2-5 The Calculus of Scalar and Vector Fields (pp. 33-55)

Fields are functions of coordinate variables (e.g., $x$, $\rho$, $\theta$)

Q: How can we integrate or differentiate vector fields?

A: There are many ways, we will study:

1. 4.
2. 5.
3. 6.

A. The Integration of Scalar and Vector Fields

1. The Line Integral
\[ \int_C \overrightarrow{A}(\overrightarrow{r_c}) \cdot d\overrightarrow{l} \]

Q1:

**HO:** Differential Displacement Vectors

A1:

**HO:** The Differential Displacement Vectors for Coordinate Systems

Q2:

A2: **HO:** The Line Integral

Q3:

A3: **HO:** The Contour C

**HO:** Line Integrals with Complex Contours

Q4:
A4: \{ 
HO: Steps for Analyzing Line Integrals

Example: The Line Integral

2. The Surface Integral

Another important integration is the surface integral:

$$\iint_S \mathbf{A}(\mathbf{r}_s) \cdot d\mathbf{s}$$

Q1:

A1:

HO: Differential Surface Vectors

HO: The Differential Surface Vectors for Coordinate Systems

Q2:

A2: HO: The Surface Integral
Q3:  

HO: The Surface S

A3:  

HO: Integrals with Complex Surfaces

Q4:  

HO: Steps for Analyzing Surface Integrals

A4:  

Example: The Surface Integral

3. The Volume Integral

The third important integration is the volume integral—it's the easiest of the 3!

\[ \iiint_{V} g(\vec{r}) \, dv \]

Q1:  

A1:

**HO: The Differential Volume Element**

**HO: The Volume \( V \)**

**Example: The Volume Integral**

**B. The Differentiation of Vector Fields**

1. **The Gradient**

   The Gradient of a scalar field \( g(\vec{r}) \) is expressed as:
   
   \[
   \nabla g(\vec{r})
   \]

   \[
   \nabla g(\vec{r}) = A(\vec{r})
   \]

   **Q:**

   **A: HO: The Gradient**
Q:

A: **HO: The Gradient Operator in Coordinate Systems**

Q: The gradient of every scalar field is a vector field—does this mean every vector field is the gradient of some scalar field?

A:

**HO: The Conservative Field**

**Example: Integrating the Conservative Field**

2. **Divergence**

The **Divergence** of a vector field \( \mathbf{A}(\vec{r}) \) is denoted as:

\[
\nabla \cdot \mathbf{A}(\vec{r})
\]

\[
\nabla \cdot \mathbf{A}(\vec{r}) = g(\vec{r})
\]
Q:

A:  **HO: The Divergence of a Vector Field**

Q:

A:  **HO: The Divergence Operator in Coordinate Systems**

**HO: The Divergence Theorem**

3. **Curl**

The **Curl** of a vector field \( \mathbf{A}(\mathbf{r}) \) is denoted as:

\[
\nabla \times \mathbf{A}(\mathbf{r})
\]

\[
\nabla \times \mathbf{A}(\mathbf{r}) = \mathbf{B}(\mathbf{r})
\]
Q:  
A:  **HO: The Curl**

Q:  
A:  **HO: The Curl Operator in Coordinate Systems**

**HO: Stoke's Theorem**

**HO: The Curl of a Conservative Vector Field**

4. **The Laplacian**

\[ \nabla^2 g(\vec{r}) \]
C. Helmholtz's Theorems

\[ \nabla \cdot \mathbf{A}(\vec{r}) \quad \text{and/or} \quad \nabla \times \mathbf{A}(\vec{r}) \]

Q:

A: **HO: Helmholtz's Theorems**