Chapter 2 – *Vector Analysis*

2-2 Physical Quantities and Units (*pp.* 7-11)

A. Types of physical quantities

HO: Examples of Physical Quantities

B. Vector Representation

HO: Vector Representations

C. The Directed Distance

HO: The Directed Distance

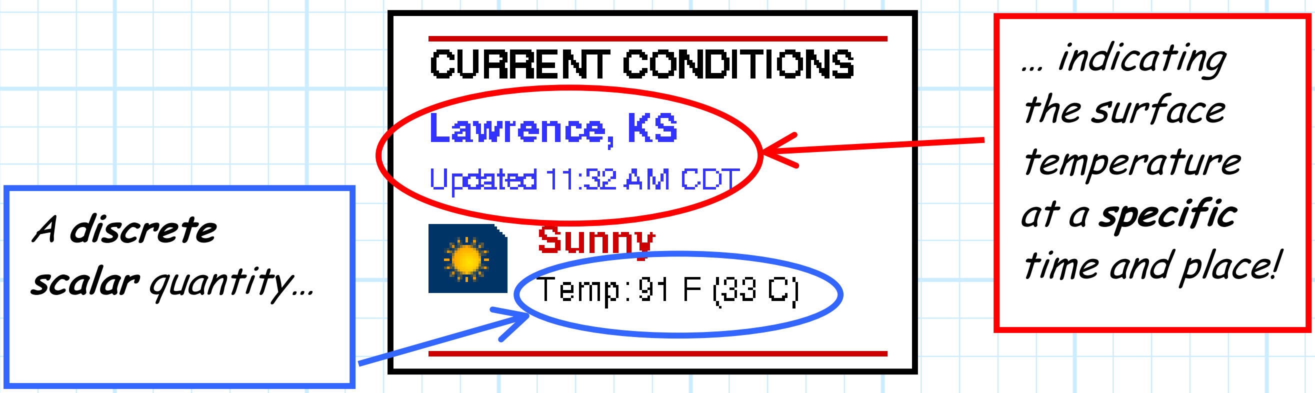
Examples of Physical Quantities

A. **Discrete Scalar Quantities** can be described with a single **numeric** value. Examples include:

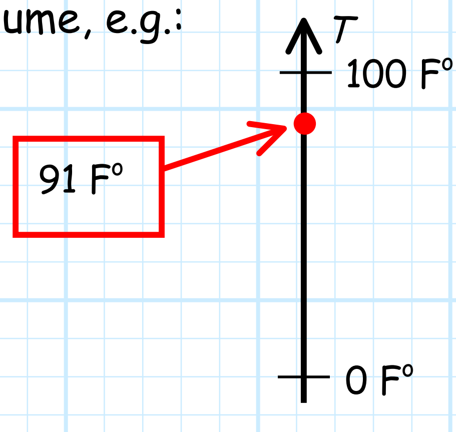
1) My **height** (~ 6 ft.).

2) The weight of your text book (~ 1.0 lbs.)

3) The **surface temperature** of a **specific** location at a **specific** time.



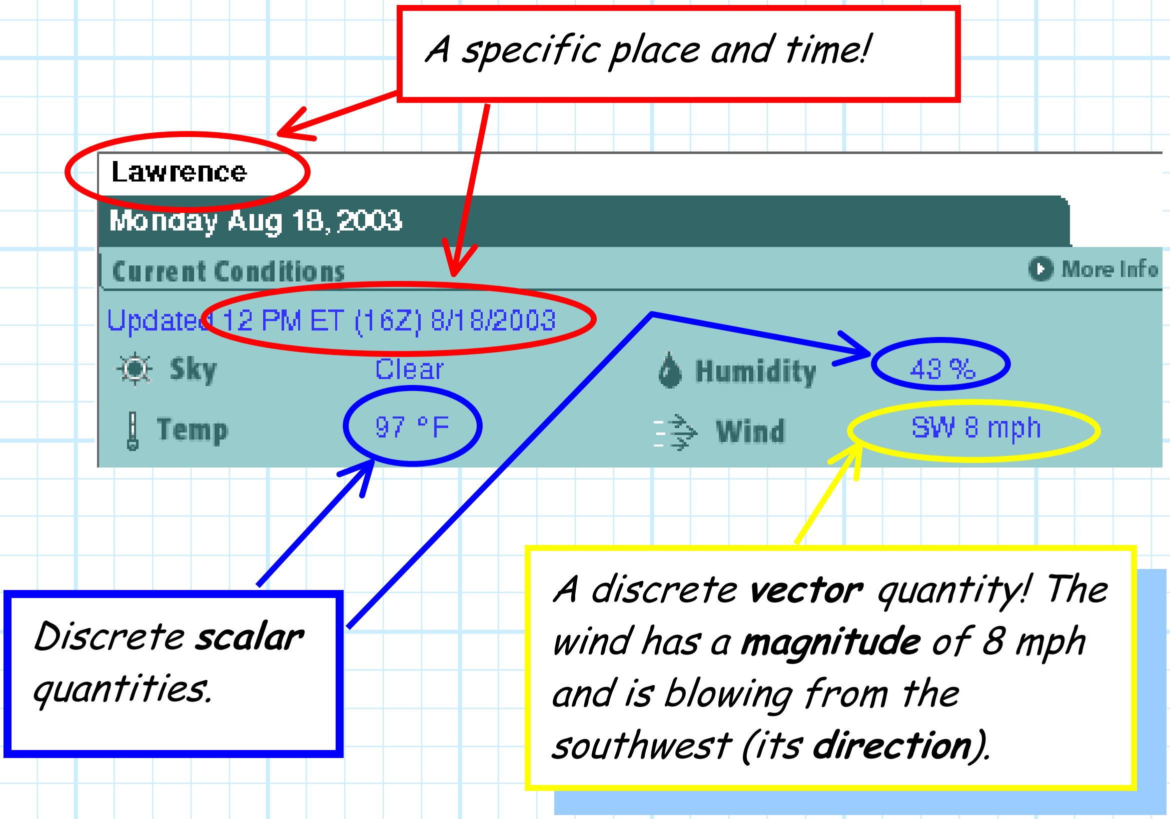
Graphically, a discrete scalar quantity can be indicated as a **point** on a line, surface or volume, e.g.:



B. <u>**Discrete Vector Quantities**</u> must be described with both a **magnitude** and a **direction**. Examples include:

1) The **force** I am exerting on the floor (180 lbs. +++, in a **direction** toward the center of the earth).

2) The wind **velocity** of a **specific** location at a **specific** time.



We will find that a discrete vector can be **graphically** represented as an **arrow**:

wherein the length of the arrow is proportional to the **magnitude** and the orientation indicates **direction**.

C. **<u>Scalar Fields</u>** are quantities that must be described as one function of (typically) **space** and/or **time**. For example:

1) My weight as a **function of time**.

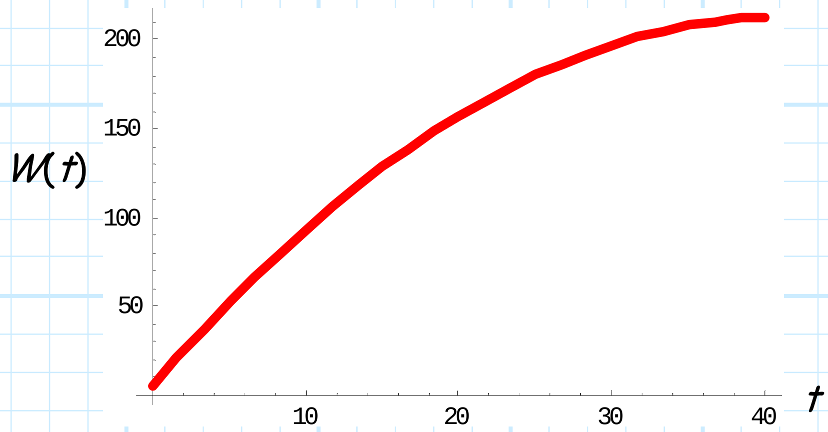
Note that we **cannot** specify this as a **single numerical value**, as my weight has **changed** significantly over the course of my life!

Instead, we must use a **function** of time to describe my weight:

$$W(t) = 5.2 + 10t - 0.12\,t^2 \quad \text{lbs.}$$

where t is my age in years.

We can likewise **graphically** represent this scalar field by plotting the function $W(t)$:



Note that we **can** use this scalar field to determine **discrete** scalar values! For example, say we wish to determine my weight **at birth**. This is a discrete scalar value—it can be expressed numerically:

$$W(t=0) = 5.2 + 10(0) - 0.12\,(0)^2$$
$$= 5.2 \text{ lbs.}$$

Why I'm ***always*** *hungry!*

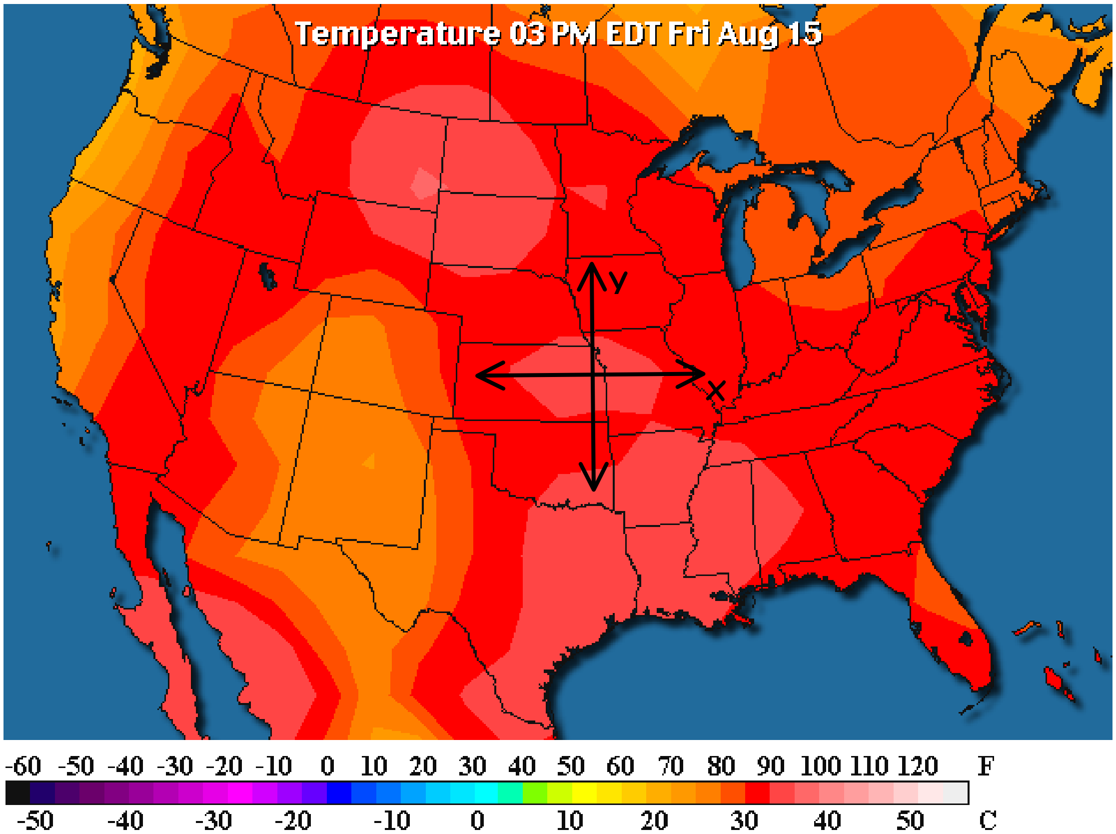
Note this **discrete** scalar value indicates my weight at a **specific** time (t=0). We likewise could determine my **current** weight (a **discrete** scalar value) by evaluating the scalar field $W(t)$ at t=41 (Doh!).

2) The current surface temperature across the **entire** the U.S.

Again, this quantity **cannot** be specified with a single numeric value. Instead, we must specify temperature as a **function** of position (location) on the surface of the U.S. , e.g.:

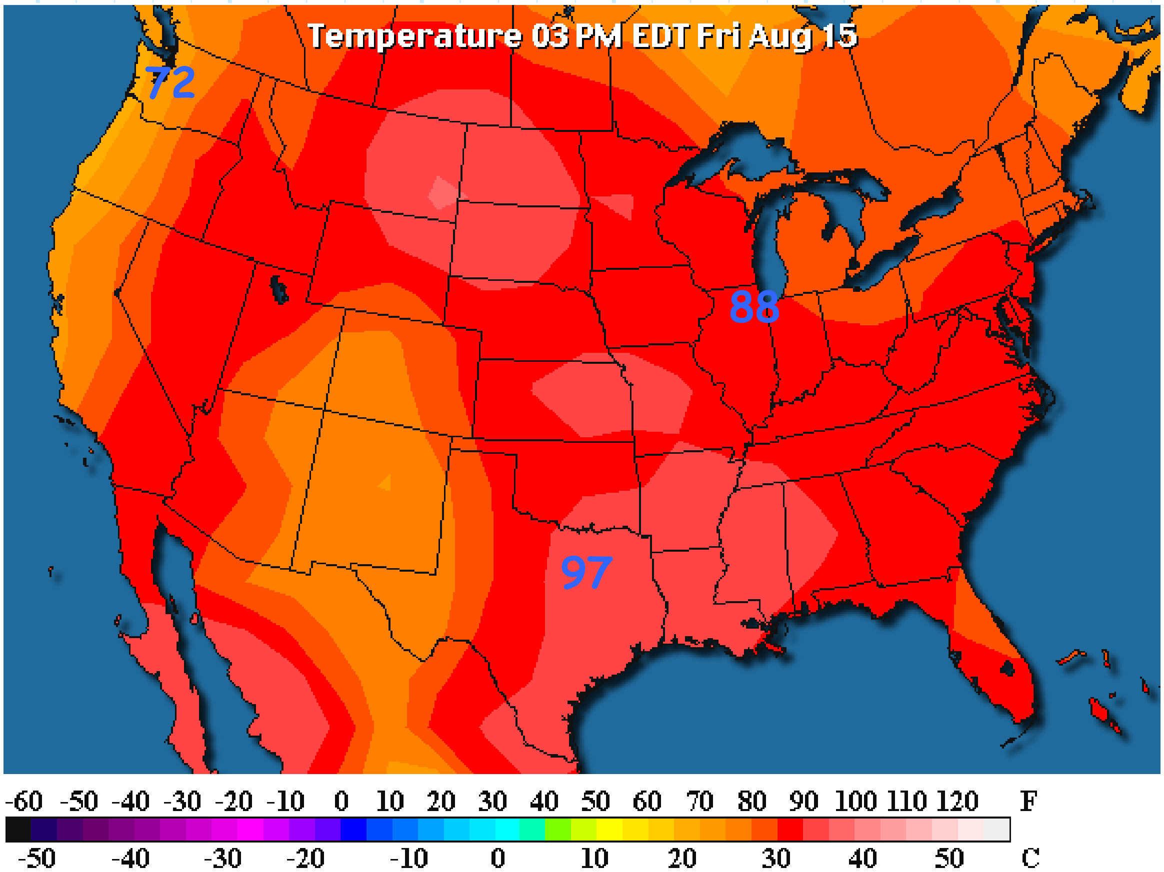
$$T(x,y) = 80.0 + 0.1x - 0.2y + 0.003xy + ...$$

where x and y are Cartesian coordinates that specify a **point** in the U.S. Often, we find it useful to **plot** this function:



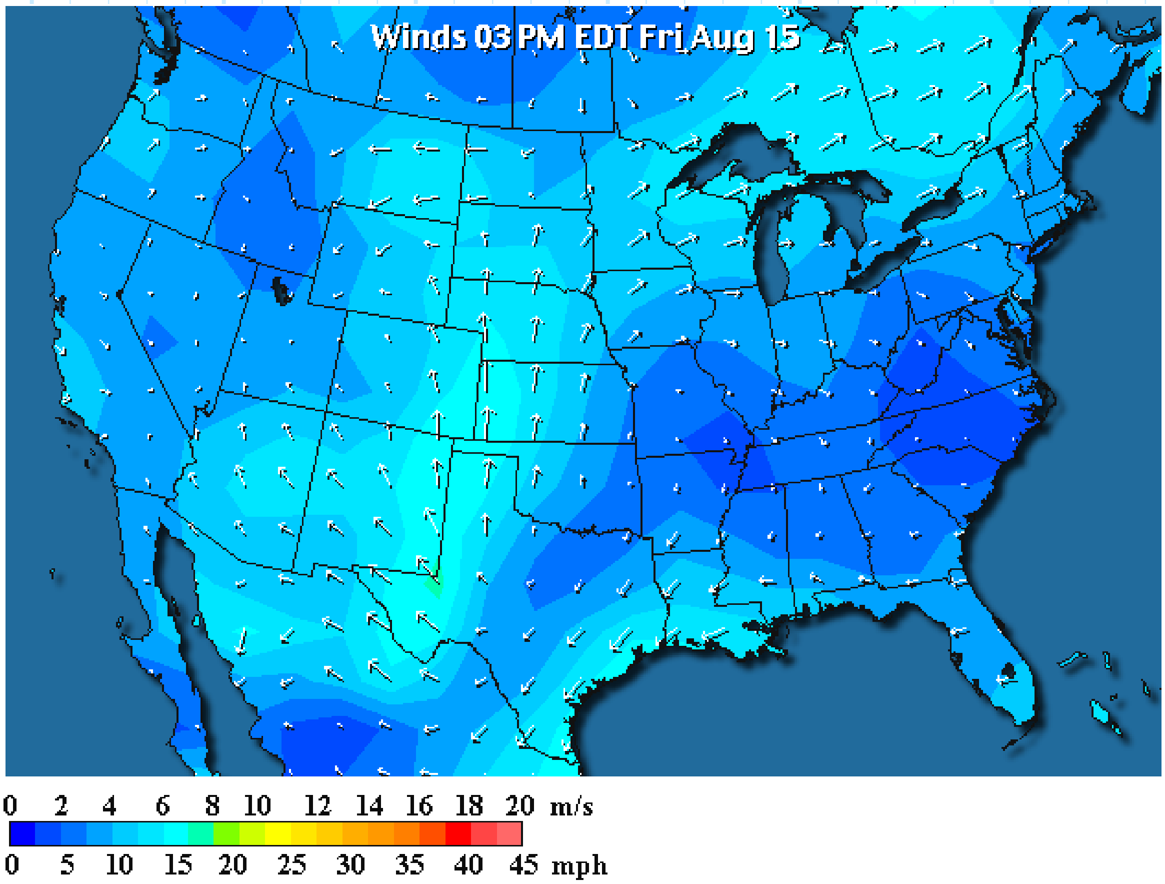
Again, we can use this scalar **field** to determine **discrete** scalar values—we must simply indicate a **specific** location (point) in the U.S. For example:

$$T(x,y = Seattle,WA) = 72\ F^{\circ}$$
$$T(x,y = Dallas,TX) = 97\ F^{\circ}$$
$$T(x,y = Chicago,IL) = 88\ F^{\circ}$$



D. **<u>Vector Fields</u>** are vector quantities that must be described as a function of (typically) space and/or time. Note that this means **both** the magnitude and direction of vector quantity are a function of time and/or space!

An example of a vector field is the **surface wind velocity** across the entire U.S. Again, it is obvious that we **cannot** express this as a **discrete** vector quantity, as **both** the magnitude and direction of the surface wind will **vary** as a function of location (x,y):



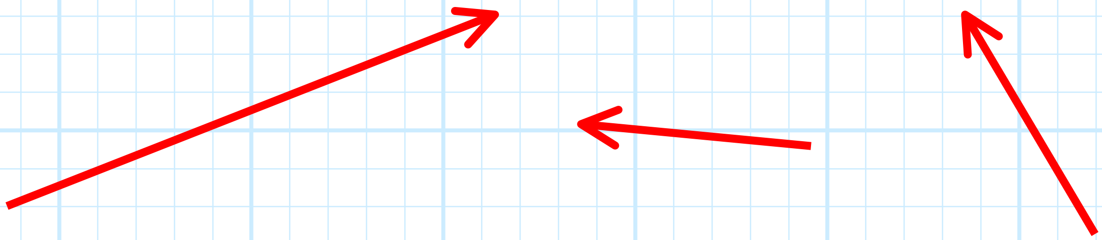
We can **mathematically** describe vector fields using **vector algebraic** notation. For example, the wind velocity across the US might be described as:

$$\mathbf{v}(x,y) = x^2 y\, \hat{a}_x + (2x - y^2)\, \hat{a}_y$$

Don't worry! You will learn what this vector field expression means in the coming weeks.

Vector Representations

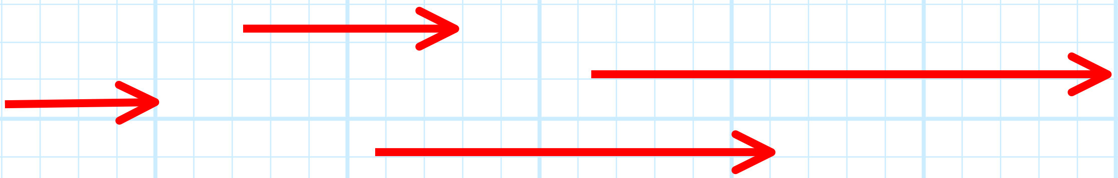
We can **symbolically** represent a discrete vector quantity as an **arrow**:



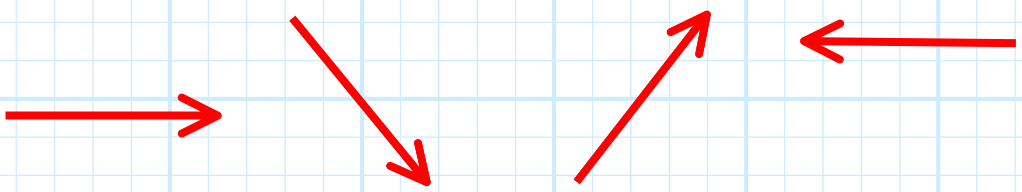
* The **length** of the arrow is proportional to the **magnitude** of the vector quantity.

* The **orientation** of the arrow indicates the **direction** of the vector quantity.

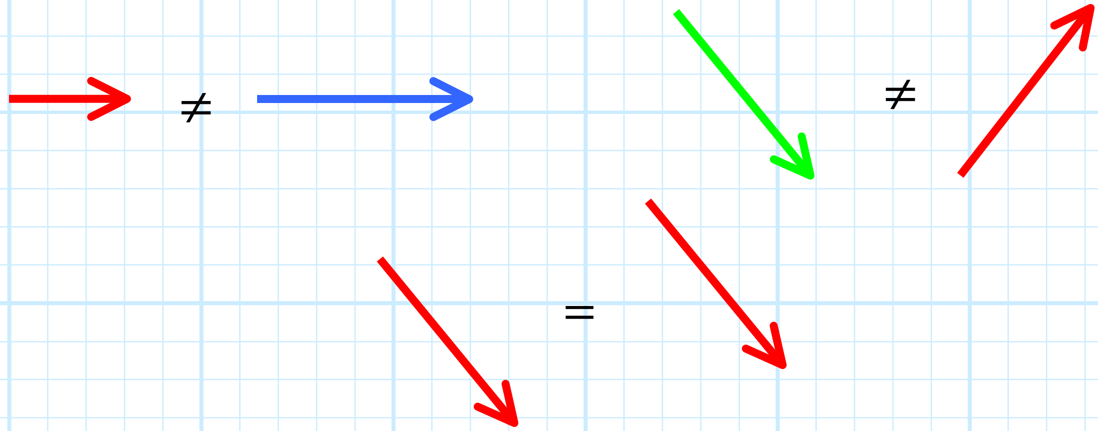
For example, these arrows **symbolize** vector quantities with **equal** direction but **different** magnitudes:



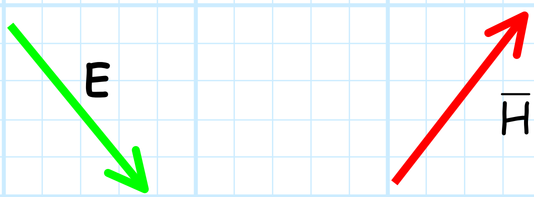
while these arrows represent vector quantities with equal **magnitudes** but different **directions**:



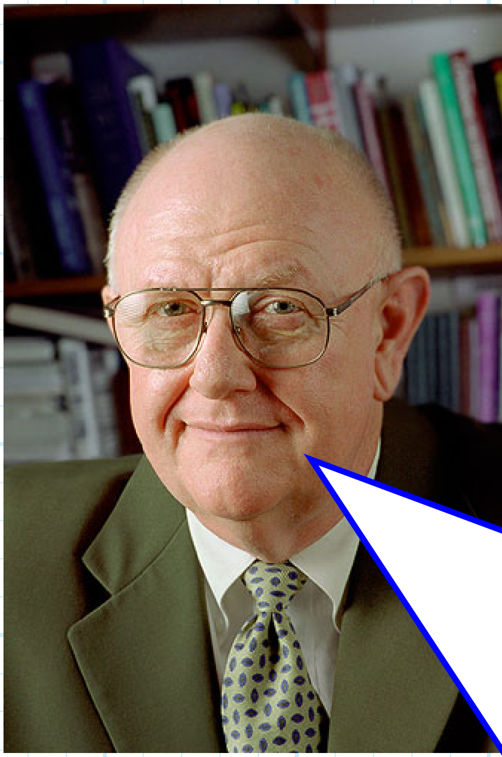
* Two vectors are **equal** only if **both** their magnitudes and directions are identical.



* The variable names of a vector quantity will always be either **boldface** (e.g., **A**, **E**, **H**) or have an **overbar** (e.g., $\overline{A}$, $\overline{B}$, $\overline{C}$).



We will learn that **vector** quantities have their own **special algebra** and **calculus**! This is why we **must** clearly identify vectors quantities in our mathematics (with boldface or overbars). By contrast, variables of **scalar** quantities will **not** be in bold face or have an overbar (e.g. I, V, x, ρ, ϕ)



Vector algebra and **vector calculus** include special operations that **cannot** be performed on scalar quantities (and vice versa).

Thus, you **absolutely must** denote (with an $\overline{\text{overbar}}$) **all vector quantities** in the vector math **you** produce in homework and on exams!!!

Vectors **not properly denoted** will be assumed scalar, and thus the mathematical result will be **incorrect**—and will be **graded appropriately** (this is **bad**)!

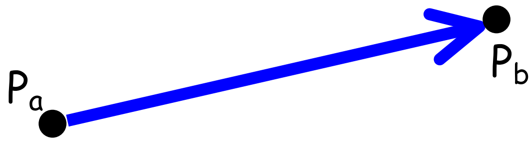
* The **magnitude** of a vector quantity is denoted as:

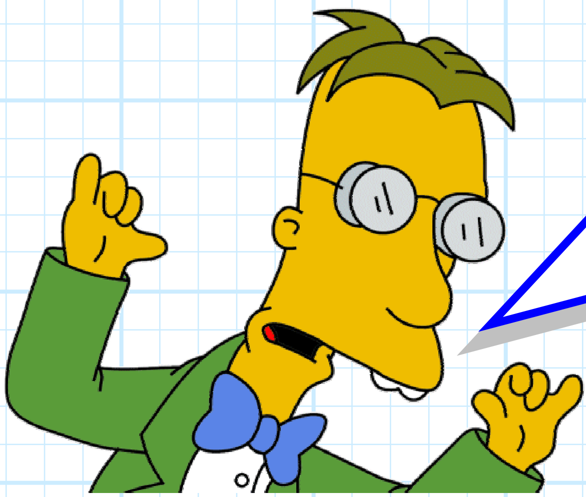
$$|\mathbf{A}| \quad \text{or} \quad |\bar{E}|$$

Note that the **magnitude** of a vector quantity is a **scalar** quantity (e.g., $|\mathbf{F}| = 6$ Newtons or $|\mathbf{v}| = 45$ mph).

The Directed Distance

Q: *It appears that a discrete vector is an* ***easy*** *concept: it's simply an arrow that extends from* ***one point*** *in space to* ***another point*** *in space—****right****?*

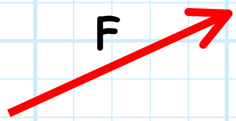




A: *Good heavens* ***NO!*** *Although this is* ***sometimes*** *a valid description of a vector, most of the time it is* ***not****.*

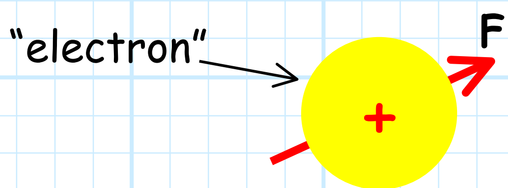
In ***most*** *physical applications, a discrete vector describes a quantity at* ***one specific point*** *in space!*

Remember the arrow representing a discrete vector is **symbolic**. The length of the arrow is **proportional** to the magnitude of the vector quantity; it generally does **not** represent a physical length!

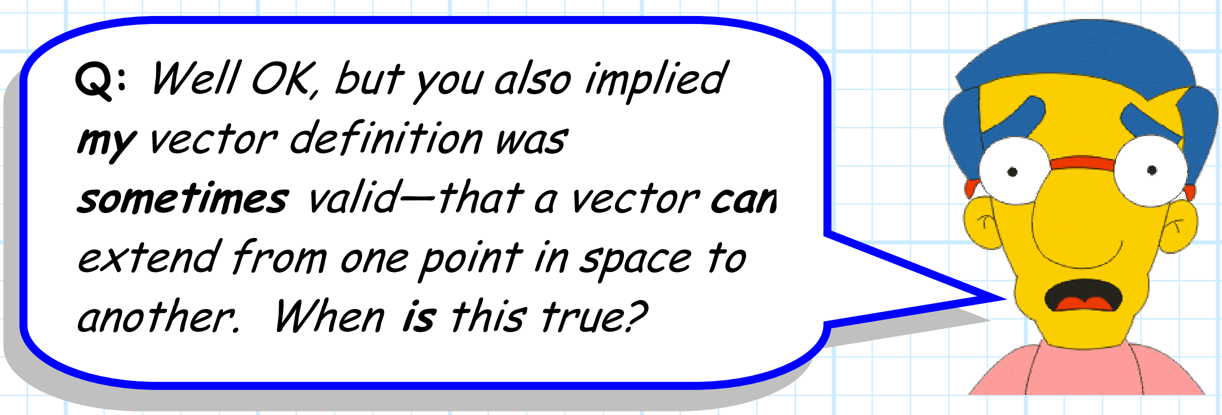


$|\mathbf{F}| = 2.0$ Newtons

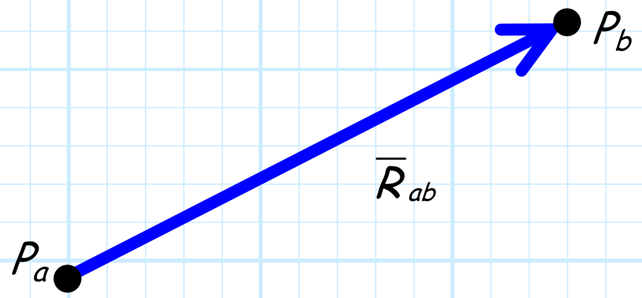
For example, consider a case where we apply a **force** to an **electron**. This force might be due to gravity, or (as we shall see later) an electric field. At any rate, this force is a **vector** quantity; it will have a **magnitude** (in Newtons), and a **direction** (e.g., up, down, left, right).



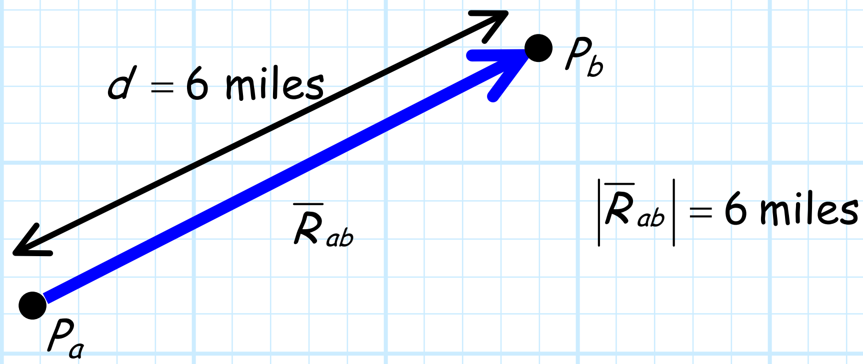
The force described by this vector is applied **at the point** in space where the electron (a **very** small object) is located. The force does **not** "extend" from **one point** in space near the electron to **another point** in space near the electron—it is applied to the electron **precisely** where the electron is located!



A: A vector that extends from one point in space (point *a*) to another point in space (point *b*) is a **special** type of **vector** called a **directed distance**!

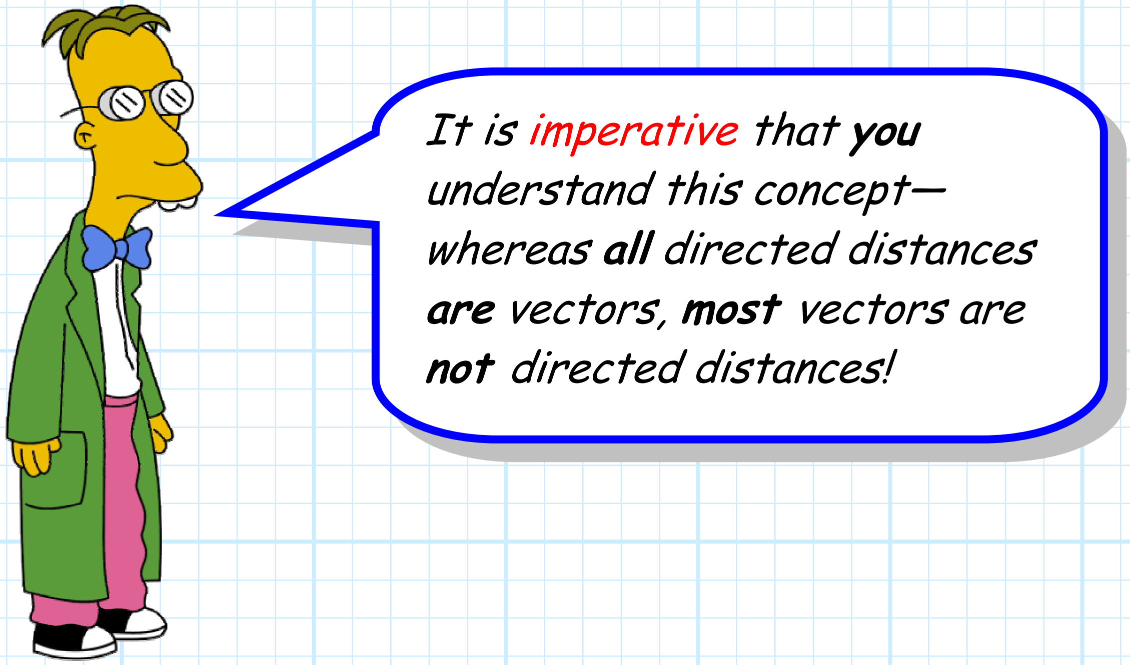


The arrow that represents a **directed distance** vector is **more** than just symbolic—its length (i.e., magnitude) is **equal** to the distance between the two points!

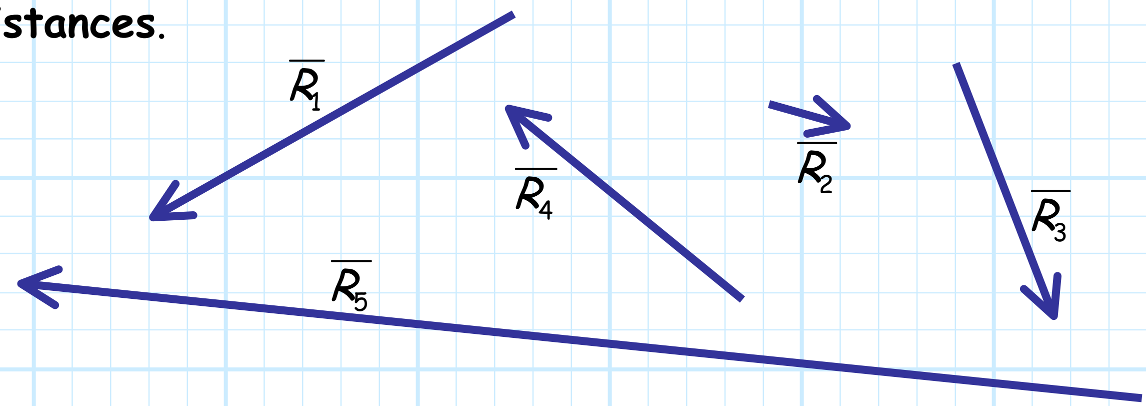


Note the **direction** of the directed distance vector $\bar{R}_{ab}$ indicates the direction of point P_b with respect to point P_a.

Thus, a directed distance vector is **used** to indicate the **location** (both its distance and direction) of one point with respect to another.



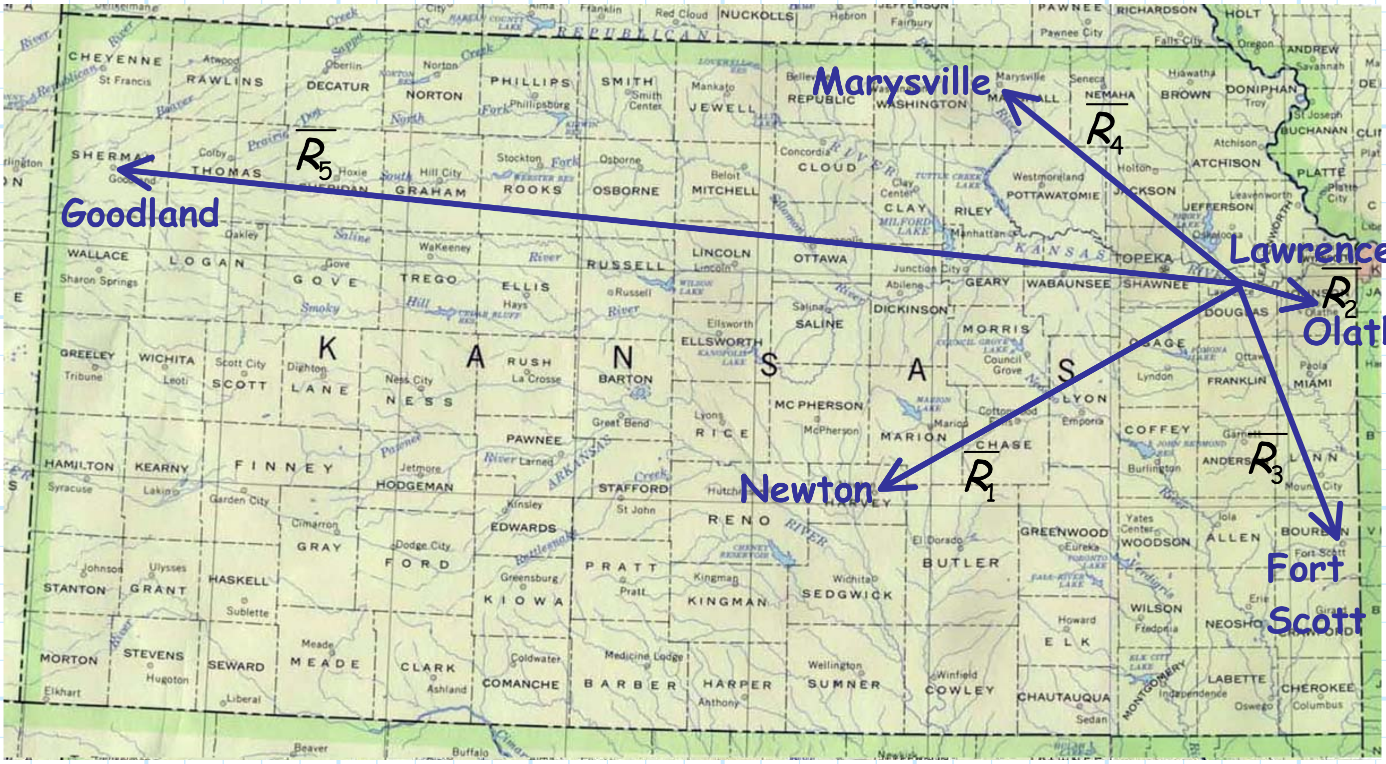
For example, the **vectors** below are all examples of **directed distances**.



Q: *What the heck do these vectors **tell** us ??*

A: The **location** of some of your hometowns !

These directed distances represent the **direction** and **distance** to towns in Kansas, with **respect to** our location here in Lawrence.



For example:

a) Newton is **150 miles southwest** of Lawrence.
b) Olathe is **30 miles east** of Lawrence.
c) Fort Scott is **100 miles south** of Lawrence.
d) Marysville is **100 miles northwest** of Lawrence.
e) Goodland is **350 miles west** of Lawrence.

The location of each town is identified with both a **distance** and **direction**. Therefore a **vector**, specifically a **directed distance**, can be used to indicate the location of each town.

Typically, we will use directed distances to identify points in **three-dimensions** of space, as opposed to the two-dimensional examples given here.