2.4 Orthogonal Coordinate Systems

Reading Assignment: *pp.16-33*

We live in a **3**-dimensional world!

Meaning:

1)

2)

Q: What 3 scalar **values** and what 3 unit **vectors** do we use ??

A: We have several options! A **set** of 3 scalar values that define position and a set of unit vectors that define direction form a **Coordinate system**. Examples of coordinate systems include:

1.

2.

3.

A. Coordinates

* The 3 **scalar** values used to define **position** are called **coordinates**.

* E.G., scalar values u_1, u_2, and u_3 can define precisely the **location** of point P in space (i.e., P(u_1, u_2, u_3)).

* All coordinates are defined with respect to an **arbitrary** point called the **origin**.

HO: Cartesian Coordinates

HO: Cylindrical Coordinates

HO: Spherical Coordinates

B. Coordinate Transformations

We can rewrite the **location** of point P(x,y,z) in terms of cylindrical coordinates (i.e, P(r,θ,ϕ)), for example.

Or, we can rewrite a **scalar field** $g(x,y,z)$ in terms of cylindrical coordinates (i.e, $g(\rho,\phi,z)$), for example.

HO: Coordinate Transformations

Example: Coordinate Transformations

C. Base Vectors

* The 3 **unit vectors** used to define **direction** are called **base vectors**.

* E.G., base vectors $\hat{a}_1, \hat{a}_2, \hat{a}_3$ can be used to precisely describe the **direction** of some **vector A**.

HO: Base Vectors

HO: Cartesian Base Vectors

D. Vector Expansion using Base Vectors

Q: **Why** are base vectors important? **How** are they used?

A: We find that any and **all** vectors can be expressed as the **sum** of **3** vectors, each pointing in the precise **direction** of one of the three base vectors!

e.g.,

$$\mathbf{B} = B_1 \hat{\boldsymbol{a}}_1 + B_2 \hat{\boldsymbol{a}}_2 + B_3 \hat{\boldsymbol{a}}_3$$

or

$$\mathbf{C} = C_x \hat{\boldsymbol{a}}_x + C_y \hat{\boldsymbol{a}}_y + C_z \hat{\boldsymbol{a}}_z$$

HO: Vector Expansion using Base Vectors

E. Spherical and Cylindrical Base Vectors

HO: Spherical Base Vectors

HO: Cylindrical Base Vectors

F. Vector Algebra and Vector Expansions

HO: Vector Algebra using Orthonormal Base Vectors

G. The Vector Field

* Recall a vector **field** is a function of **position**.

* We express position in terms of **coordinates**.

* Thus, a vector field is **function** of coordinate values (e.g., *x, y, z*).

* But, we express a vector field with **3 scalar components**.

HO: Vector Fields

HO: Expressing Vector Fields with Coordinate Systems

H. The Position Vector

In addition to coordinates (e.g., r,θ,ϕ), we can use a special **directed distance** to specify points in space.

HO: The Position Vector

HO: Applications of the Position Vector

HO: Vector Field Notation

HO: A Gallery of Vector Fields