2.4 Orthogonal Coordinate Systems

Reading Assignment: pp.16-33

We live in a 3-dimensional world!

Meaning:

1)

2)

Q: What 3 scalar values and what 3 unit vectors do we use??

A: We have several options! A set of 3 scalar values that define position and a set of unit vectors that define direction form a Coordinate system. Examples of coordinate systems include:

1.

2.

3.
A. Coordinates

* The 3 scalar values used to define position are called coordinates.

* E.G., scalar values $u_1$, $u_2$, and $u_3$ can define precisely the location of point $P$ in space (i.e., $P(u_1, u_2, u_3)$).

* All coordinates are defined with respect to an arbitrary point called the origin.

**HO: Cartesian Coordinates**

**HO: Cylindrical Coordinates**

**HO: Spherical Coordinates**

B. Coordinate Transformations

We can rewrite the location of point $P(x,y,z)$ in terms of cylindrical coordinates (i.e., $P(r,\theta,\phi)$), for example.
Or, we can rewrite a scalar field $g(x,y,z)$ in terms of cylindrical coordinates (i.e., $g(\rho,\phi,z)$), for example.

**HO: Coordinate Transformations**

**Example: Coordinate Transformations**

**C. Base Vectors**

* The 3 unit vectors used to define direction are called base vectors.

* E.G., base vectors $\hat{a}_1, \hat{a}_2, \hat{a}_3$ can be used to precisely describe the direction of some vector $A$.

**HO: Base Vectors**

**HO: Cartesian Base Vectors**
D. Vector Expansion using Base Vectors

Q: Why are base vectors important? How are they used?

A: We find that any and all vectors can be expressed as the sum of 3 vectors, each pointing in the precise direction of one of the three base vectors!

e.g.,

\[ B = B_1 \hat{a}_1 + B_2 \hat{a}_2 + B_3 \hat{a}_3 \]

or

\[ C = C_x \hat{a}_x + C_y \hat{a}_y + C_z \hat{a}_z \]

HO: Vector Expansion using Base Vectors

E. Spherical and Cylindrical Base Vectors

HO: Spherical Base Vectors

HO: Cylindrical Base Vectors
F. Vector Algebra and Vector Expansions

**HO: Vector Algebra using Orthonormal Base Vectors**

G. The Vector Field

* Recall a vector field is a function of position.

* We express position in terms of coordinates.

* Thus, a vector field is function of coordinate values (e.g., x, y, z).

* But, we express a vector field with 3 scalar components.

**HO: Vector Fields**
HO: Expressing Vector Fields with Coordinate Systems

H. The Position Vector

In addition to coordinates (e.g., $r, \theta, \phi$), we can use a special directed distance to specify points in space.

HO: The Position Vector

HO: Applications of the Position Vector

HO: Vector Field Notation

HO: A Gallery of Vector Fields