3-2 Charge and Charge Density

Reading Assignment: *pp. 61-63*

All electric phenomena can be attributed to **electric charge.**

HO: Electric Charge

Q:

A: HO: Charge Density

Q:

A: HO: Total Charge

Electric Charge

Most of classical physics can be described in terms of three fundamental units, which define our physical "reality".

1. Mass (e.g., *kg*) 

2. Distance (e.g., *meters*) 

3. Time (e.g., *seconds*) 

From these fundamental units, we can define other important physical parameters. For example, **energy** can always be described in units of $kg\ m^2/s^2$.

But, these three fundamental units alone are insufficient for describing all of classic physics—we require one more to completely describe physical reality!

This fourth fundamental unit is **Coulomb**, the unit of **electric charge**.

> All **electromagnetic** phenomena can be attributed to electric charge!

We shall find that electric charge is **somewhat** analogous to mass. However, one important difference between mass and charge is that charge can be either **positive** or **negative**!

Essentially, charge (like mass) is a property of **atomic particles**. Specifically, we find that:

The charge "on" a **proton** is $+1.6 \times 10^{-19}\ C$

The charge "on" a **neutron** is $0.0\ C$

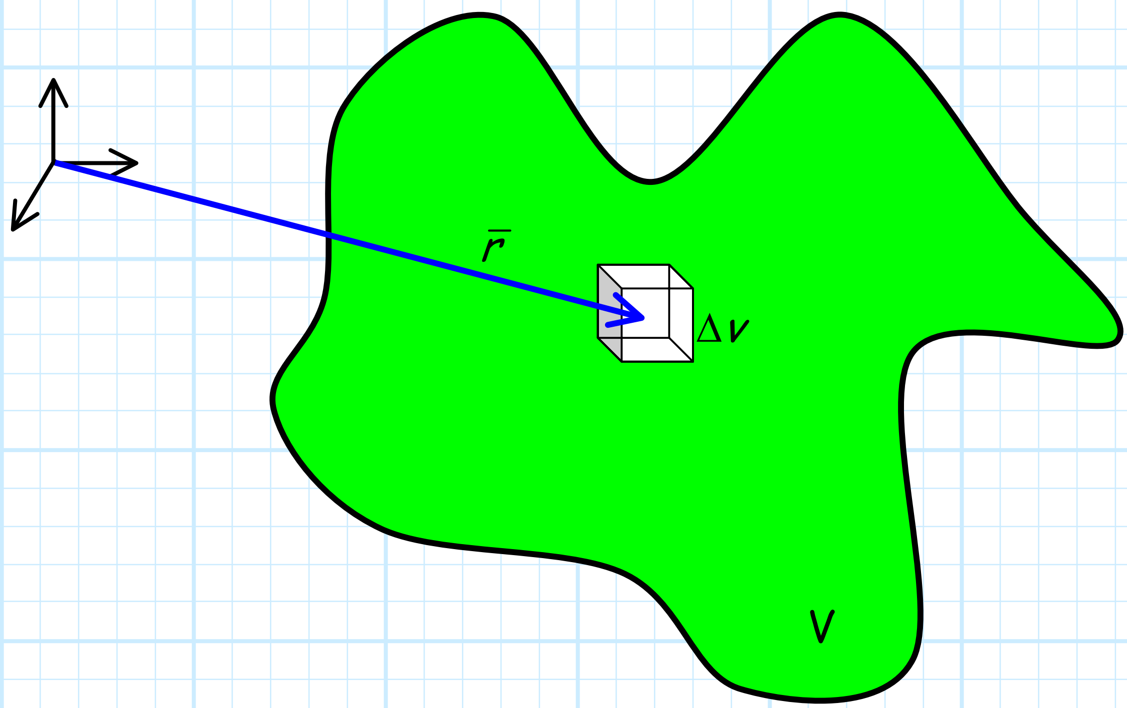
The charge "on" an **electron** is $-1.6 \times 10^{-19}\ C$

Charged particles (of all types) can be **distributed** (unevenly) across a volume, surface, or contour.

Charge Density

In many cases, charged particles (e.g., electrons, protons, positive ions) are **unevenly distributed** throughout some volume V.

We define **volume charge density** at a specific point $\bar{r}$ by evaluating the total net charge ΔQ in a small volume Δv surrounding the point.



$$\text{Volume Charge Density} = \rho_v(\bar{r}) \doteq \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v}$$

Volume charge density is therefore a **scalar field**, and is expressed with units such as **coulombs/m³**.

IMPORTANT NOTE: Volume charge density indicates the **net** charge density at each point $\bar{r}$ within volume V.

Q: *What exactly do you mean by **net** charge density ?*

A: Remember, there are positively charged particles and there are negatively charged particles, and **both** can exist at the same location $\bar{r}$.

Thus, a **positive** charge density does **not** mean that **no** negatively charged particles (e.g., electrons) are present, it simply means that there is **more** positive charge than there is negative!

It might be more instructive to define:

$$\Delta Q = \Delta Q^{+} + \Delta Q^{-}$$

where ΔQ^{+} is the amount of **positive** charge (therefore a **positive number**) and ΔQ^{-} is the amount of **negative** charge (therefore a **negative number**). We can call ΔQ the net, or **total charge**.

Volume charge density can therefore be expressed as:

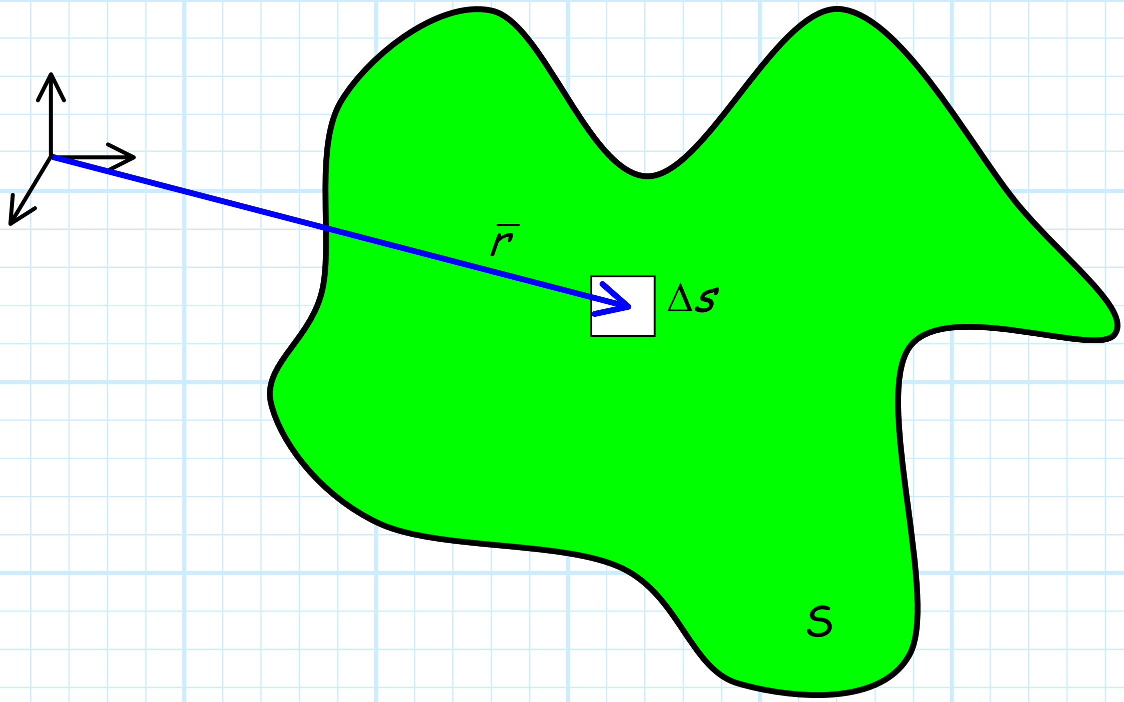
$$\rho_v(\bar{r}) \doteq \lim_{\Delta v \to 0} \frac{\Delta Q^+ + \Delta Q^-}{\Delta v} = \rho_v^+(\bar{r}) + \rho_v^-(\bar{r})$$

For example, the charge density at some location $\bar{r}$ due to negatively charged particles might be –10.0 C/m^3, while that of positively charged particles might be –5 C/m^3. Therefore, the net, or **total** charge density is:

$$\rho_v(\bar{r}) = \rho_v^+(\bar{r}) + \rho_v^-(\bar{r}) = 5 + (-10) = -5 \quad C/m^3$$

Surface Charge Density

Another possibility is that charge is unevenly distributed across some surface S. In this case, we can define a **surface charge density** as by evaluating the total charge ΔQ on a small patch of surface Δs, located at point $\bar{r}$ on surface S:



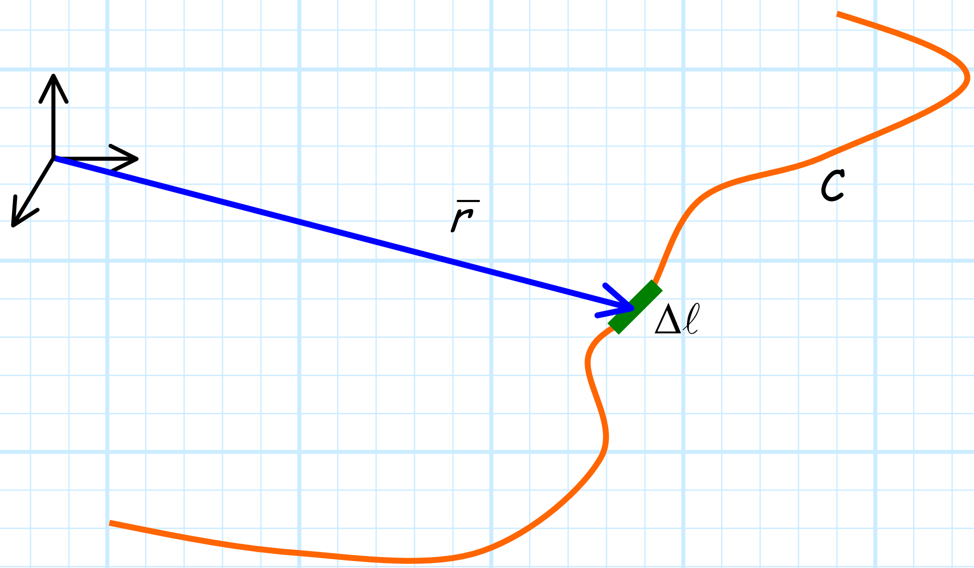
Surface charge density $\rho_s(\bar{r})$ is therefore defined as:

$$\rho_s(\bar{r}) \doteq \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta s}$$

Note the **units** for surface charge density will be **charge/area** (e.g. **C/m^2**).

Line Charge Density

Finally, we also consider the case where charge is unevenly distributed across some **contour** C. We can therefore define a **line charge density** as the charge ΔQ along a small distance $\Delta \ell$, located at point $\bar{r}$ of contour C.



We therefore define line charge density $\rho_\ell(\bar{r})$ as:

$$\rho_\ell(\bar{r}) \doteq \lim_{\Delta\ell \to 0} \frac{\Delta Q}{\Delta \ell}$$

As you might expect, the units of a line charge density is charge per length (e.g., **C/m**).

Total Charge

Q: *If we know charge density $\rho_v(\bar{r})$, describing the charge distribution throughout a* ***volume*** *V, can we determine the* ***total charge*** *Q contained within this volume?*

A: You betcha! Simply **integrate** the charge density over the entire volume, and you get the **total charge** Q contained within the volume.

In other words:

$$Q = \iiint\limits_V \rho_v(\bar{r})\, dv$$

Note this is a **volume integral** of the type we studied in Section 2-5. Therefore select the differential volume dv that is appropriate for the volume V.

Likewise, we can determine the total charge distributed across a **surface** S by integrating the surface charge density:

$$Q = \iint_S \rho_s(\bar{r})\, ds$$

Q: *Hey! This is **NOT** the surface integral we studied in Section 2-5.*

A: True! This is a **scalar** integral; sort of a two-dimensional version of the volume integral.

The differential surface element *ds* in this integral is simply the **magnitude** of the differential surface vectors we studied earlier:

$$ds = \left|\overline{ds}\right|$$

For example, if we integrate over the surface of a sphere, we would use the differential surface element:

$$ds = \left|\overline{ds}_r\right| = r^2 \sin\theta \; d\theta \, d\phi$$

Finally, we can determine the total charge on **contour** C by integrating the **line charge density** $\rho_\ell(\bar{r})$ across the entire contour:

$$Q = \int_C \rho_\ell(\bar{r})\, d\ell$$

The differential element $d\ell$ is likewise related to the differential displacement vector we studied earlier:

$$d\ell = \left|\overline{d\ell}\right|$$

For example, if the contour is a circle around the *z*-axis, then $d\ell$ is:

$$d\ell = \left|\overline{d\phi}\right| = \rho\, d\phi$$