Special Problem 2-4.5

Vectors \( \hat{a}, \hat{b}, \hat{c} \) form an orthonormal set of vectors.

Vectors \( \hat{i}, \hat{j}, \hat{k} \) also form an orthonormal set of vectors.

The following is known about these vectors:

\[
\hat{i} = \frac{1}{5\sqrt{2}} (3 \hat{a} + 4 \hat{b} + 5 \hat{c}) \\
\hat{j} = \frac{1}{5\sqrt{3}} (7 \hat{a} + \hat{b} - 5 \hat{c})
\]

\[
\hat{a} \cdot \hat{k} = \frac{1}{\sqrt{6}} \\
\hat{k} \cdot \hat{b} = -\frac{2}{\sqrt{6}} \\
\hat{c} \cdot \hat{k} = \frac{1}{\sqrt{6}}
\]

Vector \( \mathbf{A} \) can be written using the first set of orthonormal base vectors as:

\[
\mathbf{A} = \sqrt{6} \hat{a} - 2\sqrt{6} \hat{b}
\]

The same vector of course can be written in terms of the second set of orthonormal base vectors as:

\[
\mathbf{A} = A_i \hat{i} + A_j \hat{j} + A_k \hat{k}
\]

Determine the value of scalar components \( A_i, A_j \) and \( A_k \).