbf{r}}) \cdot \overline{d\ell} = \mu_0 I$$

Amperes law states that the **line integral** of $\mathbf{B}(\bar{r})$ around a **closed contour** C is proportional to the **total current** I flowing **through** this closed contour ($\mathbf{B}(\bar{r})$ is **not** conservative!).

Likewise, we can take a **volume integral** over both sides of the magnetostatic equation $\nabla \cdot \mathbf{B}(\bar{r}) = 0$:

$$\iiint_V \nabla \cdot \mathbf{B}(\bar{r})\, dv = 0$$

But wait! The left side can be rewritten using the **Divergence Theorem**:

$$\iiint_V \nabla \cdot \mathbf{B}(\bar{r})\, dv = \oiint_S \mathbf{B}(\bar{r}) \cdot \overline{ds}$$

where S is the **closed surface** that **surrounds** volume V. Therefore, we can write the integral form of $\nabla \cdot \mathbf{B}(\bar{r}) = 0$ as:

$$\oiint_S \mathbf{B}(\bar{r}) \cdot \overline{ds} = 0$$

Summarizing, the **integral form** of the magnetostatic equations are:

$$\oiint_S \mathbf{B}(\bar{r}) \cdot \overline{ds} = 0 \qquad \oint_C \mathbf{B}(\bar{r}) \cdot \overline{d\ell} = \mu_0 I$$