3-2 Charge and Charge Density (pp.61-63)

HO: Electric Charge

Q:

A: HO: Charge Density

Q:

A: HO: Total Charge

Electric Charge

Most of classical physics can be described in terms of three fundamental units, which define our physical "reality"

1. Mass (e.g., *kg*)

2. Distance (e.g., *meters*)

3. Time (e.g., *seconds*)

From these fundamental units, we can define other important physical parameters. For example, **energy** can always be described in units of $kg\ m^2/s^2$.

But, these three fundamental units alone are insufficient for describing all of classic physics—we require one more to completely describe physical reality!

This fourth fundamental unit is **Coulomb**, the unit of **electric charge**.

All **electromagnetic** phenomena can be attributed to electric charge!

We shall find that electric charge is somewhat analogous to mass. However, one important difference between mass and charge is that charge can be either **positive** or **negative**!

Essentially, charge (like mass) is a property of **atomic particles**. Specifically, we find that:

The charge "on" a **proton** is $+1.6 \times 10^{-19}$ C

The charge "on" a **neutron** is 0.0 C

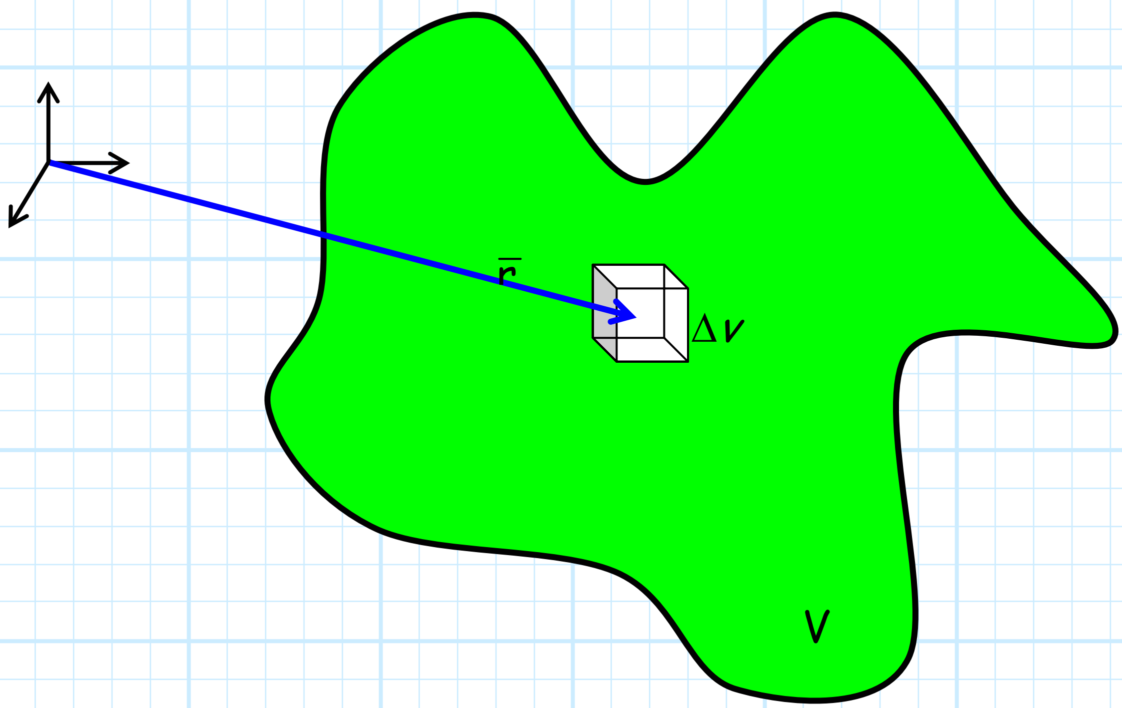
The charge "on" an **electron** is -1.6×10^{-19} C

Charged particles (of all types) can be distributed (unevenly) across a volume, surface, or contour.

Charge Density

In many cases, charged particles (e.g., electrons, protons, positive ions) are **unevenly distributed** throughout some volume V.

We define **volume charge density** at a specific point $\bar{r}$ by evaluating the total net charge ΔQ in a small volume Δv surrounding the point.



$$\text{Volume Charge Density} = \rho_v(\bar{r}) \doteq \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v}$$

Volume charge density is therefore a **scalar field**, and is expressed with units such as **coulombs/m^3**.

IMPORTANT NOTE: Volume charge density indicates the **net** charge density at each point $\bar{r}$ within volume V.

Q: *What exactly do you mean by* ***net*** *charge density ?*

A: Remember, there are positively charged particles and there are negatively charged particles, and **both** can exist at the same location $\bar{r}$.

Thus, a **positive** charge density does **not** mean that **no** negatively charged particles (e.g., electrons) are present, it simply means that there is **more** positive charge than there is negative!

It might be more instructive to define:

$$\Delta Q = \Delta Q^{+} + \Delta Q^{-}$$

where ΔQ^{+} is the amount of **positive** charge (therefore a **positive number**) and ΔQ^{-} is the amount of **negative** charge (therefore a **negative number**). We can call ΔQ the net, or **total charge**.

Volume charge density can therefore be expressed as:

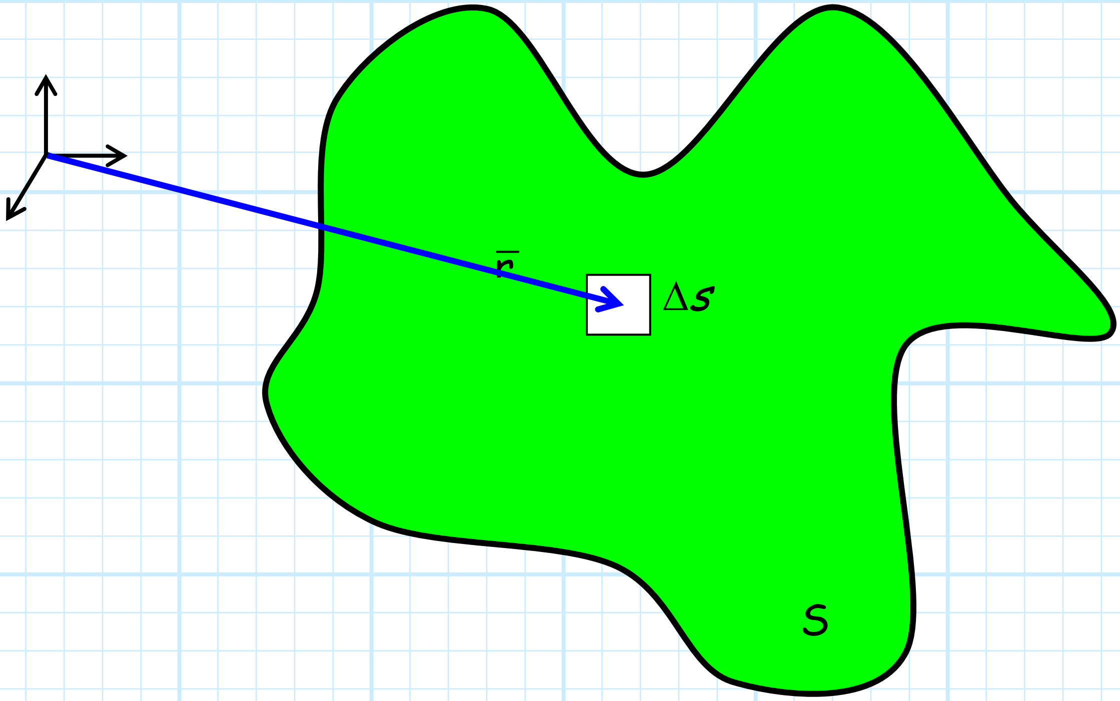
$$\rho_v(\bar{r}) \doteq \lim_{\Delta v \to 0} \frac{\Delta Q^+ + \Delta Q^-}{\Delta v} = \rho_v^+(\bar{r}) + \rho_v^-(\bar{r})$$

For example, the charge density at some location $\bar{r}$ due to negatively charged particles might be –10.0 C/m^3, while that of positively charged particles might be –5 C/m^3. Therefore, the net, or **total** charge density is:

$$\rho_v(\bar{r}) = \rho_v^+(\bar{r}) + \rho_v^-(\bar{r}) = 5 + (-10) = -5 \;\; C/m^3$$

Surface Charge Density

Another possibility is that charge is unevenly distributed across some surface S. In this case, we can define a **surface charge density** as by evaluating the total charge ΔQ on a small patch of surface Δs, located at point $\bar{r}$ on surface S:



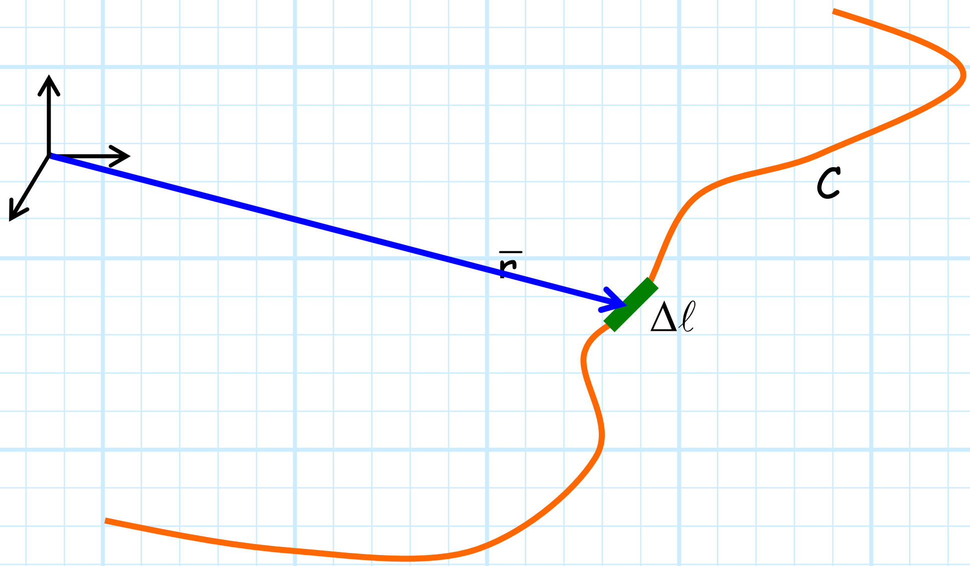
Surface charge density $\rho_s(\bar{r})$ is therefore defined as:

$$\boxed{\rho_s(\bar{r}) \doteq \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta s}}$$

Note the **units** for surface charge density will be **charge/area** (e.g. **C/m^2**).

Line Charge Density

Finally, we also consider the case where charge is unevenly distributed across some **contour** C. We can therefore define a **line charge density** as the charge ΔQ along a small distance $\Delta \ell$, located at point $\bar{r}$ of contour C.



We therefore define line charge density $\rho_\ell(\bar{r})$ as:

$$\boxed{\rho_\ell(\bar{r}) \doteq \lim_{\Delta\ell \to 0} \frac{\Delta Q}{\Delta \ell}}$$

As you might expect, the units of a line charge density is charge per length (e.g., **C/m**).

Total Charge

Q: *If we know charge density $\rho_v(\bar{\mathbf{r}})$, describing the charge distribution throughout a* ***volume*** *V, can we determine the* ***total charge*** *Q contained within this volume?*

A: You betcha! Simply **integrate** the charge density over the entire volume, and you get the **total charge** Q contained within the volume.

In other words:

$$\boxed{Q = \iiint_V \rho_v(\bar{\mathbf{r}})\, dv}$$

Note this is a **volume integral** of the type we studied in Section 2-5. Therefore select the differential volume dv that is appropriate for the volume V.

Likewise, we can determine the total charge distributed across a **surface** S by integrating the surface charge density:

$$Q = \iint_S \rho_s(\bar{\mathrm{r}})\, ds$$

Q: *Hey! This is **NOT** the surface integral we studied in Section 2-5.*

A: True! This is a **scalar** integral; sort of a two-dimensional version of the volume integral.

The differential surface element *ds* in this integral is simply the **magnitude** of the differential surface vectors we studied earlier:

$$ds = \left|\overline{ds}\right|$$

For example, if we integrate over the surface of a sphere, we would use the differential surface element:

$$ds = \left|\overline{ds_r}\right| = r^2 \sin\theta \, d\theta \, d\phi$$

Finally, we can determine the total charge on **contour** *C* by integrating the **line charge density** $\rho_\ell(\bar{\mathrm{r}})$ across the entire contour:

$$\boxed{Q = \int_C \rho_\ell(\bar{\mathbf{r}})\, d\ell}$$

The differential element $d\ell$ is likewise related to the differential displacement vector we studied earlier:

$$d\ell = \left|\overline{d\ell}\right|$$

For example, if the contour is a circle around the *z*-axis, then $d\ell$ is:

$$d\ell = \left|\overline{d\phi}\right| = \rho\, d\phi$$