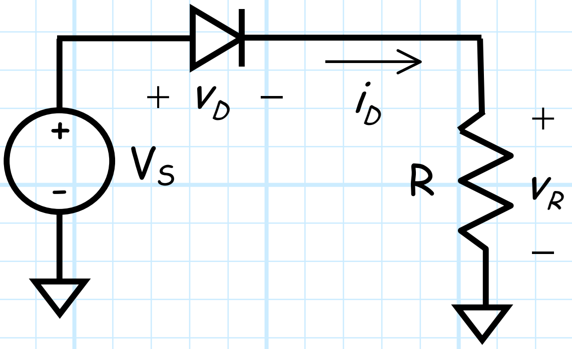
Transcendental Solutions of Junction Diode Circuits

In a previous example, we were able to use the junction diode equation to **algebraically** analyze a circuit and find **numeric** solutions for all circuit currents and voltages.

However, we will find that this type of circuit analysis is, in general, often **impossible** to achieve using the junction diode equation!

Q: *Impossible !?! **I** must intercede, and point out that **you** are clearly wrong. If **I** have an explicit mathematical description of each device in a circuit (which **I** do for a junction diode), **I** can use KVL and KCL to analyze **any** circuit.*

A: Although we can always determine a numerical solution, it is often impossible to find this solution **algebraically**. Consider this **simple** junction diode circuit:



From KVL:

Likewise, from the **junction** diode equation:

$$i_D = I_s \left(e^{v_D / nV_T} - 1 \right)$$

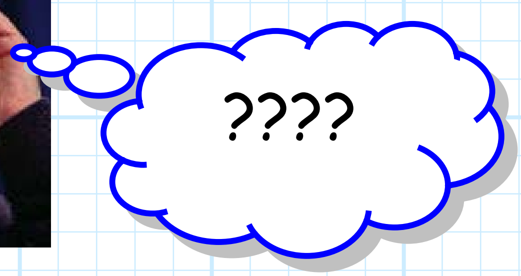
Equating these two, we have a **single** equation with a **single** unknown (v_D):

$$\frac{V_s - v_D}{R} = I_s \left(e^{v_D / nV_T} - 1 \right)$$

Q: *Precisely! Just as* ***I*** *said! You have 1 equation with 1 unknown. Go solve this equation for v_D, and then you can determine all other unknown voltages and currents (i.e., i_D and v_R). Gosh, is there any problem that* ***I*** *cannot solve?*

A: But that's the problem! What **is** the algebraic solution of v_D for the equation:

$$\frac{V_s - v_D}{R} = I_s \left(e^{v_D / nV_T} - 1 \right)$$



The above equation is known as a **transcendental equation**. It is an algebraic expression for which there is **no** algebraic solution!

Examples of transcendental equations include:

$$x = cos[x], \quad y^2 = ln[y], \quad \text{or} \quad 4 - x = 2^x$$

Q: *But, we could build that simple junction diode circuit in the lab. Therefore v_D, i_D and v_R must have **some** numeric value, right !?!*

A: Absolutely! For every value of source voltage V_s, resistance R, and junction diode parameters n and I_s, there is a specific numerical solution for v_D, i_D and v_R. However, we **cannot** find this **numerical** solution with **algebraic** methods!

Q: *Well then how the heck do we find solution??*

A: We use what is know as **numerical methods**, often implementing some **iterative** approach, typically with the help of a **computer** (see example 3.4 on pp. 154-155).

This generally involves **more work** than we wish to do when analyzing junction diode circuits!

Q: *So just how **do** we analyze junction diode circuits??*

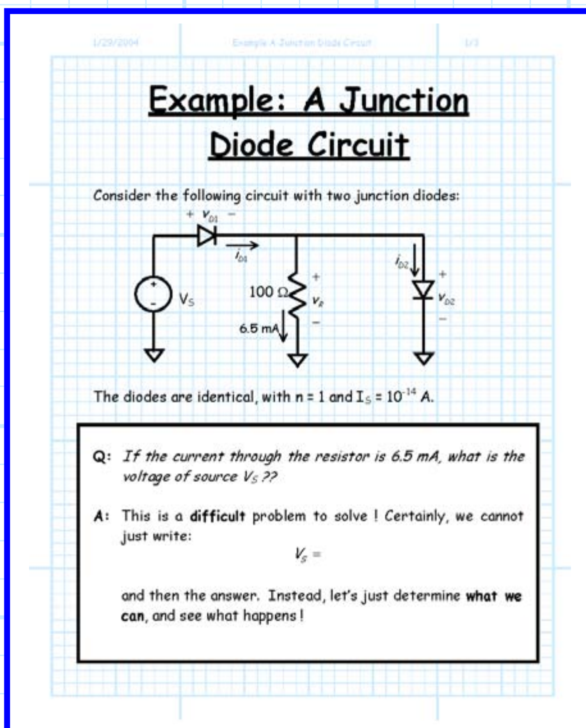
A: We replace the junction diodes with **circuit models** that **approximate** junction diode behavior!

Q: *Oh you're tricky, but you are still clearly **wrong** (thus **I** am clearly right). Recall in an **earlier example** we analyzed a junction diode circuit, but we did **not** use "approximate models" nor "numerical methods" to find the answer!*

A: This is absolutely correct; we did **not** use approximate models or numerical methods to solve that problem. However, if you look back at that example, you will find that the problem was a bit **contrived**.

* Recall that effectively, we were **given** the voltage across one diode as part of the problem statement. We were then asked to find the **source voltage** V_S.

* This was a bit of an **academic** problem, as in the "real world" it is **unlikely** that we would somehow know the voltage across the diode without knowing the value of the voltage source that produced it!



Example: A Junction Diode Circuit

Consider the following circuit with two junction diodes:

The diodes are identical, with $n = 1$ and $I_S = 10^{-14}$ A.

Q: *If the current through the resistor is 6.5 mA, what is the voltage of source V_S ??*

A: This is a **difficult** problem to solve ! Certainly, we cannot just write:

$V_S =$

and then the answer. Instead, let's just determine **what we can**, and see what happens !

* Thus, problems like this previous example are sometimes used by professors to create junction diode circuit problems that are solvable, **without** encountering a dreaded **transcendental equation**!

* In the real world, we typically know **neither** the diode voltage **nor** the diode current directly—transcendental equations are most often the **sad** result!

* **Instead** of applying numerical techniques, we will find it much faster (albeit slightly less accurate) to apply **approximate circuit models**.