Chapter 2 - Vector Analysis

2-2 Physical Quantities and Units (pp. 7-11)

A. Types of physical quantities

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B. Vector Representation
HO: Vector Representations

C. The Directed Distance

HO: The Directed Distance
Examples of Physical Quantities

A. **Discrete Scalar Quantities** can be described with a single numeric value. Examples include:

1) My height (~ 6 ft.).

2) The weight of your text book (~ 1.0 lbs.)

3) The **surface temperature** of a **specific** location at a **specific** time.

Graphically, a discrete scalar quantity can be indicated as a **point** on a line, surface or volume, e.g.:
B. **Discrete Vector Quantities** must be described with both a **magnitude** and a **direction**. Examples include:

1) The **force** I am exerting on the floor (180 lbs. +++, in a **direction** toward the center of the earth).

2) The wind **velocity** of a **specific** location at a **specific** time.

A specific place and time!

We will find that a discrete vector can be **graphically** represented as an **arrow**:

23 mph West

14 mph North

wherein the length of the arrow is proportional to the **magnitude** and the orientation indicates **direction**.
C. **Scalar Fields** are quantities that must be described as one function of (typically) space and/or time. For example:

1) My weight as a function of time.

Note that we cannot specify this as a single numerical value, as my weight has changed significantly over the course of my life!

Instead, we must use a function of time to describe my weight:

\[ W(t) = 5.2 + 10t - 0.12t^2 \text{ lbs.} \]

where \( t \) is my age in years.

We can likewise graphically represent this scalar field by plotting the function \( W(t) \):

![Graph of \( W(t) \)](image)

Note that we can use this scalar field to determine discrete scalar values! For example, say we wish to determine my weight at birth. This is a discrete scalar value—it can be expressed numerically:

\[ W(t = 0) = 5.2 + 10(0) - 0.12(0)^2 = 5.2 \text{ lbs.} \]

Why I'm always hungry!
Note this discrete scalar value indicates my weight at a specific time \((t=0)\). We likewise could determine my current weight (a discrete scalar value) by evaluating the scalar field \(W(t)\) at \(t=41\) (Doh!).

2) The current surface temperature across the entire the U.S.

Again, this quantity cannot be specified with a single numeric value. Instead, we must specify temperature as a function of position (location) on the surface of the U.S., e.g.:

\[
T(x, y) = 80.0 + 0.1x - 0.2y + 0.003xy + ....
\]

where \(x\) and \(y\) are Cartesian coordinates that specify a point in the U.S. Often, we find it useful to plot this function:
Again, we can use this scalar field to determine **discrete** scalar values—we must simply indicate a **specific** location (point) in the U.S. For example:

\[
T(x, y = Seattle, WA) = 72 \text{ F}^\circ \\
T(x, y = Dallas, TX) = 97 \text{ F}^\circ \\
T(x, y = Chicago, IL) = 88 \text{ F}^\circ 
\]

**D. Vector Fields** are vector quantities that must be described as a function of (typically) space and/or time. Note that this means **both** the magnitude and direction of vector quantity are a function of time and/or space!
An example of a vector field is the **surface wind velocity** across the entire U.S. Again, it is obvious that we *cannot* express this as a **discrete** vector quantity, as both the magnitude and direction of the surface wind will **vary** as a function of location \((x,y)\):

We can **mathematically** describe vector fields using **vector algebraic** notation. For example, the wind velocity across the US might be described as:

\[
\mathbf{v}(x,y) = x^2 y \hat{a}_x + (2x - y^2) \hat{a}_y
\]

Don’t worry! You will learn what this vector field expression means in the coming weeks.
Vector Representations

We can symbolically represent a discrete vector quantity as an arrow:

* The length of the arrow is proportional to the magnitude of the vector quantity.

* The orientation of the arrow indicates the direction of the vector quantity.

For example, these arrows symbolize vector quantities with equal direction but different magnitudes:

While these arrows represent vector quantities with equal magnitudes but different directions:
* Two vectors are equal only if both their magnitudes and directions are identical.

* The variable names of a vector quantity will always be either **boldface** (e.g., $A, E, H$) or have an **overbar** (e.g., $\bar{A}, \bar{B}, \bar{C}$).

We will learn that **vector** quantities have their own **special algebra** and **calculus**! This is why we must clearly identify vectors quantities in our mathematics (with boldface or overbars). By contrast, variables of **scalar** quantities will not be in bold face or have an overbar (e.g. $I, V, x, \rho, \phi$).
Vector algebra and vector calculus include special operations that cannot be performed on scalar quantities (and vice versa).

Thus, you absolutely must denote (with an overbar) all vector quantities in the vector math you produce in homework and on exams!!!

Vectors not properly denoted will be assumed scalar, and thus the mathematical result will be incorrect—and will be graded appropriately (this is bad)!

* The magnitude of a vector quantity is denoted as:

\[ |A| \text{ or } \overline{E} \]

Note that the magnitude of a vector quantity is a scalar quantity (e.g., \( |F| = 6 \text{ Newtons} \) or \( |v| = 45 \text{ mph} \)).
The Directed Distance

Q: It appears that a discrete vector is an easy concept: it’s simply an arrow that extends from one point in space to another point in space—right?

A: Good heavens NO! Although this is sometimes a valid description of a vector, most of the time it is not. In most physical applications, a discrete vector describes a quantity at one specific point in space!
Remember the arrow representing a discrete vector is symbolic. The length of the arrow is proportional to the magnitude of the vector quantity; it generally does not represent a physical length!

\[ |\mathbf{F}| = 2.0 \text{ Newtons} \]

For example, consider a case where we apply a force to an electron. This force might be due to gravity, or (as we shall see later) an electric field. At any rate, this force is a vector quantity; it will have a magnitude (in Newtons), and a direction (e.g., up, down, left, right).

The force described by this vector is applied at the point in space where the electron (a very small object) is located. The force does not "extend" from one point in space near the electron to another point in space near the electron—it is applied to the electron precisely where the electron is located!

Q: Well OK, but you also implied my vector definition was sometimes valid—that a vector can extend from one point in space to another. When is this true?
A: A vector that extends from one point in space (point $a$) to another point in space (point $b$) is a special type of vector called a directed distance!

The arrow that represents a directed distance vector is more than just symbolic—it’s length (i.e., magnitude) is equal to the distance between the two points!

Note the direction of the directed distance vector $\vec{R}_{ab}$ indicates the direction of point $P_b$ with respect to point $P_a$.

Thus, a directed distance vector is used to indicate the location (both its distance and direction) of one point with respect to another.
For example, the vectors below are all examples of directed distances.

- $\mathbf{R}_1$
- $\mathbf{R}_4$
- $\mathbf{R}_2$
- $\mathbf{R}_3$
- $\mathbf{R}_5$

Q: *What the heck do these vectors tell us??*

A: The *location* of some of your hometowns!

These directed distances represent the *direction* and *distance* to towns in Kansas, with *respect to* our location here in Lawrence.

*It is imperative that you understand this concept—whereas all directed distances are vectors, most vectors are not directed distances!*
For example:

a) Newton is 150 miles southwest of Lawrence.
b) Olathe is 30 miles east of Lawrence.
c) Fort Scott is 100 miles south of Lawrence.
d) Marysville is 100 miles northwest of Lawrence.
e) Goodland is 350 miles west of Lawrence.

The location of each town is identified with both a distance and direction. Therefore a vector, specifically a directed distance, can be used to indicate the location of each town.

Typically, we will use directed distances to identify points in three-dimensions of space, as opposed to the two-dimensional examples given here.