Transcendental Solutions of Junction Diode Circuits

In a previous example, we were able to use the junction diode equation to algebraically analyze a circuit and find numeric solutions for all circuit currents and voltages.

However, we will find that this type of circuit analysis is, in general, often impossible to achieve using the junction diode equation!

Q: Impossible !?! I must intercede, and point out that you are clearly wrong. If I have an explicit mathematical description of each device in a circuit (which I do for a junction diode), I can use KVL and KCL to analyze any circuit.

A: Although we can always determine a numerical solution, it is often impossible to find this solution algebraically. Consider this simple junction diode circuit:

From KVL:
Likewise, from the **junction** diode equation:

$$i_D = I_s \left( e^{v_D/nV_T} - 1 \right)$$

Equating these two, we have a **single** equation with a **single** unknown ($v_D$):

$$\frac{V_s - v_D}{R} = I_s \left( e^{v_D/nV_T} - 1 \right)$$

**Q:** Precisely! Just as I said! You have 1 equation with 1 unknown. Go solve this equation for $v_D$, and then you can determine all other unknown voltages and currents (i.e., $i_D$ and $v_R$). Gosh, is there any problem that I cannot solve?

**A:** But that’s the problem! What is the algebraic solution of $v_D$ for the equation:

$$\frac{V_s - v_D}{R} = I_s \left( e^{v_D/nV_T} - 1 \right)$$
The above equation is known as a **transcendental equation**. It is an algebraic expression for which there is **no** algebraic solution!

**Examples** of transcendental equations include:

\[ x = \cos(x), \quad y^2 = \ln(y), \quad \text{or} \quad 4 - x = 2^x \]

**Q:** But, we could build that simple junction diode circuit in the lab. Therefore \( v_D, \, i_D \) and \( v_R \) must have **some** numeric value, right!?!?

**A:** Absolutely! For every value of source voltage \( V_s \), resistance \( R \), and junction diode parameters \( n \) and \( I_s \), there is a specific numerical solution for \( v_D, \, i_D \) and \( v_R \). However, we **cannot** find this **numerical** solution with **algebraic** methods!

**Q:** Well then how the heck do we find solution??

**A:** We use what is know as **numerical methods**, often implementing some **iterative** approach, typically with the help of a **computer** (see example 3.4 on pp. 154-155).

This generally involves **more work** than we wish to do when analyzing junction diode circuits!

**Q:** So just how do we analyze junction diode circuits??

**A:** We replace the junction diodes with **circuit models** that **approximate** junction diode behavior!
A: This is absolutely correct; we did not use approximate models or numerical methods to solve that problem. However, if you look back at that example, you will find that the problem was a bit contrived.

* Recall that effectively, we were given the voltage across one diode as part of the problem statement. We were then asked to find the source voltage $V_s$.

* This was a bit of an academic problem, as in the "real world" it is unlikely that we would somehow know the voltage across the diode without knowing the value of the voltage source that produced it!
* Thus, problems like this previous example are sometimes used by professors to create junction diode circuit problems that are solvable, without encountering a dreaded transcendental equation!

* In the real world, we typically know neither the diode voltage nor the diode current directly—transcendental equations are most often the sad result!

* Instead of applying numerical techniques, we will find it much faster (albeit slightly less accurate) to apply approximate circuit models.

I wish I had a nickel for every time my software has crashed—Oh wait, I do!