2-5 The Calculus of Scalar and Vector Fields (pp.33-55)

Fields are **functions** of coordinate variables (e.g., x, ρ, θ)

Q: How can we integrate or differentiate **vector** fields ??

A: There are many ways, we will study:

1.
2.
3.
4.
5.
6.

A. The *Integration* of Scalar and Vector Fields

1. *The Line Integral*

$$\int_C \mathbf{A}(\bar{r}_c) \cdot \overline{d\ell}$$

Q1:

A1: HO: Differential Displacement Vectors

HO: The Differential Displacement Vectors for Coordinate Systems

Q2:

A2: HO: The Line Ingtegral

Q3:

A3: HO: The Contour C

HO: Line Integrals with Complex Contours

Q4:

A4:

- HO: Steps for Analyzing Line Integrals
- Example: The Line Integral

2. *The Surface Integral*

Another important integration is the **surface integral**:

$$\iint_S \mathbf{A}(\bar{r}_s) \cdot \overline{ds}$$

Q1:

A1:

HO: Differential Surface Vectors

HO: The Differential Surface Vectors for Coordinate Systems

Q2:

A2: HO: The Surface Integral

Q3:

A3:
- HO: The Surface S
- HO: Integrals with Complex Surfaces

Q4:

A4:
- HO: Steps for Analyzing Surface Integrals
- Example: The Surface Integral

3. *The Volume Integral*

The third important integration is the **volume integral**—it's the easiest of the 3!

$$\iiint_V g(\bar{r})\, dv$$

Q1:

A1:

HO: The Differential Volume Element

HO: The Volume *V*

Example: The Volume Integral

B. The *Differentiation* of Vector Fields

1. The Gradient

The **Gradient** of a **scalar** field $g(\bar{r})$ is expressed as:

$$\nabla g(\bar{r})$$

$$\nabla g(\bar{r}) = \mathbf{A}(\bar{r})$$

Q:

A: HO: The Gradient

Q:

A: HO: The Gradient Operator in Coordinate Systems

Q: The gradient of every scalar field is a vector field—does this mean every vector field is the gradient of some scalar field?

A:

HO: The Conservative Field

Example: Integrating the Conservative Field

2. Divergence

The **Divergence** of a vector field $\mathbf{A}(\bar{r})$ is denoted as:

$$\nabla \cdot \mathbf{A}(\bar{r})$$

$$\nabla \cdot \mathbf{A}(\bar{r}) = g(\bar{r})$$

Q:

A: HO: The Divergence of a Vector Field

Q:

A: HO: The Divergence Operator in Coordinate Systems

HO: The Divergence Theorem

3. Curl

The **Curl** of a vector field $\mathbf{A}(\bar{r})$ is denoted as:

$$\nabla \times \mathbf{A}(\bar{r})$$

$$\nabla \times \mathbf{A}(\bar{r}) = \mathbf{B}(\bar{r})$$

Q:

A: HO: The Curl

Q:

A: HO: The Curl Operator in Coordinate Systems

HO: Stoke's Theorem

HO: The Curl of a Conservative Vector Field

4. The Laplacian

$$\nabla^2 g(\bar{r})$$

C. Helmholtz's Theorems

$\nabla \cdot \mathbf{A}(\bar{r})$ and/or $\nabla \times \mathbf{A}(\bar{r})$

Q:

A: HO: Helmholtz's Theorems