The Triple Product

The triple product is not a “new” operation, as it is simply a combination of the dot and cross products.

The triple product of vectors $A$, $B$, and $C$ is denoted as:

$$A \cdot B \times C$$

Q: Yikes! Does this mean:

$$(A \cdot B) \times C$$

or

$$A \cdot (B \times C)$$

A: The answer is easy! Only one of these two interpretations makes sense:
In the first case, \( A \cdot B \) is a scalar value, say \( d = A \cdot B \). Therefore we can write the first equation as:

\[
(A \cdot B) \times C = d \times C
\]

But, this makes no sense! The cross product of a scalar and a vector has no meaning.

In the second interpretation, the cross product \( B \times C \) is a vector, say \( B \times C = D \). Therefore, we can write the second equation as:

\[
A \cdot (B \times C) = A \cdot D
\]

Not only does this make sense, but the result is a scalar!

The triple product \( A \cdot B \times C \) results in a scalar value.

**The Cyclic Property**

It can be shown that the triple product of vectors \( A, B, \) and \( C \) can be evaluated in three ways:

\[
A \cdot B \times C = C \cdot A \times B = B \cdot C \times A
\]
But, it is important to note that this does not mean that order is unimportant! For example:

\[ A \cdot B \times C \neq A \cdot C \times B \]
\[ A \cdot B \times C \neq C \cdot B \times A \]
\[ A \cdot B \times C \neq B \cdot A \times C \]

The cyclical rule means that the triple product is invariant to shifts (i.e., rotations) in the order of the vectors.

There are six ways to arrange three vectors. Therefore, we can group the triple product of three vectors into two groups of three products:

\[ A \cdot B \times C = C \cdot A \times B = B \cdot C \times A \]
\[ B \cdot A \times C = C \cdot B \times A = A \cdot C \times B \]

but, \[ A \cdot B \times C = -(B \cdot A \times C) \]