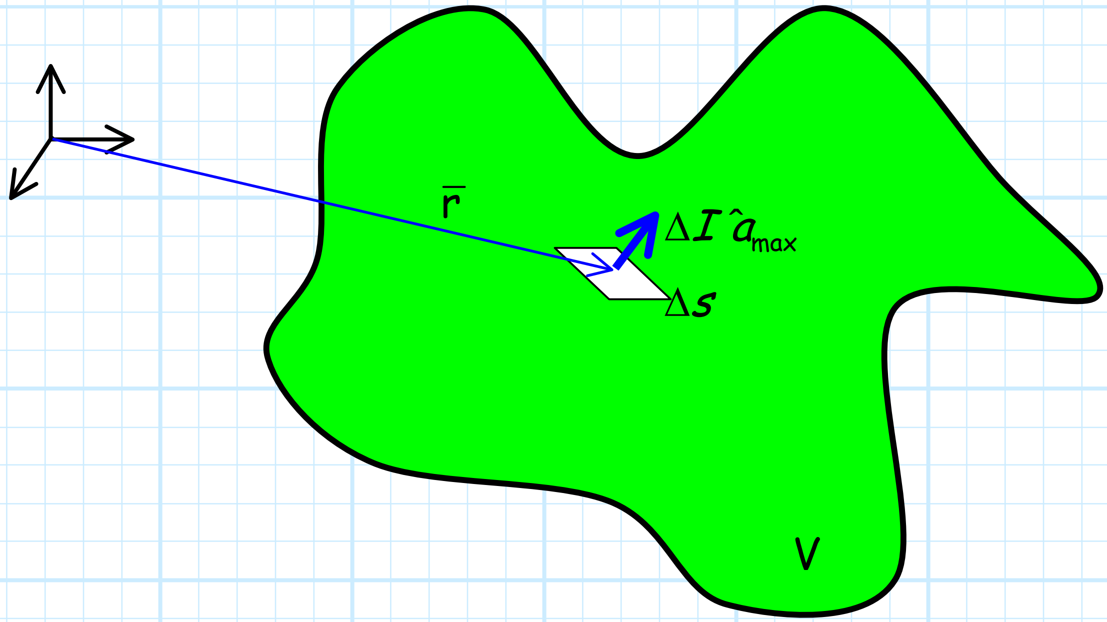
Volume Current Density

Say at a given point $\bar{r}$ located in a volume V, charge is moving in **direction** $\hat{a}_{max}$.



Now, consider a **small surface** Δs that is centered at the point denoted by $\bar{r}$, and oriented such that it is orthogonal to unit vector $\hat{a}_{max}$. Since charge is moving across this small surface at some rate (coulombs/sec), we can define a **current** $\Delta I = \Delta Q / \Delta t$ that represents the current flowing through Δs.

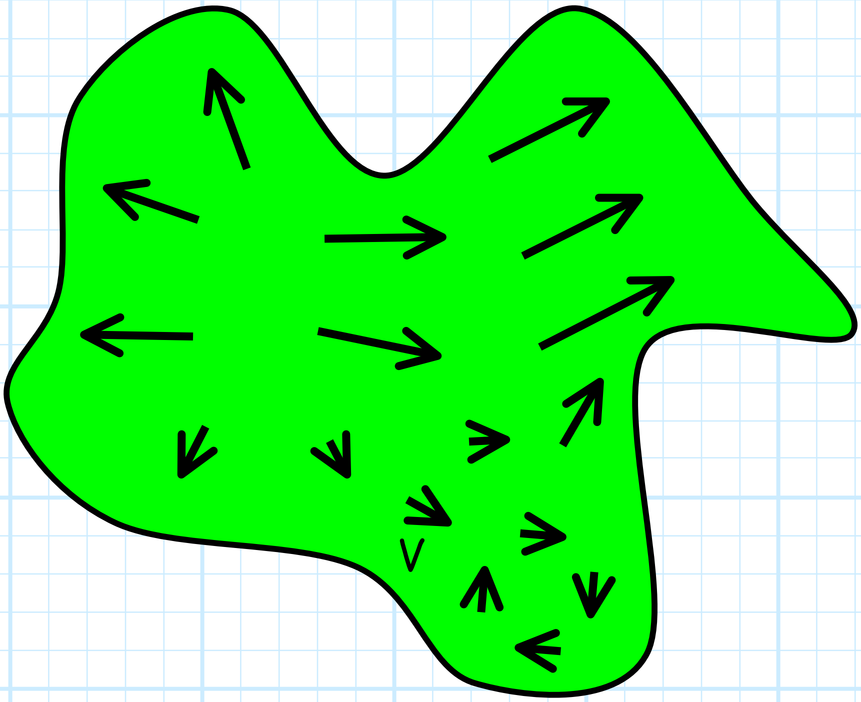
Note **vector** $\Delta I \, \hat{a}_{max}$ therefore represents both the **magnitude** (ΔI) and **direction** $\hat{a}_{max}$ of the current flowing through surface area Δs at point $\bar{r}$.

From this, we can define a **volume current density** $\mathbf{J}(\bar{r})$ at each and every point $\bar{r}$ in volume V by **normalizing** $\Delta I \, \hat{a}_{max}$ by dividing by the surface area Δs:

$$\mathbf{J}(\bar{r}) = \lim_{\Delta s \to 0} \frac{\Delta I \, \hat{a}_{max}}{\Delta s} \qquad \left[\frac{\text{Amps}}{\text{m}^2}\right]$$

The result is a **vector** field !

For example, current density $\mathbf{J}(\bar{r})$ might look like:



NOTE: The **unit** of **volume** current density is **current/area**; for example, A/m^2.