C. Noise in Microwave Systems

Bad News: Even if we **completely** reject the image and all spurious signals, there will still be an unwanted signal that will **always** appear at the detector/demodulator.

→ NOISE

Q: *What **is** noise, and where does it come from?*

A: HO: Receiver Noise

Q: *So how do we **quantify** noise?*

A: HO: The Statistics of Noise

Q: *How much **external** noise do we typically see?*

A: HO: Antenna Noise Temperature

Q: *What about **internal** noise; how much noise is generated by a microwave **component** in our receiver?*

A: HO: Equivalent Noise Temperature

Another way to specify the noise performance of a microwave component is by its **Noise Figure**.

HO: Noise Figure and SNR

Q: *What about **passive** devices; do they generate noise? What is their noise figure?*

A: HO: Noise Figure of Passive Devices

A microwave system (e.g., a receiver) is made of **many components**. We can (and must!) determine the overall system noise figure and/or equivalent noise temperature for an **entire** system.

HO: System Equivalent Noise Temperature

HO: System Noise Figure

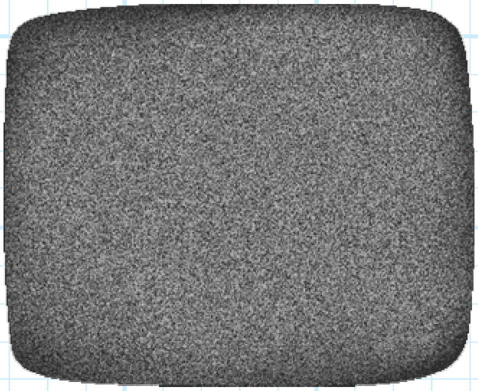
Receiver Noise

Q: *Say we tune our receiver to a frequency at which **no** signal is present. Does this mean that the output of the detector/demodulator will be **zero**?*

A: Nope! Unfortunately, **even** if we completely reject all spurious signals, there will **always** be one "unwanted" signal that reaches the demodulator/ detector.

→ This unwanted signal is called **noise**.

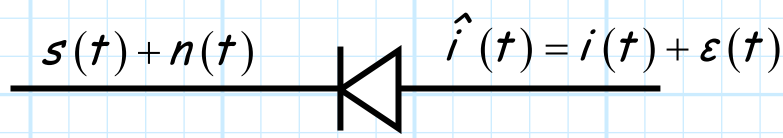
Noise is a completely **random** signal, and when it reaches the demodulator the result is a completely **random** demodulator output. This provides the familiar **"hiss"** you might here if you tune your **radio** to a frequency where no station exists, or the **"snow"** you see on your **television** if you similarly select a channel where no station exists!



Q: *Big deal! I'd never tune to a frequency where there is **no** signal. Is this "noise" really a problem?*

A: A **big** problem! Note that we said that noise will **always** be present at the detector/demodulator—there is **no way** to completely get rid of it.

As a result, the best we can hope for (if we completely suppress all spurious signals) is that only the desired signal $s(t)$ **and** noise $n(t)$ will reach the demodulator/detector.



This noise will then cause an **error** $\varepsilon(t)$ in the demodulated signal $\hat{i}(t)$!

Q: *Yikes! How* ***large*** *will this error be?*

A: It **depends** many things (e.g., modulation type, signal bandwidth, signal power), but most fundamentally it depends on the **Signal-to-Noise Ratio** (SNR) at the demodulator input. This ratio is simply the power associated with the **signal** (P_s), to the power associated with the **noise** (P_n):

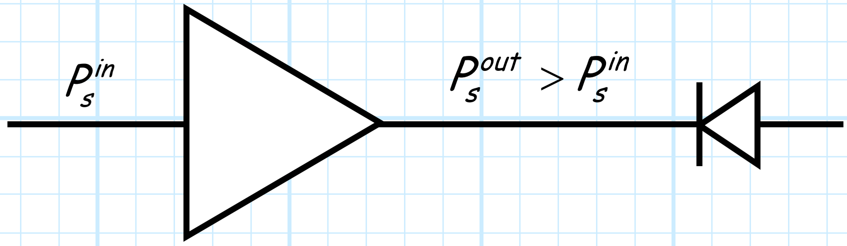
$$SNR = \frac{P_s}{P_n} \qquad\qquad SNR(dB) = P_s(dBm) - P_n(dBm)$$

Thus, if there is a lot of signal power, and just a little noise power, the SNR will be large. If the converse is true, the SNR will be small.

Then—as you might expect—the demodulator **error** $\varepsilon(t)$ **diminishes** as **SNR increases**. Thus:

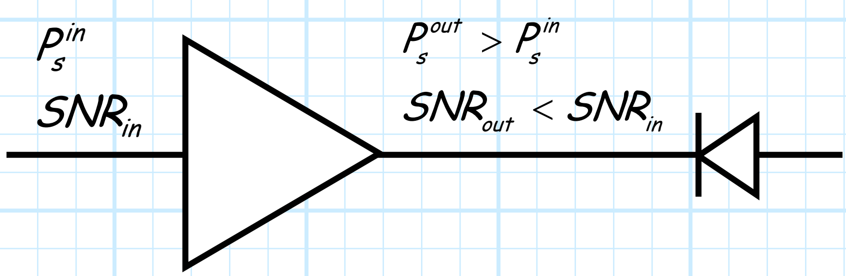
The **SNR** at the demodulator input must be **sufficiently large** in order for demodulator **error** to be **acceptably small**.

Q: *No problem! If we place enough **amplifiers** in front the demodulator/detector, then we can always make P_s **really large**—right?*



A: Yes—**but**—there's one **big** catch. Although amplifiers of course increase **signal** power P_s, they increase the **noise** power P_n **even more**!

Thus, we will find that amplifiers actually **decrease SNR**!



This is a tremendous **challenge** for radio engineers and receiver designers. We must:

1. **Increase** the **signal** power (by amplification), such that the signal is large enough to be detected/demodulated.

2. But make sure that the SNR is **not** degraded to the extent that the demodulation error is **unacceptable**.

Q: *From **where** does noise originate?*

A: Two sources: one **external** and one **internal**!

External Noise

* External noise is **coupled** into the receiver through the receiver **antenna**. It turns out that the **entire** electromagnetic spectrum is awash in **random energy** (i.e., noise).

* This random energy has neither a specific **frequency**, nor **direction**, but instead is spread across **all** directions and **all** frequencies!

* As a result, we can point our antenna in **any direction**, and we can tune our receiver to **any frequency**, but we will **always** receive a portion of the electromagnetic noise!

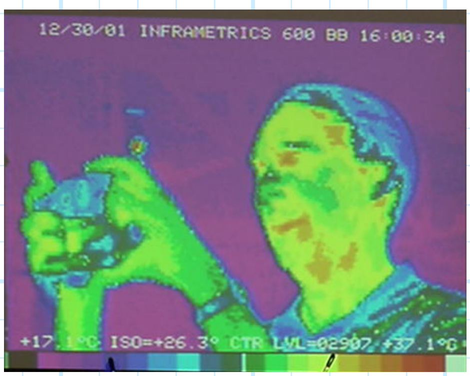
Q: *What is the **source** of this **external** noise?*

A: There are **three** sources: terrestrial, extra terrestrial and human-made.

Terrestrial Noise - Every **warm** object radiates electromagnetic energy (its one method of heat transfer)!

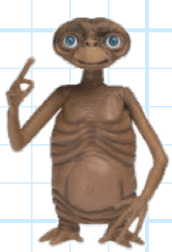
Definition of warm → Anything with a temperature **above** absolute **zero** (i.e., $> 0\ K^{\circ}$).

* The frequency spectrum of this emitted electromagnetic noise depends on the **temperature** of the object.

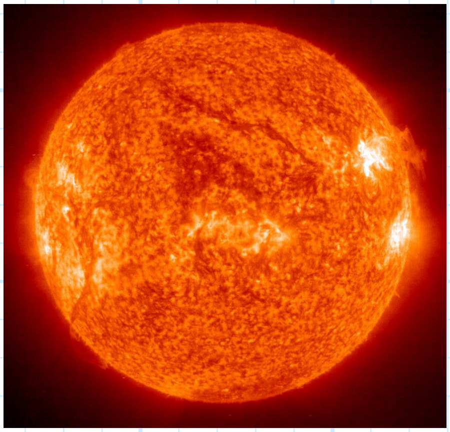


* For objects on the **Earth** (i.e., terrestrial objects), the temperature is such that the emitted energy **peaks** in the **infrared** region.

* However, terrestrial objects emit random energy across the **entire** electromagnetic spectrum—including the **RF** and **microwave** regions!



Extra-Terrestrial – There are also very warm objects in **outer space**! Among these objects are of course stars and planets, but the most significant of these extra-terrestrial objects is our very own star—the **Sun**!



The Sun—as you **may** already know—is really **hot**. As a result it radiates electromagnetic noise at an **astonishing** rate! Some of this noise is unfortunately radiated in the **RF/microwave** end of the e.m. spectrum (a great **annoyance** to us radio engineers), but the noise power radiated by the Sun **peaks** in the **"visible region"** of the electromagnetic spectrum.

Q: *Wait a second! Our **eyes** detect electromagnetic energy in the **visible** region of the e.m. spectrum. Why haven't I ever **noticed** this noise?*

A: You have! What our eyes "see" **is** this noise—**sunlight** is in fact extra-terrestrial **electromagnetic noise** produced by a very hot Sun.

Human-Made Noise – We **humans** generate a heck of a lot of random noise (both electromagnetic and otherwise)! We have built literally millions of transmitters, and each of these radiate noise that was **internally** generated!

Internal Noise

Any warm object that efficiently **absorbs** electromagnetic energy must likewise **emit** electromagnetic energy (in the form of noise).

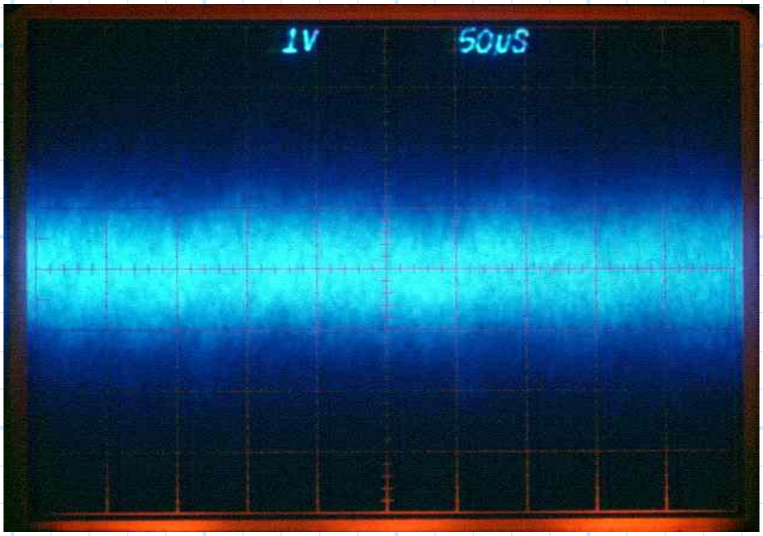
→ These objects include **resistors** and **semiconductors!**

Q: *Many of the components in our **receiver** have resistors and semiconductors; does this mean that **they** produce noise?*

A: Absolutely! This is a **major** headache for radio engineers. Not only do we end up amplifying the **external** noise coupled into the receiver through the antenna, but the receiver itself adds to this random signal by the noise it **internally** generates!

The Statistics of Thermal Noise

Noise is a completely **random** signal. It **cannot** be described deterministically, but it can be described **statistically**.



For example, consider the **frequency spectrum** of a noise process $v_n(t)$:

$$S_n(f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} v_n(t)\, e^{-j2\pi f t}\, dt$$

From this we can determine the **spectral power density** of this noise:

$$N(f) = \left| S_n(f) \right|^2 \qquad W/Hz$$

This function describes how the **energy** of a signal is **distributed** across the **frequency spectrum**. Since noise is a random function (i.e., a random process), its spectral power density is likewise **random**.

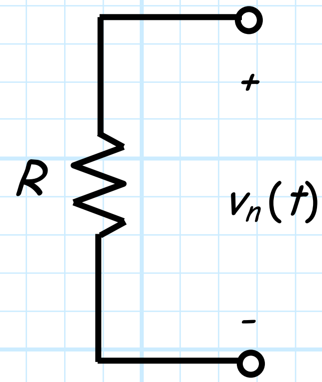
Thus, $N(f)$ likewise cannot be described **deterministically**, but it **can** be described statistically. In other words, we cannot state specifically how the noise energy is distributed across the frequency spectrum, but we can describe how it is distributed—on **average**!

The function $N(f)$ therefore is defined as the **average spectral power density** of noise.

Now, let's consider the average spectral power density of a **resistor** R at **temperature** T.

Q: *How could a resistor produce a noise signal $v_n(t)$? Isn't a resistor a **passive** device that produces **no** power?*

A: That's not quite true!



Since the resistor is a **warm** object, the free electrons within the device will be moving (due to **thermal** energy) in a random way. This creates a tiny **electric field** within the device, which in turn creates a tiny **voltage** across this resistor.

This voltage is the resistor noise voltage $v_n(t)$. We call this phenomenon **thermal noise**.

Q: *What **is** the average spectral power density $N(f)$ of this **thermal noise?***

A: Using a **bunch** quantum physics, we find that the thermal noise produced by a resistor is:

$$N(f) = kT \doteq N_0 \qquad W/Hz$$

where:

T is the temperature of the resistor in degrees **Kelvin.**
and

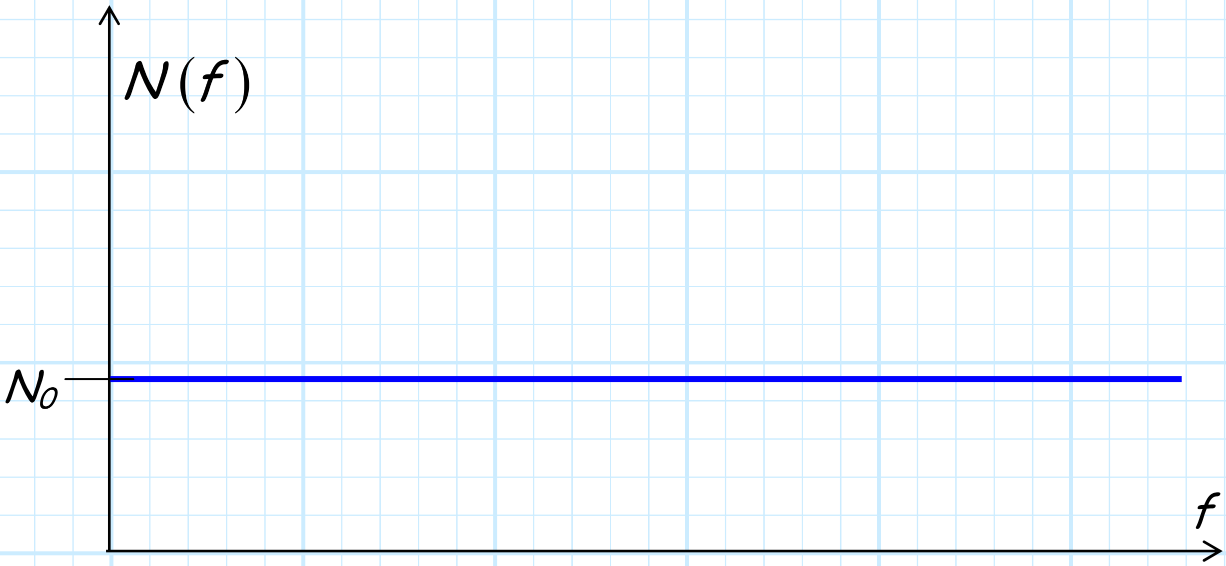
$$k = \text{Boltzman's Constant}$$
$$= 1.38 \times 10^{-23} \quad J/K^{\circ}$$

Since k is a constant, engineers often specify temperature T instead of N_0, they call this temperature the **noise temperature**:

$$T = \frac{N_0}{k}$$

Q: *Wait! This function $N(f)$ seems to be **independent of frequency** f !?!*

A: That's correct! The average spectral power density of thermal noise is theoretically a **constant** with respect to **frequency**.



In other words, the noise power is (on average) **distributed uniformly** across the frequency spectrum—no frequency will have any more or less (on average) than any other frequency.

→ Noise of this type is called **white noise.**

Q: *And this function $N(f)$ is also independent of the value of **resistance** R !?!*

A: That is again correct! The noise that a resistor produces does **not** depend on its resistance, it depends **only** on its temperature. However, this "resistor" **cannot** have the values $R = 0$ or $R = \infty$—it **must** be able to absorb power.

Q: *So $N(f) = N_0$ is the average spectral power density of the thermal noise. What simply is the **total power** P_n of the thermal noise?*

A: We can determine the total power from the average spectral power density by **integrating** the power density over all frequency:

$$P_n = \int_0^{\infty} N(f)\, df$$

Q: *Yikes! If we integrate $N(f) = N_0$ over all frequency, we get* ***infinite*** *power!*

$$P_n = \int_0^{\infty} N_0\, df = \infty \quad ??? \quad \text{(Is the energy crisis solved?)}$$

A: The **reality** is, as frequency gets **extremely** large, we find that the average spectral power density will **diminish** to zero.

$$\lim_{f \to \infty} N(f) = 0$$

In other words, the result:

$$N(f) = kT$$

is an **approximation** that is valid in the **RF/microwave region** of the electromagnetic spectrum.

Therefore:

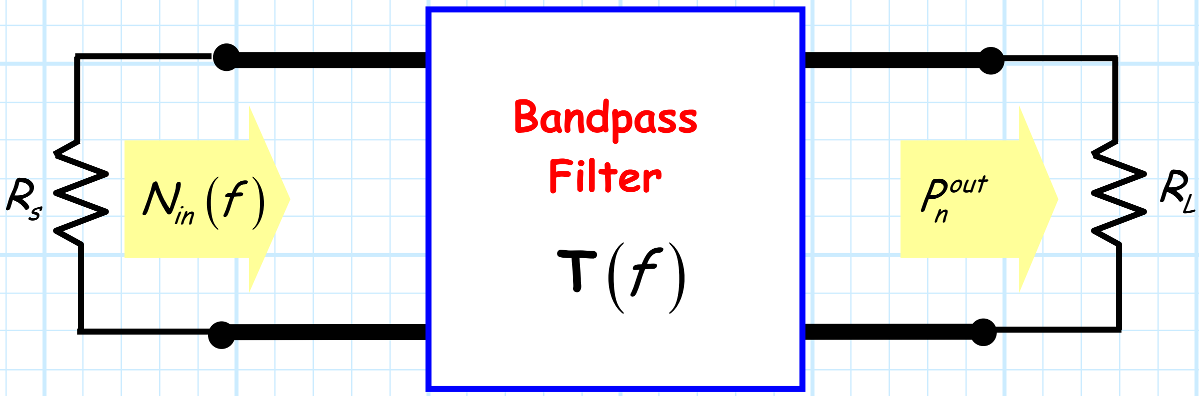
$$P_n = \int_0^{\infty} N(f)\, df < \infty$$

Q: *Still, wouldn't the resulting value of P_n* ***still*** *be quite large?*

A: Mathematically speaking, yes.

But remember, resistors reside in circuits with **reactive** elements. As a result, every microwave device has a **finite bandwidth**. This finite bandwidth will **limit** the amount of noise power that passes through the receiver to the demodulator.

For example, consider the case where a **bandpass filter** is connected between two resistors:



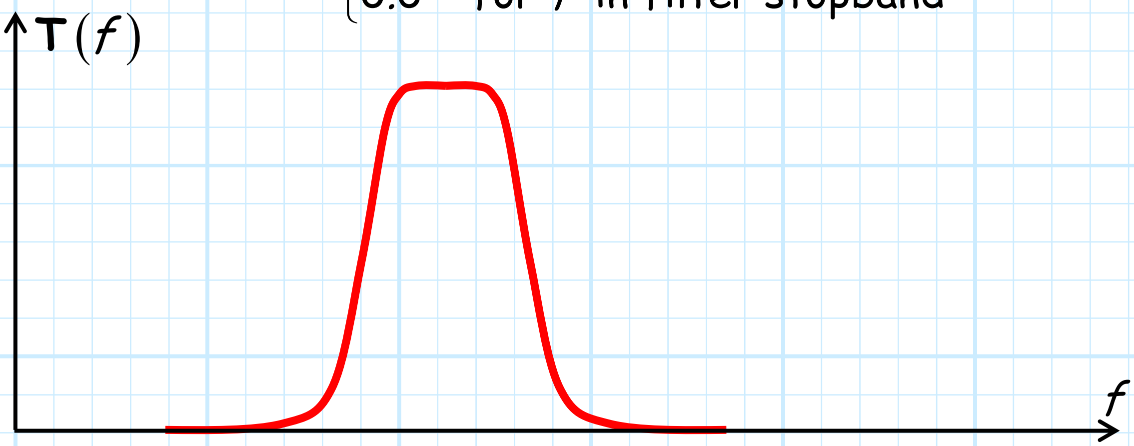
We'll consider the resistor on the **left** to be **source** of thermal noise, with average spectral power density:

$$N_{in}(f) = N_0$$

While the resistor on the **right** is a **load** that absorbs noise power P_n^{out}.

The bandpass filter has a power **transmission** coefficient $\mathbf{T}(f)$, where:

$$\mathbf{T}(f) \approx \begin{cases} 1.0 & \text{for } f \text{ in filter passband} \\ 0.0 & \text{for } f \text{ in filter stopband} \end{cases}$$

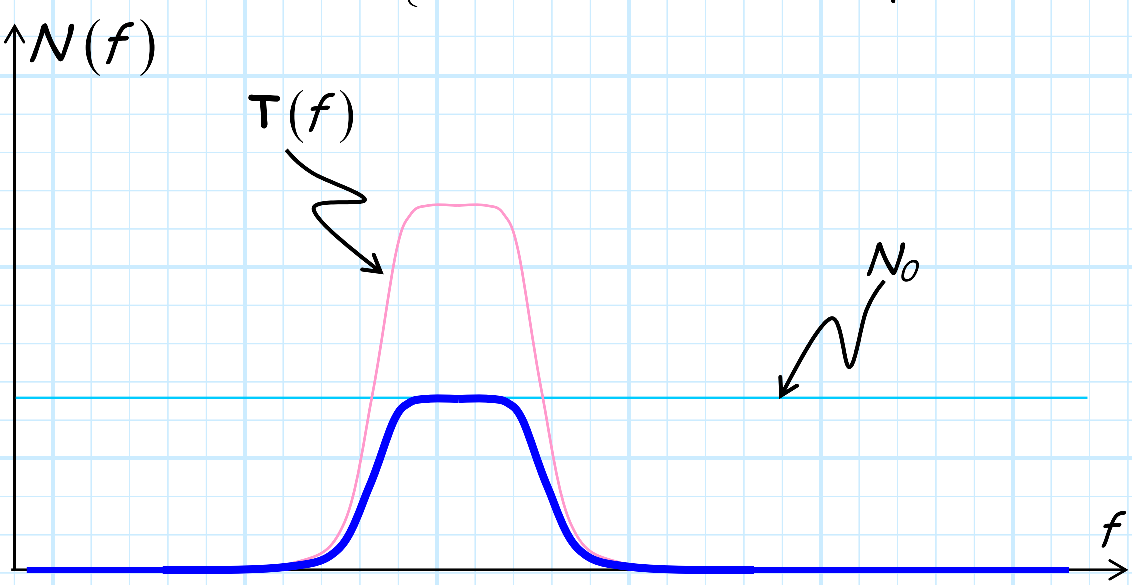


Thus, the average spectral power density of the noise at the **load** is:

$$N_{out}(f) = N_{in}(f)\,\mathbf{T}(f) = N_0\,\mathbf{T}(f)$$

And we conclude:

$$N_{out}(f) \approx \begin{cases} N_0 & \text{for } f \text{ in filter passband} \\ 0.0 & \text{for } f \text{ in filter stopband} \end{cases}$$



Now, if the filter has a **passband** that extends from frequency f_1 to frequency f_2 (i.e., bandwidth $B = f_2 - f_1$), the **total power** of the noise at the **load** is:

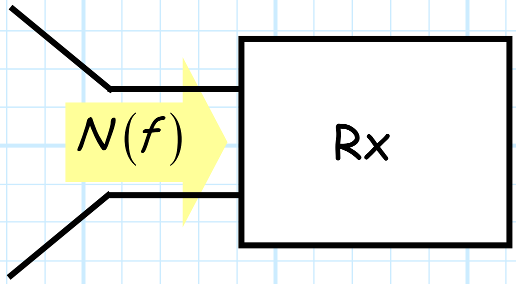
$$\begin{aligned} P_n^{out} &= \int_0^{\infty} N_0 \mathbf{T}(f)\, df \\ &= N_0 \int_0^{\infty} \mathbf{T}(f)\, df \\ &\simeq N_0 \int_{f_1}^{f_2} 1.0\, df \\ &= N_0 B \end{aligned}$$

Thus, we can conclude that the **thermal noise** produced by some resistor R at temperature T, when constrained to some finite bandwidth B (and it **always** is constrained in this way!), has **total power**:

$$P_n = kT\,B = N_0\,B$$

Antenna Noise Temperature

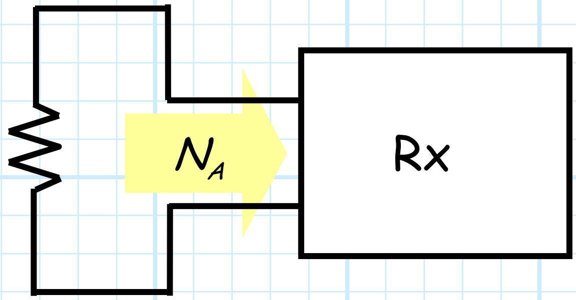
Q: *So, **external** noise will be coupled into our receiver through the receiver **antenna**. What will the average spectral power density of this noise be?*



A: Generally speaking, it will be **white noise**!

In other words, the average spectral power density of the externally generated noise will be **constant** with respect to frequency (or at least, constant across the **antenna bandwidth**).

Thus, as far as noise (and only noise) is concerned, the receiver appears to have a **warm resistor** attached to its input!



We **can** specify this input noise in terms of average spectral power density N_A, but we **typically** define it in terms of its **noise temperature**:

$$T_A \doteq \frac{N_A}{k} \qquad K^\circ$$

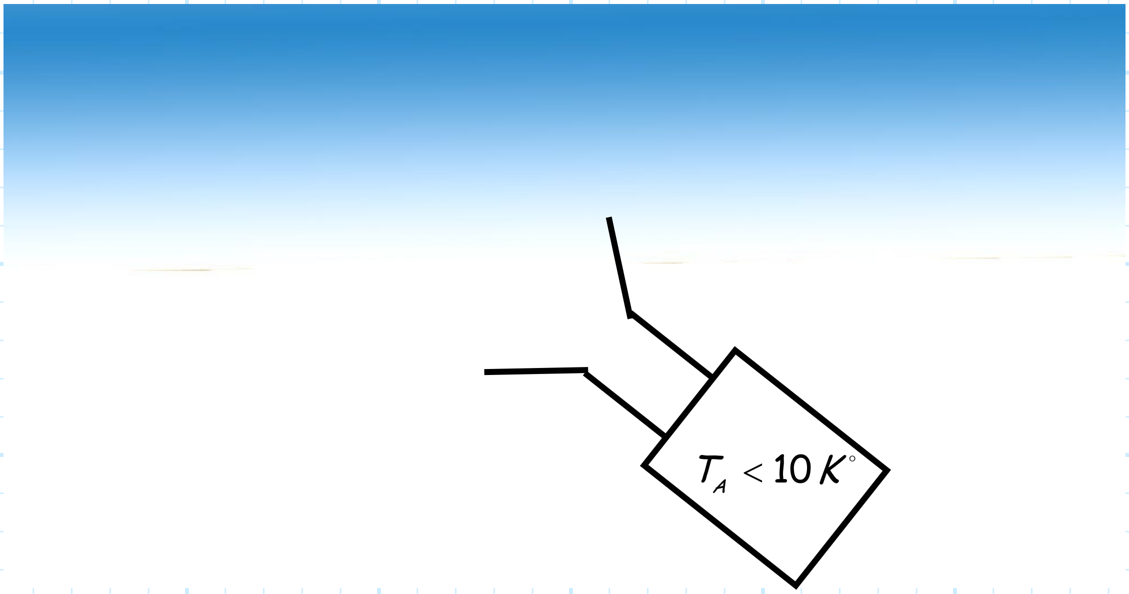
We call T_A the **antenna noise temperature**—it's the **apparent** temperature of the warm resistor "attached" to the receiver input.

Thus, we can write:

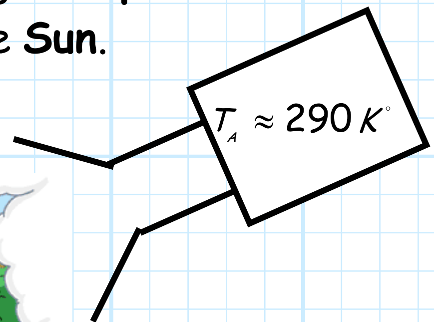
$$N_A = k\,T_A$$

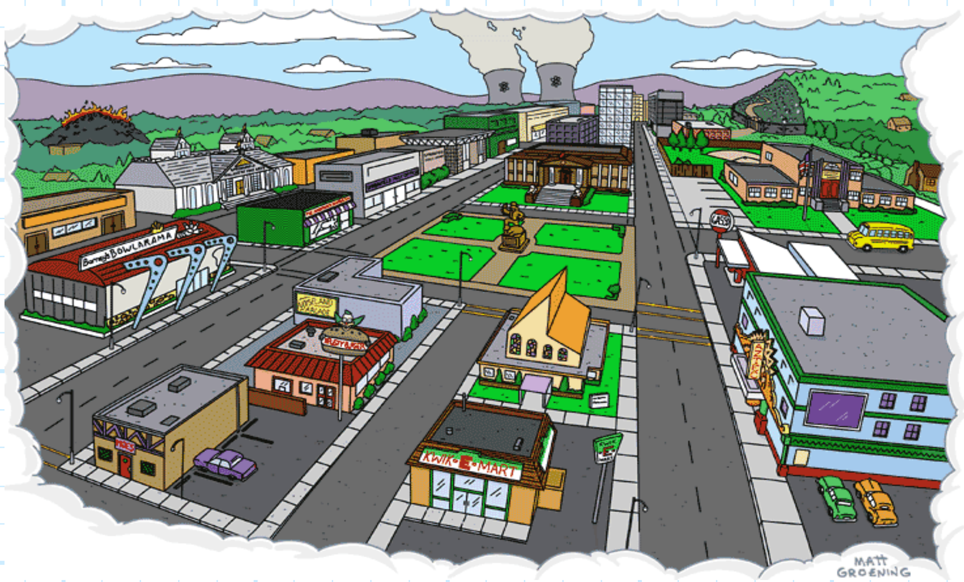
Q: *Is there some **typical** value of antenna temperature?*

A: It depends on which direction the antenna is **pointing**!



* If the antenna is pointed toward the **sky** (e.g., satellite communications), the antenna noise temperature could be very **low**, on the order of 10 K° or less. The one big **exception** to this occurs when you point your antenna at the **Sun**.





* If the antenna is **not** pointed at the sky, then most of the external noise will be generated by **terrestrial** sources. It turns out that the antenna noise temperature in this case is simply equal to the **physical temperature** at the Earth's surface!

Of course, this temperature changes somewhat, but expressed in degrees **Kelvin** (i.e., with respect to **absolute** zero) this change is **small**.

Thus, radio engineers **typically** assume an antenna noise temperature of a **standard** value of $T_o = 290\ K^\circ$.

$$T_o \doteq 290\ K^\circ$$

This is approximately "room temperature" on Earth.

Thus, the average spectral power density of the noise entering a receiver is **typically** (**if** $T_A = T_o$!) assumed to be:

$$\begin{aligned} N_A &= kT_o \\ &= \left(1.38 \times 10^{-23}\right) 290 \\ &= 4.00 \times 10^{-21} \quad W/Hz \\ &= 4.00 \times 10^{-24} \quad mW/Hz \end{aligned}$$

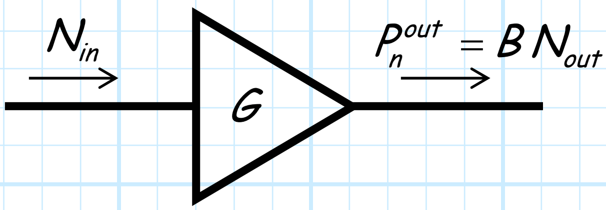
Often, this value is expressed in terms of "dBm/Hz" (←yuck!) as:

$$N_A\left(dBm / Hz\right) = -174.0$$

Equivalent Noise Temperature

In addition to the **external** noise coupled into the receiver through the antenna, each **component** of a receiver generates its own **internal** noise!

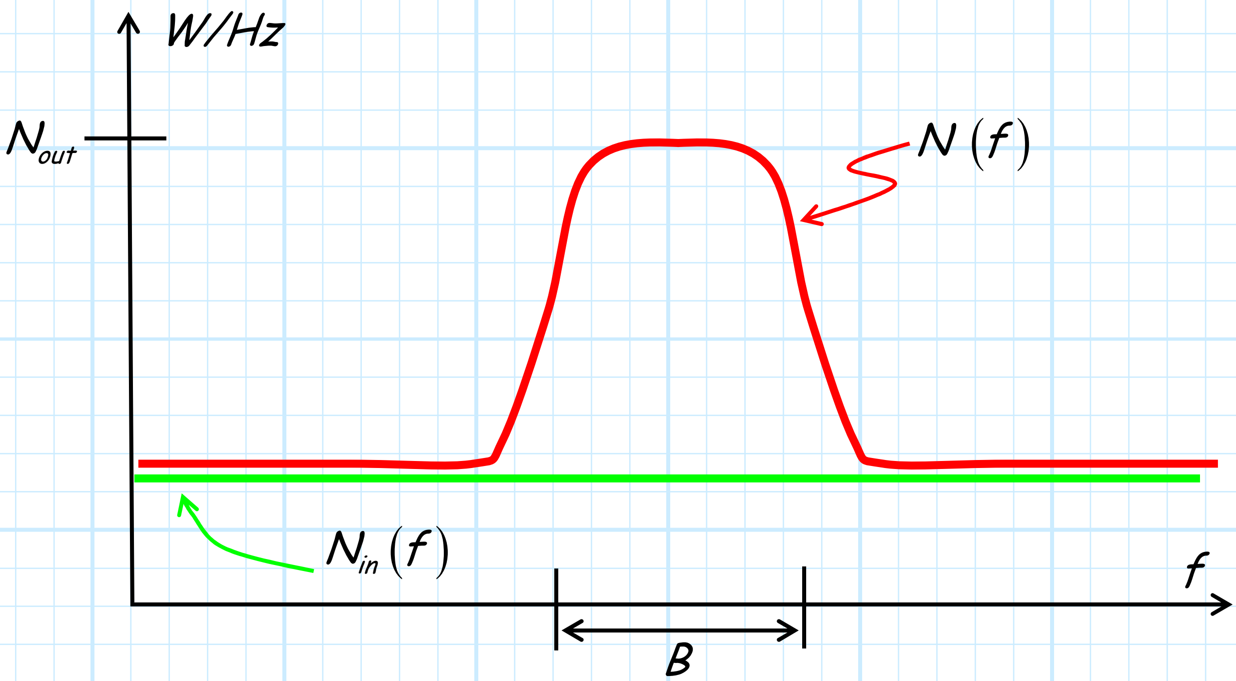
For example, consider an **amplifier** with gain G and bandwidth B:



Here there is no input signal at the amplifier input, other than some **white** (i.e., uniform across the RF and microwave spectrum) **noise** with average spectral power density N_{in}. At the **output** of the amplifier is likewise noise, with an average spectral power density of N_{out}.

This **output** average spectral power density N_{out} is typically **not** wideband, but instead is uniform only over the **bandwidth** of the amplifier:

$$N(f) \approx \begin{cases} N_{out} & \text{for } f \text{ in bandwidth } B \\ \ll N_{out} & \text{for } f \text{ outside bandwidth } B \end{cases}$$



Thus, the **noise power** at the output is:

$$\begin{aligned} P_n^{out} &= \int_0^\infty N(f)\, df \\ &\cong \int_{f_1}^{f_2} N_{out}\, df \\ &= B\, N_{out} \end{aligned}$$

Q: *The amplifier has **gain** G. So isn't* $N_{out} = G\, N_{in}$, *and thus* $P_n^{out} = G\, B\, N_{in}$ *??*

A: NO!! This is **NOT** correct!

We will find that the output noise is typically **far greater** than that provided by the amplifier gain:

$$N_{out} \gg G\, N_{in}$$

Q: *Yikes! Does an amplifier **somehow** amplify noise **more** than it amplifies **other** input signals?*

A: Actually, the amplifier **cannot** tell the difference between input noise and any other input signal. It **does** amplify the input noise, increasing its magnitude by gain G.

Q: *But you **just** said that $N_{out} \gg G\,N_{in}$!?!*

A: This **is** true! The reason that $N_{out} \gg G\,N_{in}$ is because the amplifier additionally **generates** and **outputs** its own noise signal! This **internally** generated amplifier noise has an average spectral power density (at the **output**) of N_n.

Thus, the output noise N_{out} consists of **two** parts: the **first** is the noise at the **input** that is amplified by a factor G (i.e., $G\,N_{in}$), and the **second** is the noise generated **internally** by the amplifier (i.e., N_n).

Since these two noise sources are **independent**, the average spectral power density at the output is simply the **sum** of each of the two components:

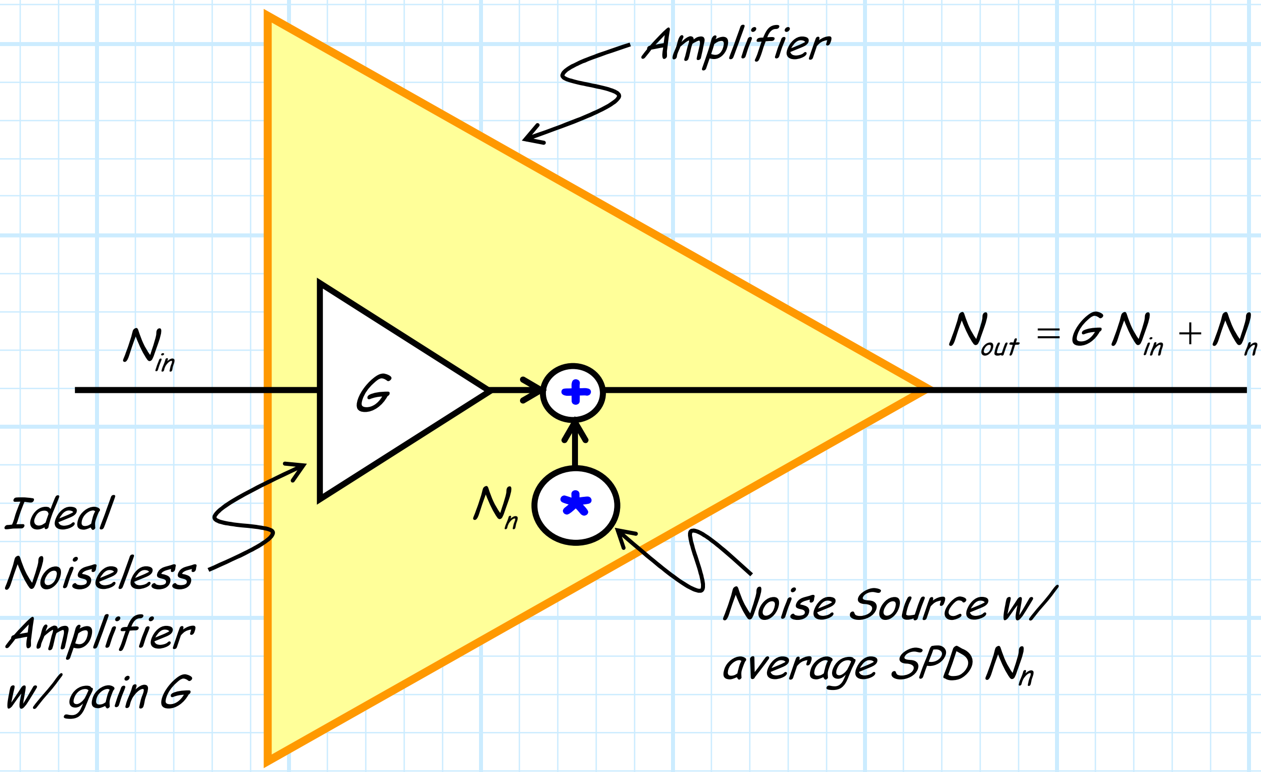
$$N_{out} = G\,N_{in} + N_n$$

Q: *So does this noise generated **internally** in the amplifier actually get **amplified** (with a gain G) or not?*

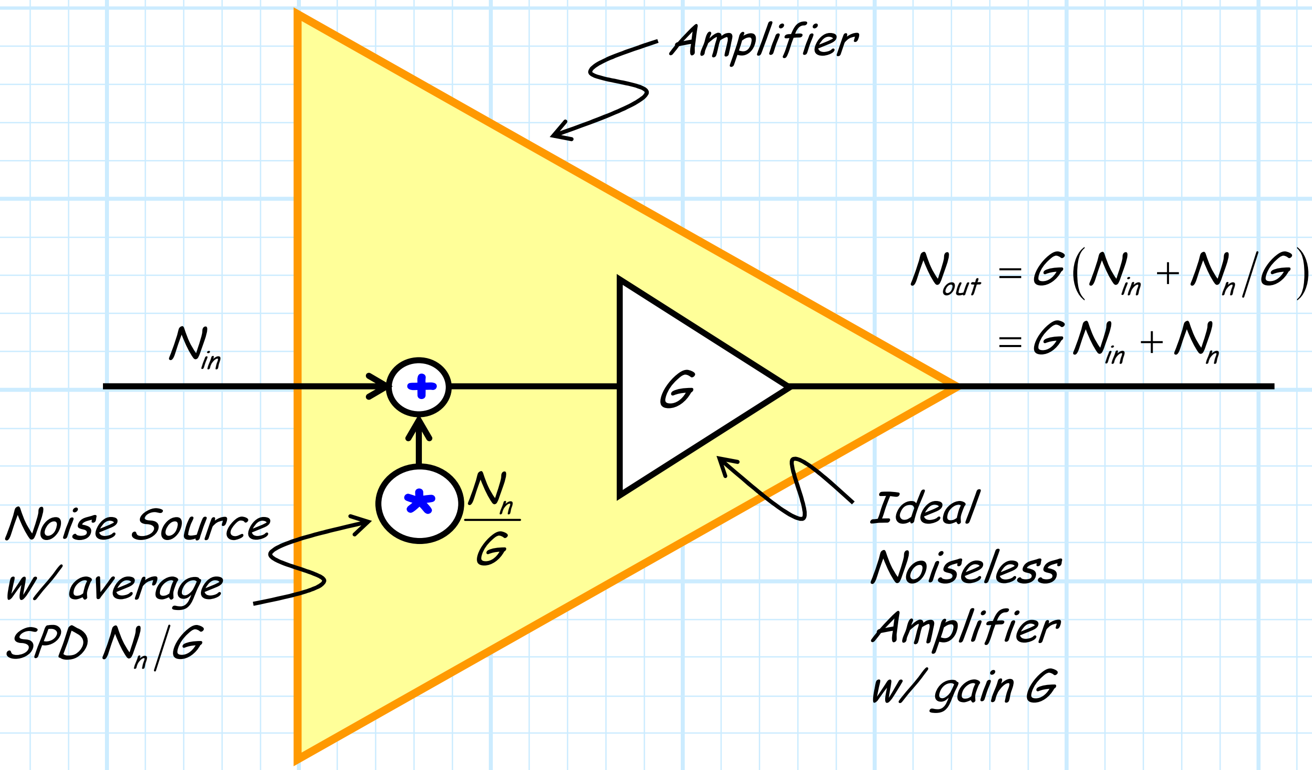
A: The internal amplifier noise is generated by **every** resistor and semiconductor element **throughout** the amplifier. Some of the noise undoubtedly is generated near the **input** and thus **amplified**, other noise is undoubtedly generated near the **output** and thus is **not** amplified at all, while still more noise might be generated somewhere in the **middle** and thus only **partially** amplified (e.g., by 0.35 G).

However, it does not matter, as the value N_n does **not** specify the value of the noise power generated at any point within the amplifier. Rather it specifies the **total** value of the noise generated throughout the amplifier, as this total noise **exits the amplifier output**.

As a result, we can **model** a "noisy" amplifier (and they're **all** noisy!) as an **noiseless** amplifier, followed by an output **noise source** producing an average spectral power density N_n:



Note however that this is **not** the **only** way we can model internally generated noise. We could **alternatively** assume that **all** the internally generated noise occurs near the amplifier **input**—and thus **all** this noise is amplified with gain G!



Note here that the noise source near the **input** of the amplifier has an average spectral power density of N_n/G.

It is in fact **this** model (where the internal noise is assumed to be created by the input) that we more **typically** use when considering the internal noise of an amplifier!

To see **why**, recall that we can alternatively express the average SPD of noise in terms of a **noise temperature** T (in degrees Kelvin):

$$N = kT$$

Thus, we can express the input noise in terms of an **input noise temperature**:

$$N_{in} = kT_{in} \qquad \Rightarrow \qquad T_{in} \doteq N_{in}/k$$

or the **output noise temperature** as:

$$N_{out} = kT_{out} \qquad \Rightarrow \qquad T_{out} \doteq N_{out}/k$$

Similarly, we can describe the **internal** amplifier noise, when modeled as being generated near the amplifier **input,** as:

$$\frac{N_n}{G} = kT_e$$

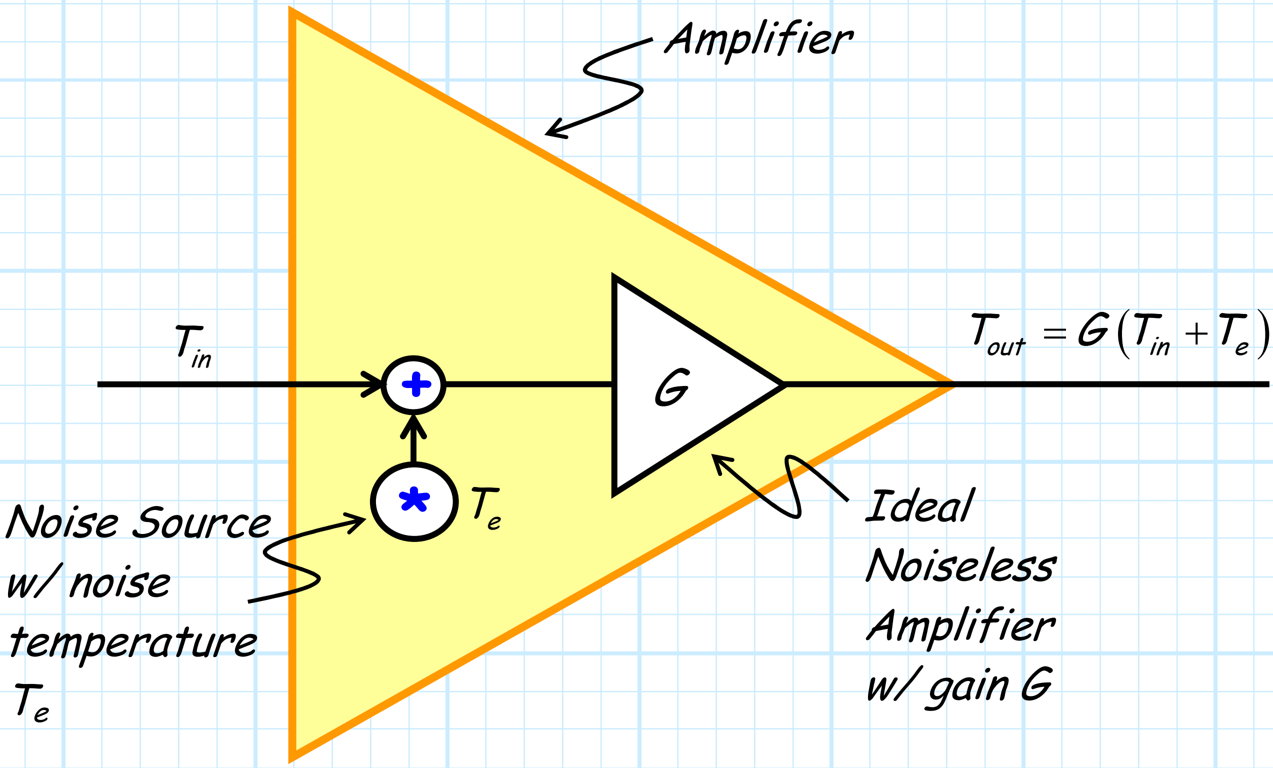
Where noise temperature T_e is defined as the **equivalent (input) noise temperature** of the amplifier:

$$\boxed{T_e \doteq \frac{N_n}{kG}}$$

Note this equivalent noise temperature is a **device parameter** (just like gain!)—it tells us how noisy our amplifier is.

Of course, the **lower** the equivalent noise temperature, the **better**. For example, an amplifier with $T_e = 0\ K^{\circ}$ would produce **no** internal noise at all!

Specifying the internal amplifier noise in this way allows us to relate **input** noise temperature T_{in} and **output** noise temperature T_{out} in a very straightforward manner:



$$T_{out} = G(T_{in} + T_e)$$

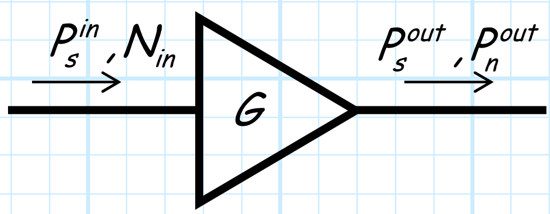
Thus, the noise **power** at the output of this amplifier is:

$$\begin{aligned} P_n^{out} &\approx N_{out} B \\ &= kT_{out} B \\ &= G k (T_{in} + T_e) B \end{aligned}$$

Noise Figure and SNR

Of course, in addition to noise, the input to an amplifier in a receiver will typically include our desired **signal**.

Say the **power** of this input signal is P_s^{in}. The output of the amplifier will therefore include **both** a signal with power P_s^{out}, and noise with power P_n^{out}:



where:

$$P_s^{out} = G\, P_s^{in}$$

and:

$$\begin{aligned} P_n^{out} &= N_{in} + G\, k T_e\, B \\ &= G\, k \left(T_{in} + T_e\right) B \end{aligned}$$

In order to accurately demodulate the signal, it is important that signal power be **large** in comparison to the noise power. Thus, a fundamental and important measure in radio systems is the **Signal-to-Noise Ratio** (SNR):

$$SNR \doteq \frac{P_s}{P_n}$$

The **larger** the SNR, the **better**!

At the **output** of the amplifier, the SNR is:

$$\begin{aligned} SNR_{out} &= \frac{P_s^{out}}{P_n^{out}} \\ &= \frac{G\,P_s^{in}}{G\,k\left(T_{in}+T_e\right)B} \\ &= \frac{P_s^{in}}{k\left(T_{in}+T_e\right)B} \end{aligned}$$

Moreover, we can define an **input noise power** as the total noise power across the **bandwidth of the amplifier**:

$$P_n^{in} = N_{in}\,B = k\,T_{in}\,B$$

And thus the **input** SNR as:

$$SNR_{in} = \frac{P_s^{in}}{P_n^{in}} = \frac{P_s^{in}}{k\,T_{in}B}$$

Now, let's take the **ratio** of the input SNR to the output SNR:

$$\begin{aligned} \frac{SNR_{in}}{SNR_{out}} &= \frac{P_s^{in}}{k\,T_{in}B}\left(\frac{k\left(T_{in}+T_e\right)B}{P_s^{in}}\right) \\ &= \frac{T_{in}+T_e}{T_{in}} \\ &= 1+\frac{T_e}{T_{in}} \end{aligned}$$

Since $T_e > 0$, it is evident that:

$$\frac{SNR_{in}}{SNR_{out}} = 1 + \frac{T_e}{T_{in}} > 1$$

In other words, the SNR at the **output** of the amplifier will be **less** than the SNR at the **input**.

→ This is very **bad** news!

This result means that the SNR will always be **degraded** as the signal passes through **any** microwave component!

As a result, the SNR at the **input** of a receiver will be the largest value it will **ever** be within the receiver. As the signal passes through each component of the receiver, the SNR will get steadily **worse**!

Q: *Why is that? After all, if we have several amplifiers in our receiver, the* ***signal*** *power will significantly* ***increase****?*

A: True! But remember, this gain will likewise increase the receiver input **noise** by the **same** amount. Moreover, each component will add **even more noise**—the internal noise produced by each receiver component.

Thus, the power of a signal traveling through a receiver increases—but the **noise** power increases **even more**!

Note that the ratio SNR_{in}/SNR_{out} essentially quantifies the degradation of SNR by an amplifier—a ratio of **one** is **ideal**, a **large** ratio is very **bad**.

So, let's go back and look **again** at ratio SNR_{in}/SNR_{out}:

$$\frac{SNR_{in}}{SNR_{out}} = 1 + \frac{T_e}{T_{in}}$$

Note what this ratio **depends** on, and what it does **not**.

This ratio **depends** on:

1. T_e (a device parameter)

2. T_{in} (**not** a device parameter)

This ratio does not **depend** on:

1. The amplifier gain G.

2. The amplifier bandwidth B.

We thus might be tempted to use the ratio SNR_{in}/SNR_{out} as another **device parameter** for describing the **noise** performance of an amplifier. After all, SNR_{in}/SNR_{out} depends on T_e, but does **not** depend on other device parameters such as G or B.

Moreover, SNR is a value that can generally be easily **measured**!

But the problem is the **input** noise temperature T_{in}. This can be **any** value—it is **independent** of the amplifier itself.

For **example**, it is event that as the input noise increases to **infinity**:

$$\lim_{T_{in}\to\infty} \frac{SNR_{in}}{SNR_{out}} = \lim_{T_{in}\to\infty}\left(1+\frac{T_e}{T_{in}}\right) = 1$$

In other words, if the input noise is large enough, the internally generated amplifier noise will become **insignificant**, and thus will degrade the SNR **very little**!

Q: *Degrade the SNR very little! This means $SNR_{out} = SNR_{in}$! Isn't this **desirable**?*

A: Not in this instance. Note that if T_{in} increases to infinity, then:

$$\lim_{T_{in}\to\infty} SNR_{in} = \lim_{T_{in}\to\infty}\left(\frac{P_s^{in}}{kT_{in}B}\right) = 0$$

In other words, the SNR does is not degraded by the amplifier **only** because the SNR is already as bad (i.e., $SNR = 0$) as it can possibly get!

Conversely, as the input noise temperature decreases toward **zero**, we find:

$$\lim_{T_{in}\to 0} \frac{SNR_{in}}{SNR_{out}} = \lim_{T_{in}\to 0}\left(1+\frac{T_e}{T_{in}}\right) = \infty$$

Q: *Yikes! The amplifier degrades the SNR by an infinite percentage! Isn't this* ***undesirable?***

A: Not in this instance. Note that if T_{in} decreases to zero, then:

$$\lim_{T_{in}\to 0} SNR_{in} = \lim_{T_{in}\to 0}\left(\frac{P_s^{in}}{k\,T_{in}\,B}\right) = \infty$$

Note this is the **perfect** SNR, and thus the ratio SNR_{in}/SNR_{out} will likewise be infinity, **regardless** of the amplifier.

Anyway, the **point** here is that although the degradation of SNR by the amplifier does depend on the **amplifier** noise characteristics (i.e., T_e), it **also** on the noise input to the amplifier (i.e., T_{in}).

This input noise is a variable that is unrelated to amplifier performace

Q: *So there is* ***no way*** *to use* SNR_{in}/SNR_{out} *as a* ***device parameter?***

A: Actually there **is**! In fact, it is the most **prevalent** parameter for specifying microwave device noise performance. This measure is called **noise figure.**

The noise figure of a device **is** simply the measured ratio SNR_{in} / SNR_{out} exhibited by a device, **for a specific input noise temperature** T_{in}.

I **repeat**:

→ "**for a specific input noise temperature** T_{in}**.**"

This specific noise temperature is almost **always** taken as the standard "room temperature" of $T_o = 290\ K^{\circ}$. Note this was likewise the standard **antenna noise temperature** assumption.

Thus, the **Noise Figure** (F) of a device is defined as:

$$
\begin{aligned}
F &\doteq \left.\frac{SNR_{in}}{SNR_{out}}\right|_{T_{in}=290K^{\circ}} \\
&= \left.\left(1 + \frac{T_e}{T_{in}}\right)\right|_{T_{in}=290K^{\circ}} \\
&= 1 + \frac{T_e}{290K^{\circ}}
\end{aligned}
$$

It is critically important that **you** understand the **definition** of noise figure. A common **mistake** is to assume that:

$$SNR_{out} = \frac{SNR_{in}}{F}$$ ← This is **not** generally true!

Note this would only be true **if** $T_{in} = 290K^{\circ}$, but this is almost **never** the case (i.e., $T_{in} \neq 290K^{\circ}$ generally speaking).

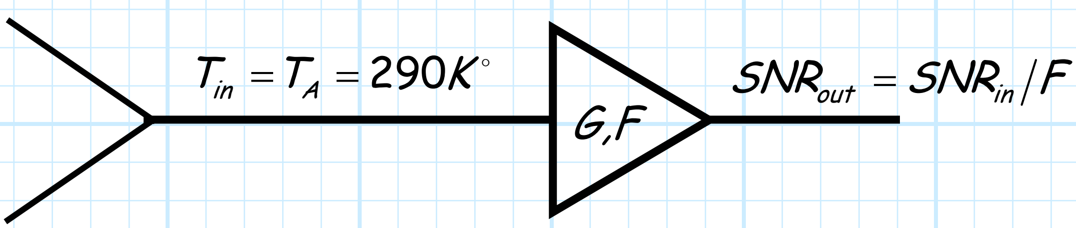
Thus, an **incorrect** (but **widely repeated**) statement would be:

" *The noise figure specifies the degradation of SNR."*

Whereas, a **correct** statement is:

" *The noise figure specifies the degradation of SNR, for the specific condition when $T_{in} = 290K^{\circ}$, and for that specific condition **only**"*

The one **exception** to this is when an **antenna** is connected to the input of an amplifier. For this case, it is evident that the input temperature **is** $T_A = T_{in} = 290K^{\circ}$:



Note that since the noise figure F of a given device is dependent on its equivalent noise temperature T_e, we can **determine** the equivalent noise temperature T_e of a device with knowledge F:

$$F = 1 + \frac{T_e}{290K^\circ} \quad \Leftrightarrow \quad T_e = (F-1)290K^\circ$$

One **more** point. Note that noise figure F is a **unitless** value (just like gain!). As such, we can easily express it in terms of **decibels** (just like gain!):

$$F(dB) = 10\,log_{10} F$$

Like gain, the noise figure of an amplifier is **typically** expressed in dB.

Noise Figure of Passive Devices

Recall that passive devices are typically **lossy**. Thus, they have a **"gain"** that is **less than one**—we can define this in terms of device **attenuation** A:

$$A = \frac{1}{G}$$

where for a lossy, passive device $G < 1$, therefore $A > 1$.

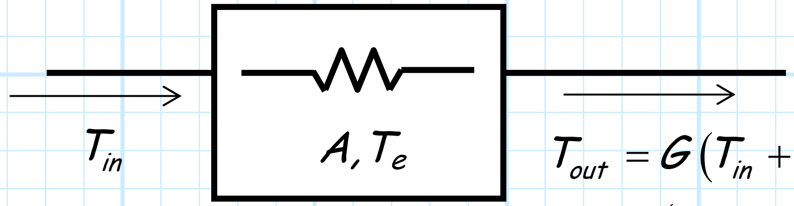
Q: *What is the equivalent noise temperature T_e or noise figure F of a **passive** device (i.e., **not** an amplifier) ?*

A: The **equivalent noise temperature** of a **passive** device can be shown to be approximately (trust me!):

$$T_e = (A - 1)T$$

where T is the **physical** temperature of the passive device. Typically we assume this physical temperature to be $290K^\circ$, so that:

$$T_e = (A - 1)\, 290K^\circ$$



$$\begin{aligned} T_{out} &= G\left(T_{in} + T_e\right) \\ &= \left(T_{in} + T_e\right)/A \end{aligned}$$

Thus, we find that the **output** noise temperature of a **passive** device is:

$$\begin{aligned} T_{out} &= G\left(T_{in} + T_e\right) \\ &= \frac{T_{in} + T_e}{A} \\ &= \frac{T_{in}}{A} + \frac{(A-1)290\,K^\circ}{A} \\ &= \frac{T_{in}}{A} - \frac{290\,K^\circ}{A} + 290\,K^\circ \end{aligned}$$

This result is **very** interesting, and **makes sense** physically. As attenuation A approaches the **lossless** case $A = 1$, we find that $T_{out} = T_{in}$. In other words the noise passes through the device **unattneuated**, and the device produces **no** internal noise!

→ Just like a length of lossless transmission line!

On the other hand, as A gets **very large**, the input noise is completely **absorbed** by the device. The noise at the device output is entirely generated **internally**, with a noise temperature $T_{out} = 290\,K^\circ$ equal to its physical temperature.

→ Just like the output of a **resistor** at physical temperature $T = 290\,K^{\circ}$

Q: *So, what is the **noise figure** F of a **passive** device?*

Now, we determined earlier that the **noise figure** of a two-port device is related to its equivalent noise temperature as:

$$F = 1 + \frac{T_e}{290\,K^{o}}$$

Therefore, the noise figure of a **passive** device is:

$$\begin{aligned} F &= 1 + \frac{(A-1)\,290K^{\circ}}{290K^{\circ}} \\ &= 1 + (A-1) \\ &= A \end{aligned}$$

Thus, for a **passive** device, the noise figure is **equal** to its attenuation!

$$F = 1/G = A$$

So, for an **active** two-port device (e.g., an amplifier), we find that two important and **independent device parameters** are gain G and noise figure F—**both** values must be specified.

However, for **passive** two-port devices (e.g., an attenuator), we find that attenuation *A* and noise figure *F* are not only completely **dependent**—they are in fact **equal**!

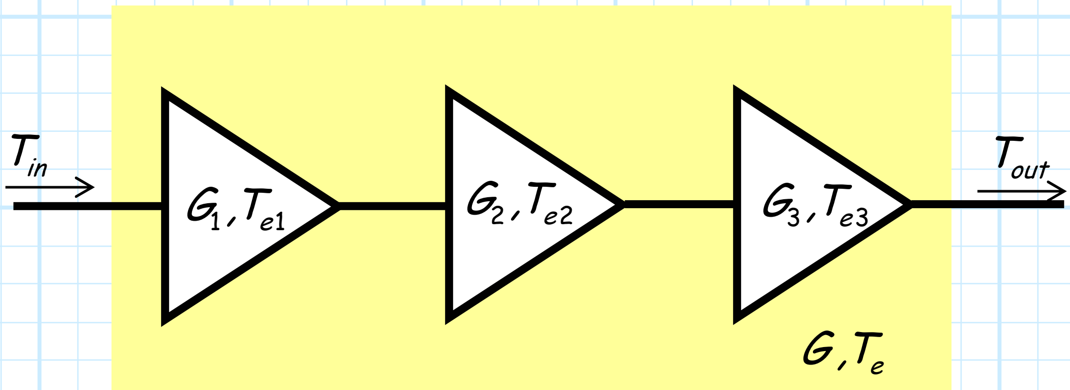
Finally, we should not that the value *A* represents the attenuation (i.e., loss) of **any** passive device—**not** just an attenuator.

For example, *A* would equal the **insertion loss** for a switch, filter, or coupler. Likewise, it would equal the **conversion loss** of a mixer.

Thus, **you** should now be able to specify the noise figure and equivalent noise temperature of each and **every** two-port device that we have studied!

System Equivalent Noise Temperature

Say we **cascade** three microwave devices, each with a different **gain** and **equivalent noise temperature**:



These three devices together can be thought of as **one** new microwave device.

Q: *What is the equivalent noise temperature T_e of this* ***overall*** *device?*

A: First of all, we must **define** this temperature as the value T_e such that:

$$T_{out} = G\left(T_{in} + T_e\right)$$

or specifically:

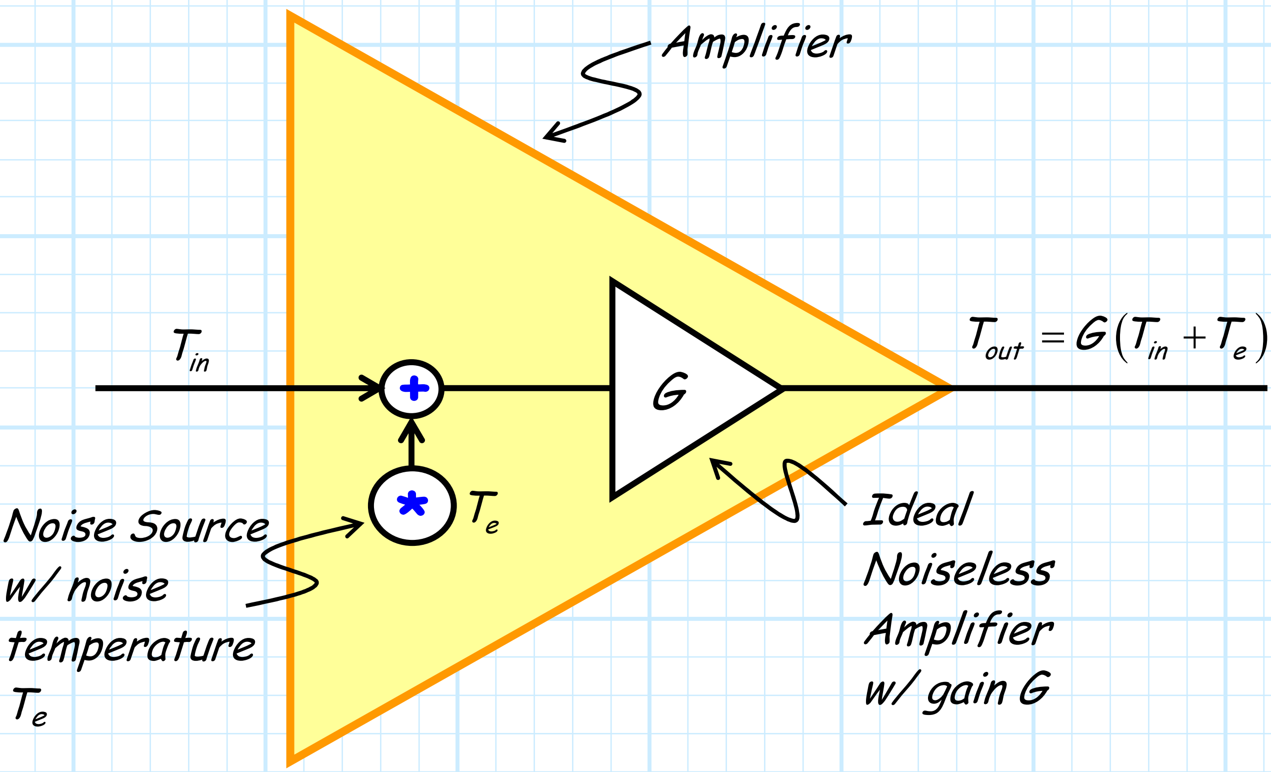
$$T_e = \frac{T_{out}}{G} - T_{in}$$

Q: *Yikes! What is the value of G ?*

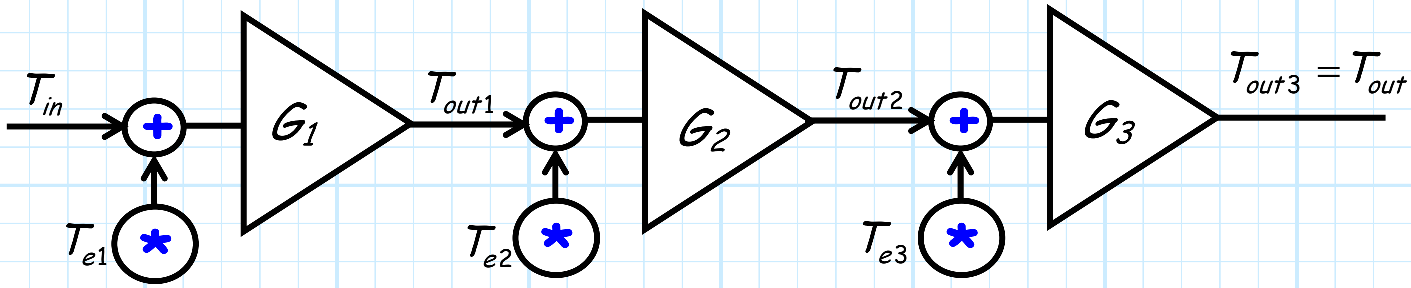
A: The value G is the **total system gain**; in other words, the overall gain of the three cascaded devices. This gain is particularly easy to determine, as is it simply the **product** of the three gains:

$$G = G_1 G_2 G_3$$

Now for the **hard part**! To determine the value of T_{out}, we must use our **equivalent noise model** that we studied earlier:



Thus, we cascade three **models**, one for each amplifier:



We can **observe** our model and note three things:

$$T_{out1} = G_1 \left(T_{in} + T_{e1} \right)$$

$$T_{out2} = G_1 \left(T_{out1} + T_{e1} \right)$$

$$T_{out3} = G_1 \left(T_{out2} + T_{e1} \right)$$

Combining these three equations, we find:

$$T_{out3} = G_1 G_2 G_3 \left(T_{in} + T_{e1} \right) + G_2 G_3 \left(T_{e2} \right) + G_3 \left(T_{e3} \right)$$

a result that is likewise **evident** from the model.

Now, since $T_{out} = T_{out3}$, we can determine the **overall** (i.e., system) equivalent noise temperature T_e:

$$\begin{aligned} T_e &= \frac{T_{out}}{G} - T_{in} \\ &= \frac{G_1 G_2 G_3 \left(T_{in} + T_{e1}\right) + G_2 G_3 \left(T_{e2}\right) + G_3 \left(T_{e3}\right)}{G_1 G_2 G_3} - T_{in} \\ &= T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} \end{aligned}$$

Moreover, we will find if we cascade an N number of devices, the overall noise equivalent temperature will be:

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \frac{T_{e4}}{G_1 G_2 G_3} + \cdots + \frac{T_{eN}}{G_1 G_2 G_3 \cdots G_{N-1}}$$

I assume that you can use the above equation to get the correct answer—but I want to know if you understand **why** your answer is correct!

Make sure **you** understand where this expression comes from, and what it means.

Look closely at the above expression, for it tells us something very **profound** about the noise in a complex microwave system (like a receiver!).

Recall that we want the equivalent noise temperature to be as **small** as possible. Now, look at the equation above, **which** terms in this summation are likely to be the **largest**?

* Assuming this system has large gain G, we will find that the **first** few terms of this summation will **typically dominate** the answer.

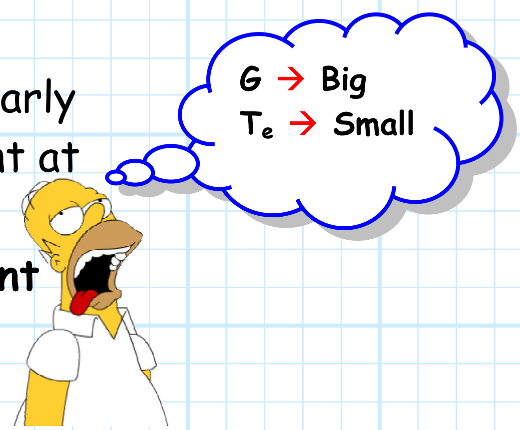
* Thus, it is evident that to make T_e as small as possible, we should start by making the **first term** as small as possible. Our **only** option is to simply make T_{e1} as small as we can.

* To make the **second** term small, we could likewise make T_{e2} small, but we have **another** option!

→ We could likewise make gain G_1 **large!**

Note that making G_1 large has **additional** benefits, as it likewise helps minimize **all** the other terms in the series!

Thus, good receiver designers are particularly careful about placing the proper component at the **beginning** of a receiver. They **covet** a device that has **high gain** but **low equivalent noise temperature** (or noise figure).



→ The ideal **first** device for a receiver is a **low-noise amplifier**!

Q: *Why* ***don't*** *the devices at the* ***end*** *of the system make much of a difference when it comes to noise?*

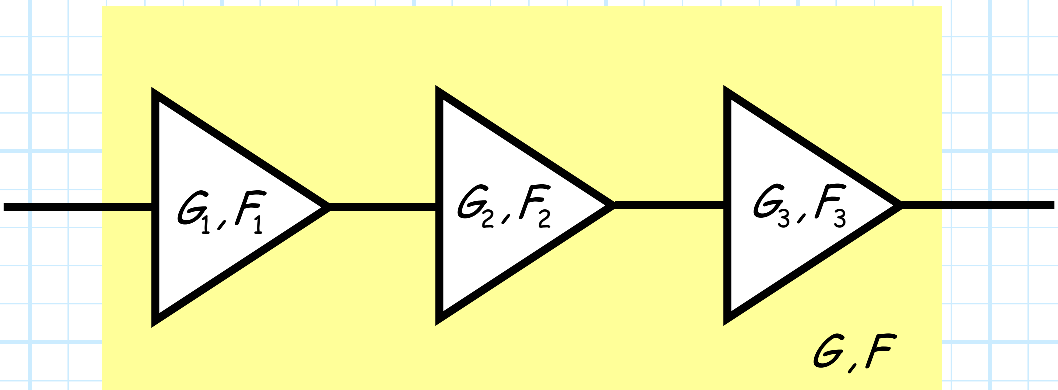
A: Recall that each microwave device **adds** more noise to the system, As a result, noise will generally **steadily increase** as it moves through the system.

* By the time it reaches the end, the noise power is typically **so large** that the additional noise generated by the devices there are **insignificant** and make **little** increase in the overall noise level.

* Conversely, the noise generated by the **first** device is amplified by **every** device in the overall system—this first device thus typically has the **greatest** impact on system noise temperature and system noise figure.

System Noise Figure

Say we **again** cascade three microwave devices, each with a different **gain** and **noise figure**:



These three devices together can be thought of as **one** new microwave device.

Q: *What is the noise temperature of this **overall** device?*

A: Recall that we found the overall **equivalent noise temperature** of this system to be:

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2}$$

Likewise, the equivalent noise temperature of each device is related to its **noise figure** as:

$$T_e = (F-1)290K^\circ$$

Combining these two expressions we find:

$$(F-1)290K^\circ = (F_1-1)290K^\circ + \frac{(F_2-1)290K^\circ}{G_1} + \frac{(F_3-1)290K^\circ}{G_1G_2}$$

and thus **solving** for F:

$$\begin{aligned} F &= \frac{1}{290K^\circ}\left((F_1-1)290K^\circ + \frac{(F_2-1)290K^\circ}{G_1} + \frac{(F_3-1)290K^\circ}{G_1G_2}\right)+1 \\ &= F_1 - 1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2} + 1 \\ &= F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2} \end{aligned}$$

Therefore, the **overall** noise figure for the "device" consisting of three cascade amplifiers can be determined **solely** from the knowledge of the gain and noise figure of **each** individual amplifier!

$$F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2}$$

Moreover, we find that if we construct a system of N cascaded microwave components, then the **overall noise figure** of the system can be determined as:

$$F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2} + \frac{F_4-1}{G_1G_2G_3} + \cdots + \frac{F_N-1}{G_1G_2G_3\cdots G_{N-1}}$$

* It is again evident from inspection of this equation that the **first device** in the cascaded chain will likely by the **most significant** device in terms of the overall system noise figure.

* We come to the same conclusion as for T_e—make the first device one with **low internal noise** (small noise figure F_1) and **high gain** G.

* In other words, make the **first** device in your receiver a **Low-Noise Amplifier (LNA)**!

One other **very important** note:

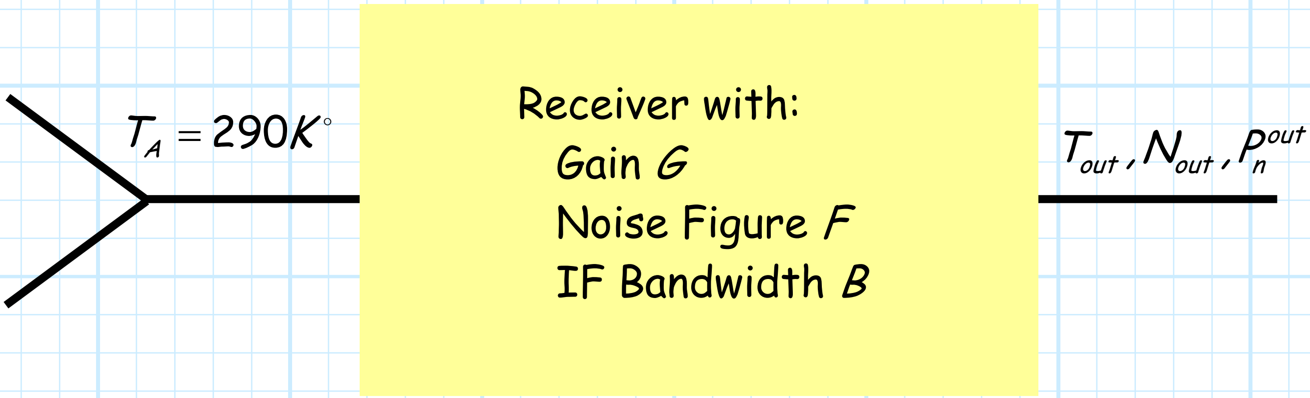
Although we used **only** amplifiers in our examples for system equivalent noise temperature and system noise figure, the results are likewise valid for **passive** devices!

Just remember, the gain G of a passive device is simply the **inverse** of its attenuation A:

$$G = \frac{1}{A}$$

Now, let's examine one important system made up of cascaded microwave components—a **receiver**!

At the **input** of every receiver is an **antenna**. This antenna, among other signals, delivers **noise power** to the input, with a temperature that is typically $T_A = 290K^\circ$.



Q: *What is the **output noise** (i.e. T_{out}, N_{out}, or P_n^{out}) of this receiver?*

A: Recall that the output noise temperature is:

$$\begin{aligned} T_{out} &= G\left(T_{in} + T_e\right) \\ &= G\left(T_A + T_e\right) \\ &= G\left(290\,K^\circ + T_e\right) \end{aligned}$$

and since:

$$T_e = (F-1)290K^\circ$$

we conclude that the **output noise temperature** is:

$$\begin{aligned} T_{out} &= G\left(290\,K^\circ + T_e\right) \\ &= G\left(290\,K^\circ + (F-1)290K^\circ\right) \\ &= G\,F\left(290K^\circ\right) \end{aligned}$$

Therefore, the **average spectral power density** at the output is:

$$\begin{aligned} N_{out} &= kT_{out} \\ &= kGF\left(290K^{\circ}\right) \end{aligned}$$

while the **output noise power** is:

$$\begin{aligned} P_n^{out} &= N_{out}B \\ &= k\,G\,F\left(290K^{\circ}\right)B \end{aligned}$$

Now, compare these values to their respective **input** values:

$$T_{in} = T_A = 290K^{\circ}$$

$$N_{in} = k\left(290K^{\circ}\right)$$

$$P_n^{in} = k\left(290K^{\circ}\right)B$$

Note for each of the values, the output is a factor ***G F*** greater than the input:

$$\frac{T_{out}}{T_{in}} = \frac{N_{out}}{N_{in}} = \frac{P_n^{out}}{P_n^{in}} = G\,F$$

→ However, I again emphasize, this expression is only valid **if** $T_{in} = 290\,K^{\circ}$!!

Of course, the ratio of the **signal** output power to the **signal** input power is:

$$\frac{P_s^{out}}{P_s^{in}} = G$$

Thus, the **signal** power is increased by a **factor** G, while the **noise** power is increased by a **factor** GF. This is why there is reduction in SNR by a **factor** F!