3-6 The Lorentz Force Law and the Field Concept

Reading Assignment: *pp. 75-81*

A. Field Theory

HO: Field Theory

B. E.M. Fields and Forces

$$\mathbf{E}(\bar{r}) \doteq$$

$$\mathbf{B}(\bar{r}) \doteq$$

HO: The Electric Force

HO: The Magnetic Force

C.   The Lorentz Force Law

HO: The Lorentz Force Law

HO: The Differential Lorentz Force Law

Example: The Lorentz Force Law

Field Theory

Coulomb's and Ampere's Laws of Force describe an **"action at a distance"** approach to describing the basic electromagnetic forces. In this approach, we can state:

1. A **charge** at one location in space will exert a force on a **charge** at another location in space.

2. **Current** flowing at one location in space will exert a force on **current** flowing at another location in space.

Alternatively, we can use a **"field theory"** of electromagnetics. In this field theory, we describe the forces in this manner:

1. A **charge** at one location in space will create an **electric** (vector) **field** at any and all other locations in space.

   This **field** will then exert a force on any other **charges** that exist in space.

2. **Current** flowing at one location in space will create a **magnetic flux density** (vector) field at any and all other locations in space.

   This **field** will then exert a force on any other **current** that exist in space.

In other words, in **field theory**, we state that **sources create fields** (everywhere)—and then the **fields apply the force** to other charges or currents!
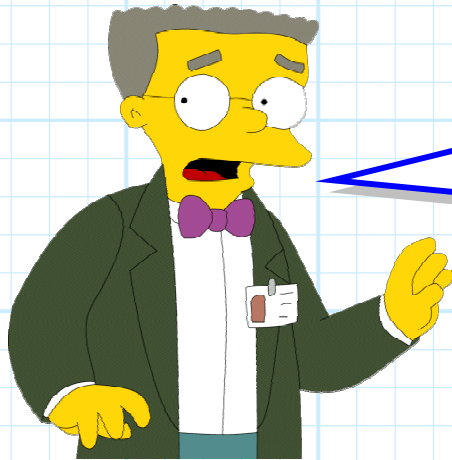
Q: *Good Heavens! This seems to make electromagnetics even more difficult!* **Why** *do we use the concept of electric and magnetic* **fields**??

A: Field theory is helpful because it allows us to **divide** the electromagnetic force problem into **two** more manageable pieces. Specifically:

We can determine the **fields** generated by **source** charges or currents, **without** every having to consider the other charges or currents that they affect!

We can determine the effect (i.e., **forces**) of fields on charges or currents, **without** every having to consider the sources that created those fields!

The Electric Force

Say a **charge** Q is located at some **point** in space, a point denoted by position vector $\bar{r}$.

Likewise, there exists **everywhere** in space an **electric field** (we neither know nor care **how** this electric field was **created**).

The value (both magnitude and direction) of the electric field vector at point $\bar{r}$ is $\mathbf{E}(\bar{r})$:
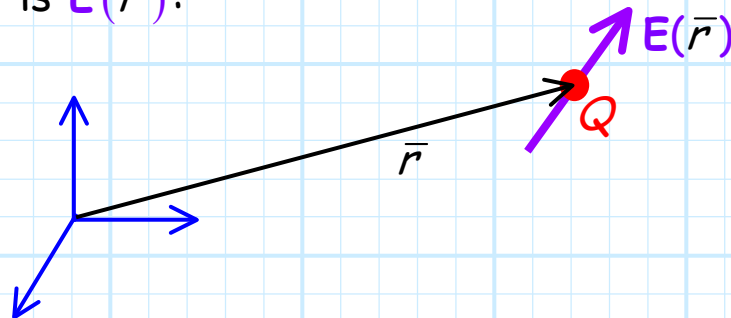

Q: *Our "**field theory**" of electromagnetics says that the electric field will apply a **force** on the charge. Precisely what **is** this force (i.e., its magnitude and direction)?*

A: Fortunately, the answer is rather **simple**! The **force** $\mathbf{F_e}$ on charge Q is the **product** of the **charge** (a scalar) and the value of the **electric field** (a vector) at the point where the charge is located:

$$\mathbf{F}_e = Q\,\mathbf{E}(\bar{r}) \qquad [\text{N}]$$

Note therefore, that the force vector will be **parallel** (or **anti-parallel**) to the electric field!

$E(\bar{r})$

(+)

F_e
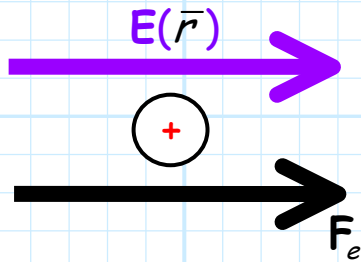
$Q > 0$ (charge is **positive**) so F_e points in the **same** direction as the electric field.

$E(\bar{r})$

(−)

F_e
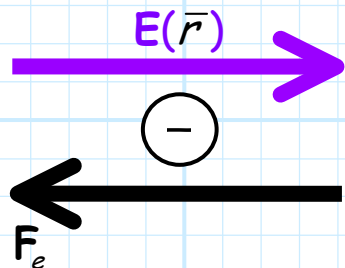
$Q < 0$ (charge is **negative**) so F_e points in the **opposite** direction as the electric field.
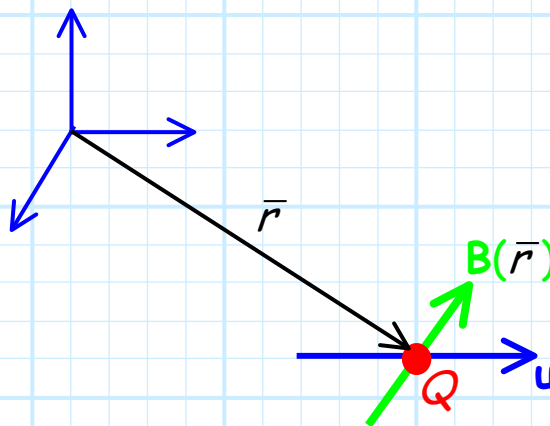
Note the **magnitude** of the electric force will increase **proportionally** with an increase in **charge** and/or and increase in the electric field **magnitude**.

The Magnetic Force

Say a **charge** Q is located at some **point** in space (denoted by position vector $\bar{r}$), and is moving with velocity **u**.

Likewise, there exists **everywhere** in space a magnetic flux density (we neither know nor care **how** this field was **created**).

The value (both magnitude and direction) of the magnetic flux density vector **at point** $\bar{r}$ is $\mathbf{B}(\bar{r})$:


Q: Our "**field theory**" of electromagnetics says that the magnetic flux density will apply a **force** on the moving charge (i.e., **current**). Precisely what **is** this force (i.e., its magnitude and direction)?

A: The answer is not **quite** as simple the electric force equation. The **force** $\mathbf{F_m}$ on charge Q moving at velocity **u** is :

$$\mathbf{F}_m = Q\,\mathbf{u} \times \mathbf{B}(\bar{r})$$

Note therefore, that the resulting force $\mathbf{F_m}$ will be **orthogonal** to both the **velocity** vector $\mathbf{u}$ and the **magnetic flux density** vector $\mathbf{B}(\bar{r})$. E.G.,:
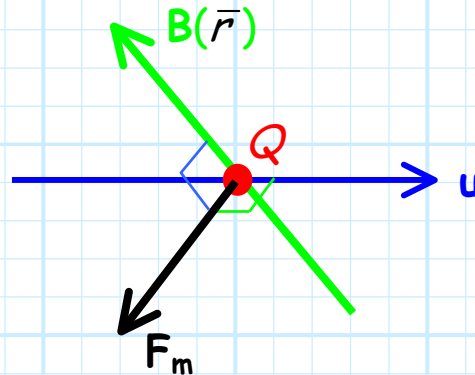
Note the **maximum** force is applied when the magnetic flux density vector is **orthogonal** to the velocity vector (i.e., $\theta = 90°$).

Alternatively, the force on the charge will actually be **zero** if the magnetic flux density is **parallel** to the velocity vector (i.e., $\theta = 0°$) :
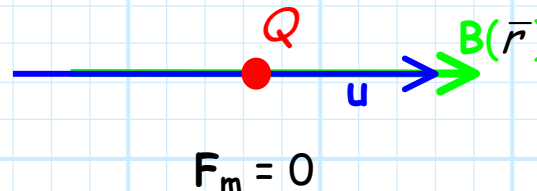
Note there is **no** equivalent situation for the **electric** force—the only way $\mathbf{F_e}$ can be zero is if the electric field $\mathbf{E}(\bar{r})$ is zero!
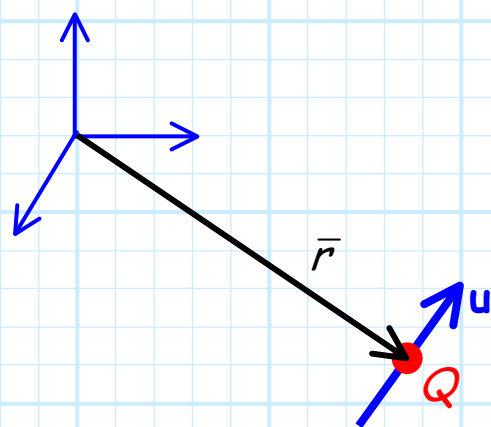
The Lorentz Force Law

Say a charge Q resides at point $\bar{r}$, and is moving at a **velocity u**.

Somewhere, other charges and currents have generated an **electric field** $\mathbf{E}(\bar{r})$ and **magnetic flux density** $\mathbf{B}(\bar{r})$.
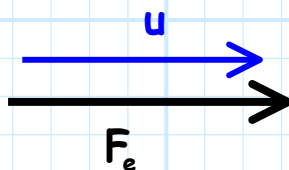
These fields exert a **force** on charge Q equal to:

$$\mathbf{F} = Q\left(\mathbf{E}(\bar{r}) + \mathbf{u} \times \mathbf{B}(\bar{r})\right)$$

Note the force due to $\mathbf{E}(\bar{r})$ (i.e., $\mathbf{F}_e$), **could** be parallel to velocity vector **u**.

For that case, $\mathbf{E}(\bar{r})$ will apply a force on the charge in the direction of its velocity. This will **speed up** (i.e., accelerate) the charge, essentially adding **kinetic energy** to the charged particle.

Or, the force due to $\mathbf{E}(\bar{r})$ could be **anti-parallel** to velocity vector **u**. For this case, the electric field $\mathbf{E}(\bar{r})$ applies a force on the charge in the **opposite direction** of its movement. This will **slow down** the charge, essentially **extracting** kinetic energy from the charged particle.

u →

F$_e$ ←
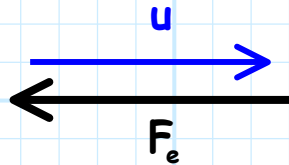
Now, contrast this with the force applied by the **magnetic flux density**. We know that:

$$\mathbf{F_m} = \left(\mathbf{u} \times \mathbf{B}(\bar{r})\right)Q$$

Therefore, the force **F**$_m$ is **always** orthogonal to velocity vector **u** (do **you** see why?).

F$_m$ ↑

u →
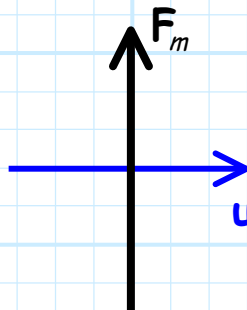
As a result, the force due to the magnetic flux density $\mathbf{B}(\bar{r})$ can change the **direction** of velocity **u** (i.e., change particle path), but **not** the magnitude of the velocity $|\mathbf{u}|$.

In other words, the force **F**$_m$ **neither** speeds up or slows down a charged particle, although it **will** change its **direction**. As a result, the magnetic flux density $\mathbf{B}(\bar{r})$ **cannot** modify the **kinetic energy** of the charged particle.

The Differential Form of the Lorentz Force Law

Say that instead of a single charged particle, we have a small **volume** dv of charge, with volume **charge density** $\rho_v(\bar{r})$. Therefore:

$$dQ = \rho_v(\bar{r}) \; dv$$

If this volume of charge is also moving with a velocity **u**, then the force $\mathbf{dF}(\bar{r})$ on charge dQ will be:

$$\mathbf{dF}(\bar{r}) = dQ\big(\mathbf{E}(\bar{r}) + \mathbf{u} \times \mathbf{B}(\bar{r})\big)$$
$$= \rho(\bar{r})\,\mathbf{E}(\bar{r})\,dv + \rho(\bar{r})\,\mathbf{u} \times \mathbf{B}(\bar{r})\,dv$$
$$= \rho(\bar{r})\,\mathbf{E}(\bar{r})\,dv + \mathbf{J}(\bar{r}) \times \mathbf{B}(\bar{r})\,dv$$

where we recall that $\mathbf{J}(\bar{r}) = \rho_v(\bar{r})\mathbf{u}$.

Therefore we can state that:

$$\mathbf{dF_e}(\bar{r}) = \rho_v(\bar{r})\,\mathbf{E}(\bar{r})\,dv$$

$$\mathbf{dF_m}(\bar{r}) = \mathbf{J}(\bar{r}) \times \mathbf{B}(\bar{r})\,dv$$

Look what this means!

It means that not only does an electric field apply a force to a single charge Q, but it also applies a force on a whole **collection** of charges, described by $\rho_v(\bar{r})$.

Likewise, the magnetic flux density not only applies a force on a moving charge particle, it also applies a force to **any** current distribution described by $\mathbf{J}(\bar{r})$.

To determine the **total** force on some **volume** V enclosing charges and currents, we must **integrate** over the entire volume:

$$\mathbf{F} = \iiint_V d\,\mathbf{F}(\bar{r})\,dv$$

$$= \iiint_V \left(\rho(\bar{r})\,\mathbf{E}(\bar{r}) + \mathbf{J}(\bar{r}) \times \mathbf{B}(\bar{r}) \right) dv$$

The Lorentz force equation tells us how fields $\mathbf{E}(\bar{r})$ and $\mathbf{B}(\bar{r})$ **affect** charges $\rho_v(\bar{r})$ and currents $\mathbf{J}(\bar{r})$.

But remember, charges $\rho_v(\bar{r})$ and currents $\mathbf{J}(\bar{r})$ likewise **create** fields $\mathbf{E}(\bar{r})$ and $\mathbf{B}(\bar{r})$!

> **Q:** *How can we determine what fields $\mathbf{E}(\bar{r})$ and $\mathbf{B}(\bar{r})$ are **created** by charges $\rho_v(\bar{r})$ and currents $\mathbf{J}(\bar{r})$?*
>
> **A:** Ask Jimmy **Maxwell**!

Example: The Lorentz Force Law

You are all very aware of the practical importance of the Lorentz force law! In fact, you likely **observe** its effects several times each day.

An almost perfect application of the Lorentz Force Law is the **Cathode Ray Tube** (CRT)—the device at the heart of every **television** and computer **monitor** (or, at least, the non-plasma/LED kind!).

At the very back of a CRT is an element that is heated to very high temperatures. This hot element begins to **"emit" electrons** into the vacuum of the CRT.


CRT

hot element

emitted electrons
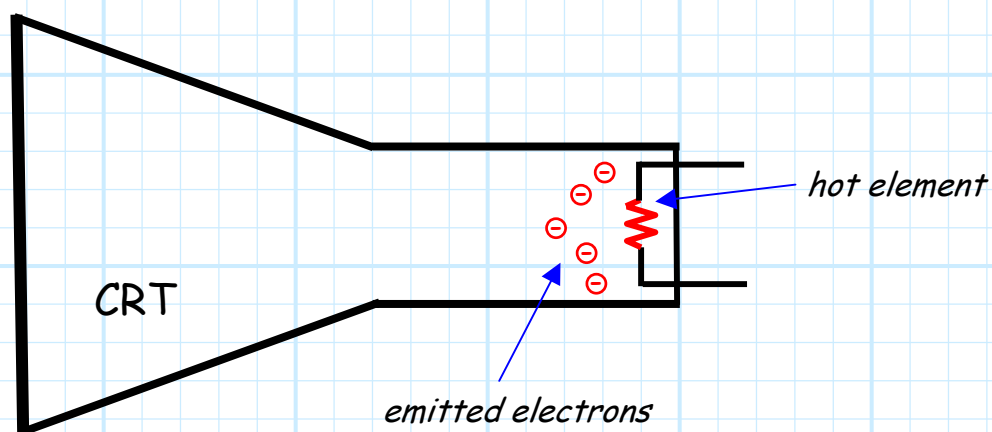
Then, an **electric field $\mathbf{E}(\bar{r})$** with **large** magnitude is created within the tube. The resulting force ($\mathbf{F} = Q_e\,\mathbf{E}(\bar{r})$) accelerates the electrons toward the front, creating a stream of electrons—a "**cathode ray**".
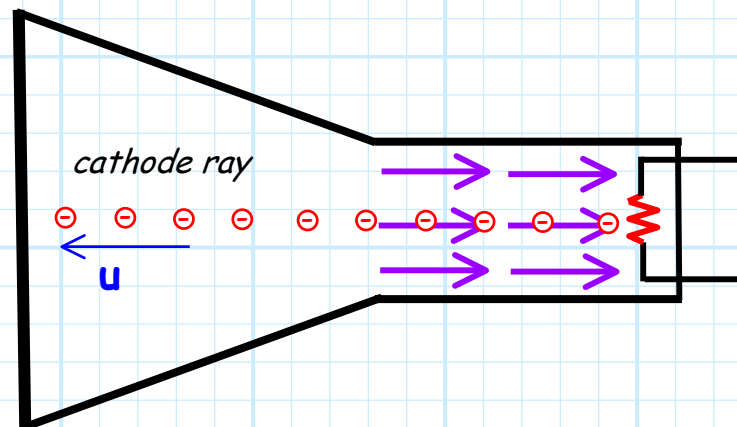
Since the electric field strength (i.e., the **force** on the electrons) is large, the electrons have great velocity (i.e., **kinetic energy**) when they reach the front of the CRT.

The front of the CRT is coated with **phosphorous**, and so the cathode ray "slams" into this phosphorous coating. The kinetic energy of each electron must be **dissipated** in some way. Typically, energy is dissipated as **heat** (the coating heats up), but for phosphorous, this energy is likewise dissipated in the form of **light**—the phosphorous begins to **glow**!
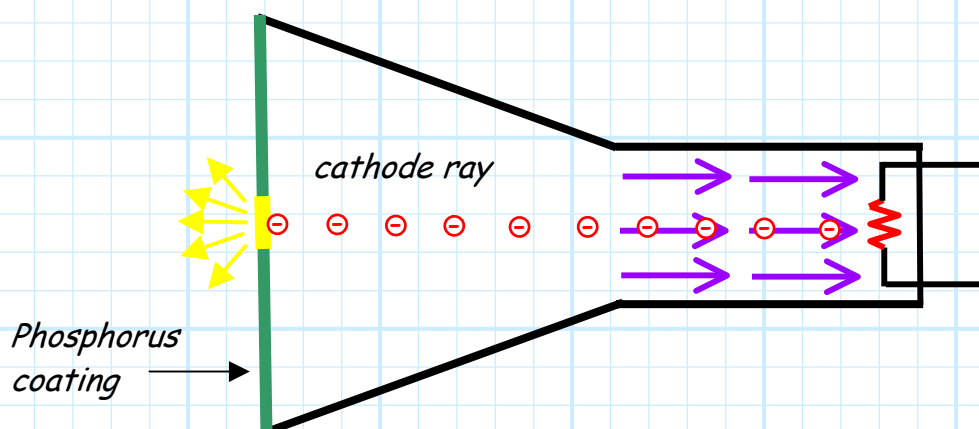
More specifically, the **spot** on the coating where the cathode ray strikes begins to glow (i.e., **fluoresces**). The **larger** the electric field, the higher the energy and thus the **brighter** the glow. Thus, we can **modulate** the light intensity by **changing** the electric field magnitude as a function of **time**.

Q: *But this electron beam would cause only **one** "pixel" to glow. How do we get the **other** pixels to emit light?*

A: This is where the **other** component of the Lorentz force Law is used!

We additionally can place a **magnetic flux density B**$(\bar{r})$ within the CRT—this will cause the cathode ray to **deflect**!

Phosphorus coating →

⊗ =magnetic flux density **B**$(\bar{r})$, pointing **into** the page.
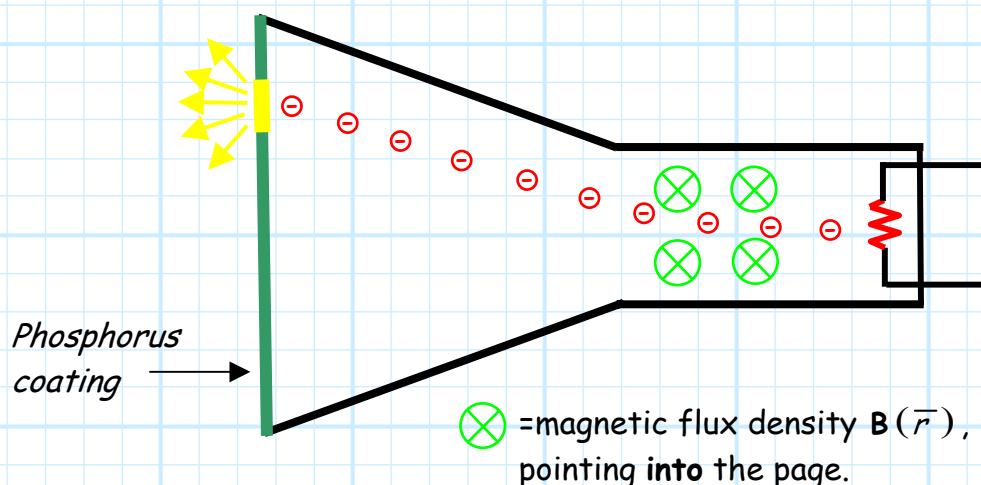
Q: *So, now we've caused a **different** pixel to glow—but how do we get all the pixels to **simultaneously** (but independently) glow? Does it require **multiple** cathode rays within the CRT?*

A: Nope! We simply **change** the magnitude and direction of $\mathbf{B}(\bar{r})$ within the CRT, and the magnitude and direction of the cathode ray **deflection** will change.

In other words, we **"scan"** the cathode ray across the screen, starting at the left end of the first "line" of pixels, and ending up at the right end of the last. As we scan, we simultaneous modulate the **electric** field, so that each pixel is the proper brightness.

The whole process is similar to how an **ink-jet printer** works (scans left to right, one line at a time)!
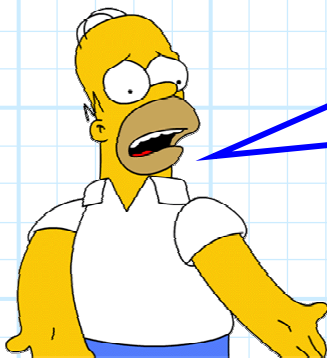
Q: *Wait a second!* ***I've*** *watched a lot of TV, and I've **never** noticed anything like that happening on my screen!*

A: There are **two** reasons that we do not **perceive** this scanning on a CRT screen. The first is that this scanning happens much too **fast** for our **feeble** minds to comprehend—30 times every second for standard TV.

The second reason is something called "persistence". When a cathode ray is removed from a spot on the phosphorous coating, it does **not** immediately turn off! It **"persists"** for a little bit, staying bright just long enough for the next cathode ray "scan" to come along.

Because of these two effects, a TV picture scan seems to **meld** into one continuous, uninterrupted 2-dimensional image!

Philo T Farnsworth (1906-1971)

On September 7th, 1927, the first successful demonstration of television occurred. A mostly self-taught genius named Philo T. Farnsworth transmitted an image of a horizontal line to a receiver in the next room. Farnsworth was born on August 19th, 1906, at Indian Creek in Beaver County, Utah. At age six he told his Dad that he'd been born an inventor. An amazing young boy, he built an electric motor at age 12 and won his first national contest at age 13. In the contest sponsored by *Science and Invention Magazine*, Farnsworth created a thief proof lock. He didn't stop there.

During the summer of 1921 (**he was 15!**), Farnsworth was leading a horsedrawn plowing machine when he stopped to look over his work. He suddenly realized that just as he was plowing the field into parallel rows, he could scan an image row by row. He figured that by doing this one line at a time, with a beam of electrons inside a cathode ray tube, he might successfully transmit an image to a receiver.   He went on to demonstrate the first operational, all electronic television system in 1927.

With an extension on his funding, it was May 1928 when Farnsworth transmitted a two-dimensional image of his wife and assistant Pem, to a receiver for viewing by an audience. Using a sketch kept by his high school chemistry teacher, Justin Tolman, he went on to win a legal suit against RCA. They claimed, to no avail, that their man, Vladimir Zworykin, had documented this concept first.

The monumental contribution made by Philo T. Farnsworth was not recognized before he died of emphysema on March 11, 1971, in Salt Lake City, Utah.

By Sarah Lane, from:
www.classbrain.com/artholiday/publish/article_168.shtml