The Volume V

As we might expect from out knowledge about how to specify a **point** P (3 equalities), a **contour** C (2 equalities and 1 inequality), and a **surface** S (1 equality and 2 inequalities), a **volume** V is defined by **3 inequalities**.

Cartesian

The inequalities:

$$c_{x1} \le x \le c_{x2} \qquad c_{y1} \le y \le c_{y2} \qquad c_{z1} \le z \le c_{z2}$$

define a **rectangular volume**, whose sides are parallel to the x-y, y-z, and x-z planes.

The differential volume dv used for constructing this Cartesian volume is:

$$dv = dx\ dy\ dz$$

Cylindrical

The inequalities:

$$c_{\rho 1} \le \rho \le c_{\rho 2} \qquad c_{\phi 1} \le \phi \le c_{\phi 2} \qquad c_{z1} \le z \le c_{z2}$$

defines a **cylinder**, or some **subsection** thereof (e.g. a **tube**!).

The differential volume dv is used for constructing this cylindrical volume is:

$$dv = \rho \; d\rho \; d\phi \; dz$$

Spherical

The equations:

$$c_{r1} \le r \le c_{r2} \qquad c_{\theta 1} \le \theta \le c_{\theta 2} \qquad c_{\phi 1} \le \phi \le c_{\phi 2}$$

defines a **sphere**, or some subsection thereof (e.g., an "**orange slice**" !).

The differential volume dv used for constructing this spherical volume is:

$$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

* Note that the three inequalities become **the limits of integration** for a volume integral. For example, integrating over a spherical volume would result in an integral of the form:

$$\iiint_V g(\bar{r}) \, dv = \int_{c_{\phi 1}}^{c_{\phi 2}} \int_{c_{\theta 1}}^{c_{\theta 2}} \int_{c_{r1}}^{c_{r2}} g(\bar{r}) \, r^2 \sin\theta \, dr \, d\theta \, d\phi$$

For this example, **if** the scalar field $g(\bar{r})$ is **not** expressed in terms of **spherical** coordinates, it must first be **transformed** before the integral can be explicitly **evaluated**.

* Note also that we can construct **complex volumes** by combining the simple volumes discussed here.

$$V = V_1 + V_2 + V_3 + V_4$$