The Magnetization Vector

Recall that we defined the Polarization vector of a dielectric material as the electric dipole density, i.e.:

\[ p(\vec{r}) \equiv \lim_{\Delta \nu \to 0} \frac{1}{\Delta \nu} \sum p_n \left[ \frac{\text{electric dipole moment}}{\text{unit volume}} \right] \]

Similarly, we can define a Magnetization vector \( M(\vec{r}) \) of a material to be the density of magnetic dipole moments at location \( \vec{r} \):

\[ M(\vec{r}) \equiv \lim_{\Delta \nu \to 0} \frac{1}{\Delta \nu} \sum m_n \left[ \frac{\text{magnetic dipole moment}}{\text{unit volume}} \right] = \frac{A}{m} \]

Note if the dipole moments of atoms/molecules within a material are completely random, the Magnetization vector will be zero (i.e., \( M(\vec{r}) = 0 \)).

However, if the dipoles are aligned, the Magnetization vector will be non-zero (i.e., \( M(\vec{r}) \neq 0 \)).
Recall a magnetic dipole will create a **magnetic vector potential** equal to:

\[ A(\mathbf{r}) = \frac{\mu_0 \mathbf{m} \times (\mathbf{r} - \mathbf{r}')}{4\pi |\mathbf{r} - \mathbf{r}'|^3} \]

Since the magnetic dipole moment of some small (i.e., differential) volume \( dv \) of the material is:

\[ \mathbf{m} = \mathbf{M}(\mathbf{r})\, dv \]

we find that the magnetic vector potential created by a **volume** \( V \) of material with magnetization vector \( \mathbf{M}(\mathbf{r}) \) is:

\[ A(\mathbf{r}) = \iiint_V \frac{\mu_0 \mathbf{M}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')} {4\pi |\mathbf{r} - \mathbf{r}'|^3} \, dv' \]

**Q:** This is freaking me out!! I thought that **currents** \( \mathbf{J}(\mathbf{r}) \) were responsible for creating magnetic vector potential. In fact, I could have sworn that:

\[ A(\mathbf{r}) = \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \, dv' \]

**A:** Relax, **both** expressions are correct!
Recall that we could attribute the electric field created by Polarization Vector $P(\vec{r})$ to polarization (i.e., bound) charges $\rho_{vp}(\vec{r})$ and $\rho_{sp}(\vec{r})$, i.e.,:

$$\rho_{vp}(\vec{r}) = -\nabla \cdot P(\vec{r}) \quad \rho_{sp}(\vec{r}) = P(\vec{r}) \cdot \hat{a}_n$$

Similarly, we can attribute the magnetic vector potential (and therefore the magnetic flux density) created by Magnetization Vector $M(\vec{r})$ to Magnetization Currents $J_m(\vec{r})$ and $J_{sm}(\vec{r})$. 