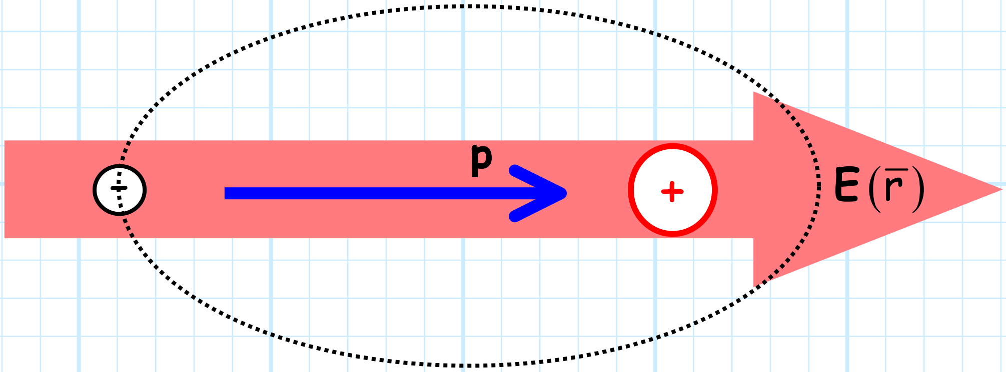
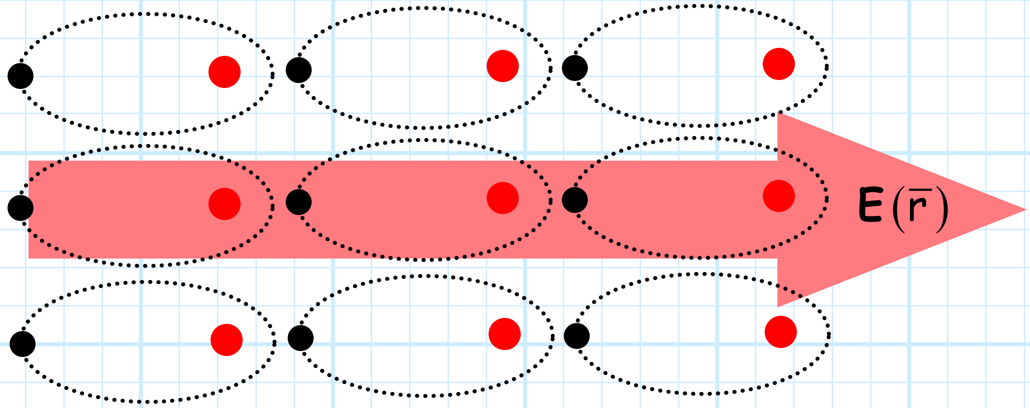
The Polarization Vector

Recall that in **dielectric materials** (i.e., insulators), the charges are **bound**.



As a result, atoms/molecules form **electric dipoles** when an electric field is present!

Note that even for some **small** volume Δv, there are **many** atoms/molecules present; therefore there will be **many** electric dipoles.



We will therefore define an **average** dipole moment, per unit volume, called the **Polarization Vector** $\mathbf{P}(\bar{r})$.

$$\mathbf{P}(\bar{r}) \doteq \frac{\sum \mathbf{p}_n}{\Delta v} \qquad \left[\frac{\text{dipole moment}}{\text{unit volume}} = \frac{C}{m^2}\right]$$

where $\mathbf{p}_n$ is one of N dipole moments in volume Δv, centered at position $\bar{r}$. Note the polarization vector is a **vector field**. As a result, the direction and magnitude of the Polarization vector can change as function of position (i.e., a function of $\bar{r}$).

Q: *How are vector fields* $\mathbf{P}(\bar{r})$ *and* $\mathbf{E}(\bar{r})$ *related??*

A: Recall that the direction of each dipole moment is the same as the polarizing electric field. Thus $\mathbf{P}(\bar{r})$ and $\mathbf{E}(\bar{r})$ have the same direction. There magnitudes are related by a unitless scalar value $\chi_e(\bar{r})$, called **electric susceptibility:**

$$\mathbf{P}(\bar{r}) = \varepsilon_0\, \chi_e(\bar{r})\, \mathbf{E}(\bar{r})$$

Electric susceptibility is a **material parameter** indicating the "stretchability" of the dipoles.

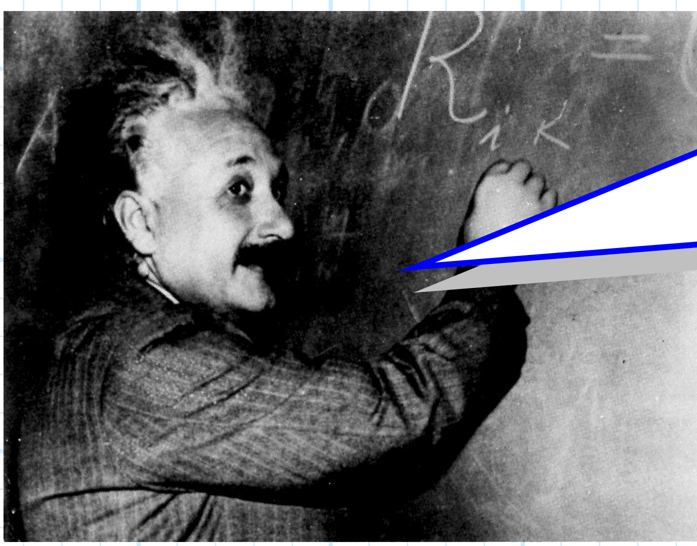
Q: *Can we determine the **fields** created by a polarized material?*

A: Recall the **electric potential field** created by **one** dipole is:

$$V(\bar{r}) = \frac{\mathbf{p} \cdot (\bar{r}-\bar{r}')}{4\pi\varepsilon_0 \left|\bar{r}-\bar{r}'\right|^3}$$

Therefore, using $d\,\mathbf{p} = \mathbf{P}(\bar{r})\,dv$, the electric potential field created by a **distribution of** dipoles (i.e., $\mathbf{P}(\bar{r})$) across some volume V is (see fig. 5.9):

$$V(\bar{r}) = \iiint_V \frac{\mathbf{P}(\bar{r}') \cdot (\bar{r}-\bar{r}')}{4\pi\varepsilon_0 \left|\bar{r}-\bar{r}'\right|^3}\, dv'$$



Q: *But I thought **scalar** charge distributions $\rho_v(\bar{r})$ and $\rho_s(\bar{r})$ created the electric potential field $V(\bar{r})$. Now you are saying that electric fields are created by the **vector** field $\mathbf{P}(\bar{r})$!?!*

A: As we will soon see, the polarization **vector** $\mathbf{P}(\bar{r})$ creates equivalent charge **distributions**—we will get the correct answer for $V(\bar{r})$ from **either** source!