2-5 The Calculus of Scalar and Vector Fields (pp.33-55)

Fields are **functions** of coordinate variables (e.g., x, ρ , θ)

Q: How can we integrate or differentiate vector fields?

A: There are many ways, we will study:

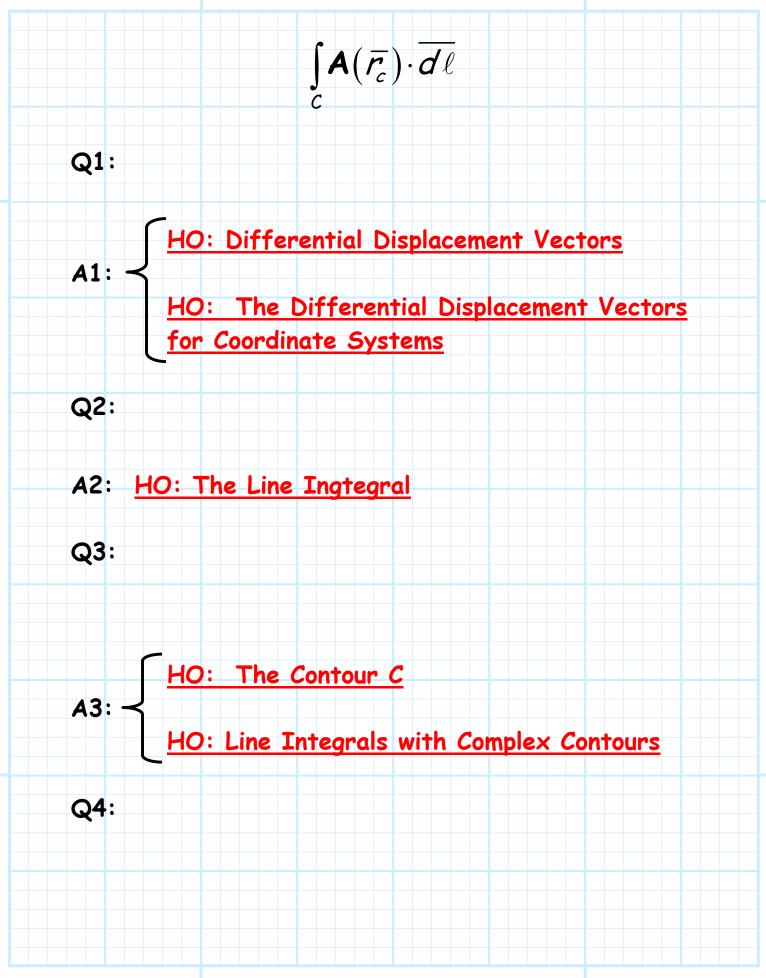
1. 4.

2. 5.

3. 6.

A. The *Integration* of Scalar and Vector Fields

1. The Line Integral



HO: Steps for Analyzing Line Integrals

Example: The Line Integral

2. The Surface Integral

Another important integration is the surface integral:

$$\iint_{S} \mathbf{A}(\bar{r}_{s}) \cdot \overline{ds}$$

Q1:

A1:

HO: Differential Surface Vectors

HO: The Differential Surface Vectors for Coordinate Systems

Q2:

A2: HO: The Surface Integral



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HO: The Surface 5

HO: Integrals with Complex Surfaces

Q4:

HO: Steps for Analyzing Surface Integrals

Example: The Surface Integral

3. The Volume Integral

The third important integration is the volume integral—it's the easiest of the 3!

$$\iiint g(\bar{r}) dv$$

Q1:

A1:

HO: The Differential Volume Element

HO: The Volume V

Example: The Volume Integral

B. The Differentiation of Vector Fields

1. The Gradient

The **Gradient** of a scalar field $g(\bar{r})$ is expressed as:

$$abla g(ar{r})$$

$$\nabla g(\bar{r}) = \mathbf{A}(\bar{r})$$

Q:

A: HO: The Gradient

Q:

A: HO: The Gradient Operator in Coordinate

Systems

Q: The gradient of every scalar field is a vector field—does this mean every vector field is the gradient of some scalar field?

A:

HO: The Conservative Field

Example: Integrating the Conservative Field

2. Divergence

The **Divergence** of a vector field $\mathbf{A}(\bar{r})$ is denoted as:

$$abla \cdot oldsymbol{A}(ar{r})$$

$$abla \cdot \mathbf{A}(ar{r}) = \mathbf{g}(ar{r})$$

Q:

A: HO: The Divergence of a Vector Field

Q:

A: HO: The Divergence Operator in Coordinate

Systems

HO: The Divergence Theorem

3. Curl

The **Curl** of a vector field $\mathbf{A}(\bar{r})$ is denoted as:

$$abla imes \mathbf{A}(ar{r})$$

$$\nabla \times \mathbf{A}(\bar{r}) = \mathbf{B}(\bar{r})$$

Q:

A: HO: The Curl

Q:

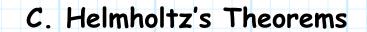
A: HO: The Curl Operator in Coordinate Systems

HO: Stoke's Theorem

HO: The Curl of a Conservative Vector Field

4. The Laplacian

 $abla^2 g(ar{r})$



 $\nabla \cdot \mathbf{A}(\bar{r})$ and/or $\nabla \times \mathbf{A}(\bar{r})$

Q:

A: HO: Helmholtz's Theorems

Dept. of EECS The Univ. of Kansas Jim Stiles