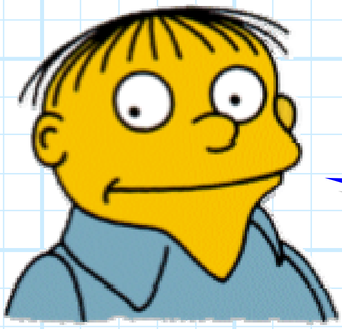
Introduction: Analysis of Electronic Circuits

Reading Assignment: *KVL and KCL text from EECS 211*

Just like EECS 211, the majority of problems (hw and exam) in EECS 312 will be **circuit analysis** problems. Thus, a key to doing well in 312 is to thoroughly **know the material from 211**!!

So, before we get started with 312, let's **review** 211 and see how it applies to **electronic circuits**.



Q: *I* ***aced*** *EECS 211 last semester; can I just* ***skip*** *this "review"??*

A: Even if you did extremely well in 211, you will want to pay attention to this review. You will see that the concepts of 211 are applied a little **differently** when we analyze **electronic** circuits.

Both the conventions and the approach used for analyzing electronic circuits will **perhaps** be unfamiliar to you at first—I thus imagine that everyone (I hope) will find this review to be **helpful**.

Electronic Circuit Notation

KVL and Electronic Circuit Notation

Analysis of Electronic Circuits

Even the **quantities** of current and resistance are a **little** different for electronic circuits!

Q: *You mean we don't use* ***Amperes*** *and* ***Ohms****??*

A: Not exactly!

Volts, Milli-Amps, Kilo-Ohms

Now let's try an **example**!

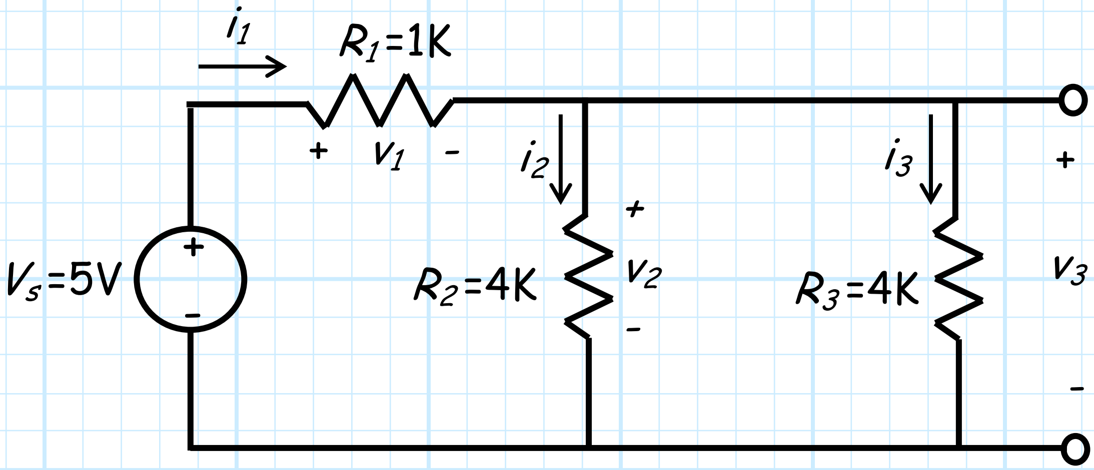
Example: Circuit Analysis using Electronic Circuit Notation

Electronic Circuit Notation

The standard **electronic circuit notation** may be a little **different** that what you became used to seeing in in EECS 211.

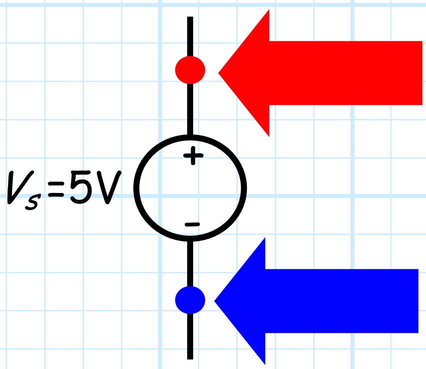
The **electronic** circuit notation has a few **"shorthand"** standards that can **simplify** circuit schematics!

Consider the circuit below:



Note the voltage values in this circuit (i.e., V_s, v_1, v_2, v_3) provide values of potential **difference** between two points in the circuit.

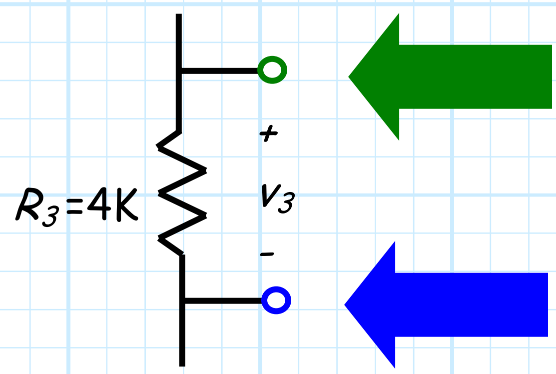
For example, from the **voltage source** we can conclude:



The electric potential at **this** point in the circuit is **5 volts greater** than:

the electric potential at **this** point in the circuit.

Or the **resistor voltage** v_3 means:

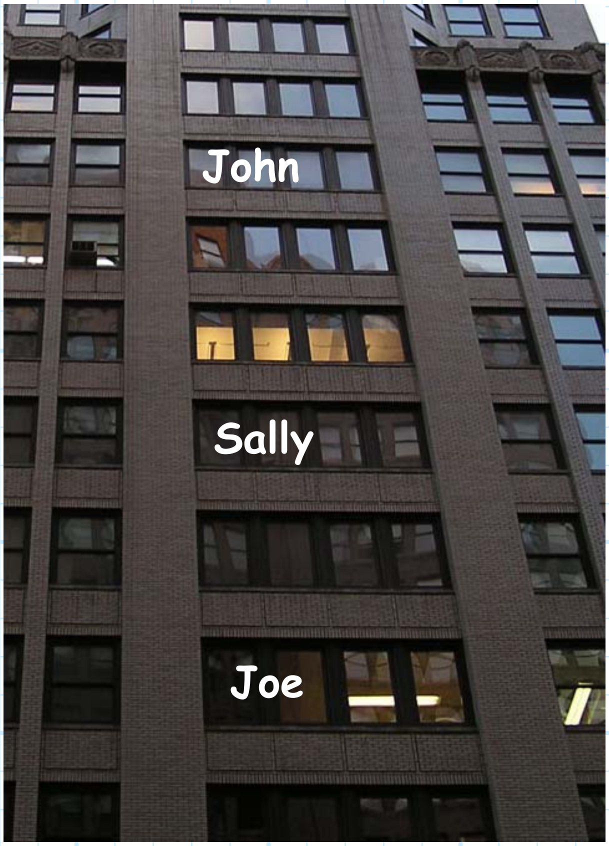


The electric potential at **this** point in the circuit is **v_3 volts greater** than:

the electric potential at **this** point in the circuit.

But remember, v_3 could be a **negative** value!

Thus, the values of voltages are **comparative**—they tell us the **difference** in electric potential between two points with in the circuit.



As an **analogy**, Say John, Sally, and Joe work in a very **tall building**. Our circuit voltages are little like saying:

"John is 5 floors above Joe"

"Sally is 2 floors above Joe"

From this **comparative** information we can deduce that John is **3 floors** above Sally.

What we **cannot** determine is on **what floor** John, Sally, or Joe are actually located. They could be located at the **highest** floors of the building, or at the **lowest** (or anywhere in between).

Similarly, we **cannot** deduce from the values V_s, v_1, v_2, v_3 the electric potential at each point in the circuit, only the **relative** values—relative to other points in the circuit. E.G.:

"Point R has an electric potential 5V higher than point B"

"Point G has an electric potential v_3 higher than point B"

Q: *So how do we determine **the** value of electric potential at a specific point in a circuit?*

A: Recall that electric potential at some point is equal to the **potential energy** possessed by 1 Coulomb of charge if located **at** that point.

Thus to determine the "**absolute**" (as opposed to relative) value of the electric potential, we first must determine **where** that electric potential is **zero**.

The problem is similar to that of the **potential energy** possessed by 1.0 *kg* of mass in a **gravitational field**. We ask ourselves: **Where** does this potential energy equal **zero**?

The answer of course is when the mass is located **on the ground**!

But this answer is a bit **subjective**; is the "ground":

A. where the carpet is located?

B. where the sidewalk is located?

C. The basement floor?

D. Sea level?

E. The center of the Earth?

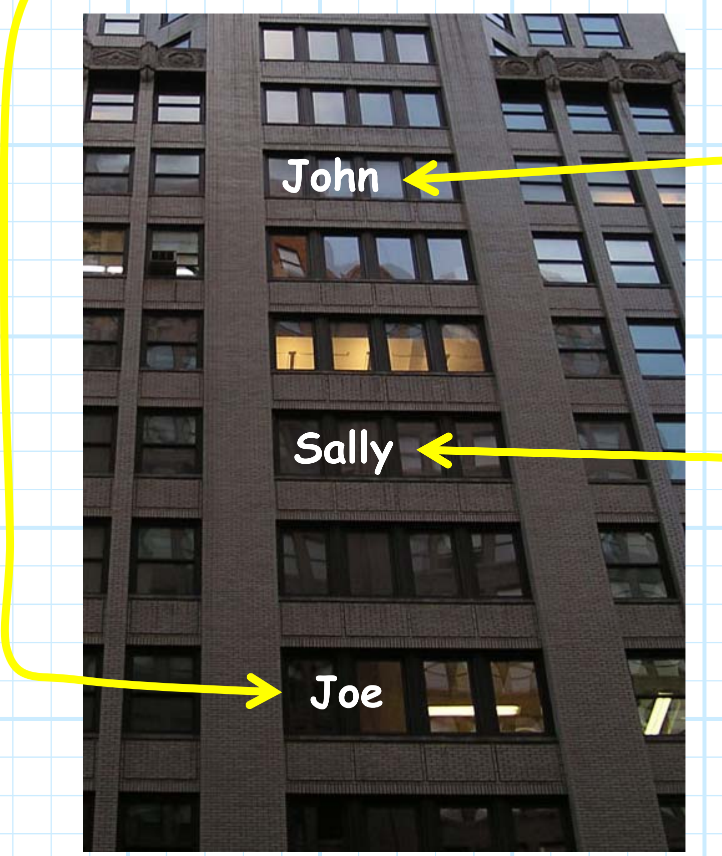
The answer is—it can be **any** of these things!

We can rather **arbitrarily** set some point as the location of ground. The potential energy is therefore described in **reference** to this ground point.

For tall buildings, the ground floor is usually defined as the floor containing the **front door** (i.e. the sidewalk)—but it doesn't have to be (just look at **Eaton Hall!**).

Now, having **defined** a ground reference, if we add to our earlier statements:

*"Joe is **32 floors** above ground"*



We can deduce:

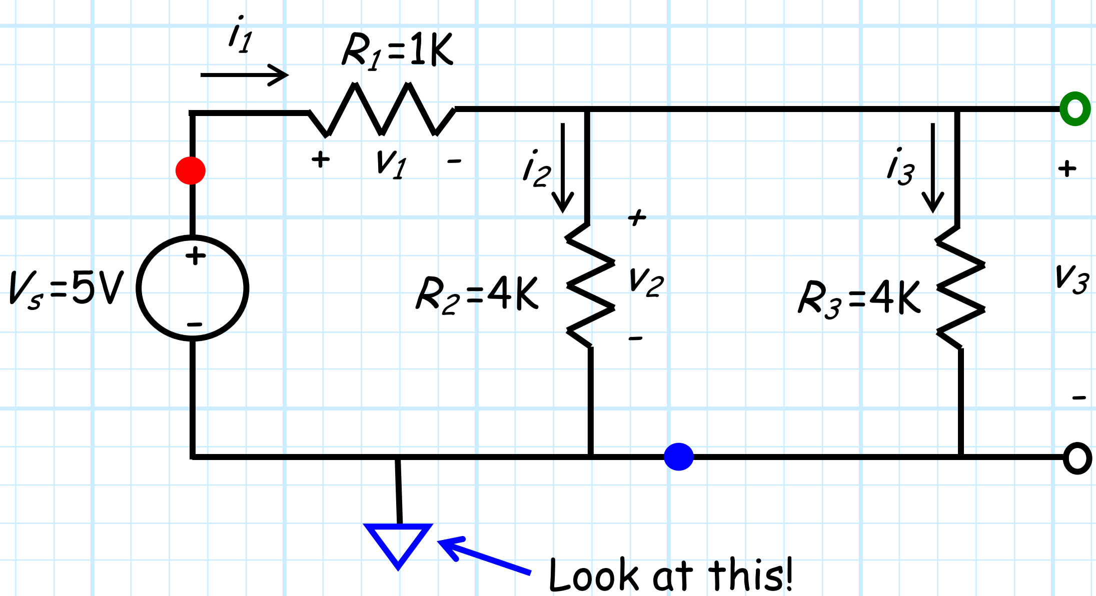
*"John is 5 floors above Joe—**therefore** John is on the **37th floor**"*

*"Sally is 2 floors above Joe—**therefore** Sally is on the **34th floor**"*

Q: *So, can we **define** a ground potential for our **circuit**?*

A: Absolutely! We just **pick** a point on the circuit and **call it** the **ground potential**. We can then reference the electric potential at every point in the circuit with **respect** to this ground potential!

Consider **now** the circuit:



Note we have added an **"upside-down triangle"** to the circuit—this denotes the location we define as our **ground potential**!

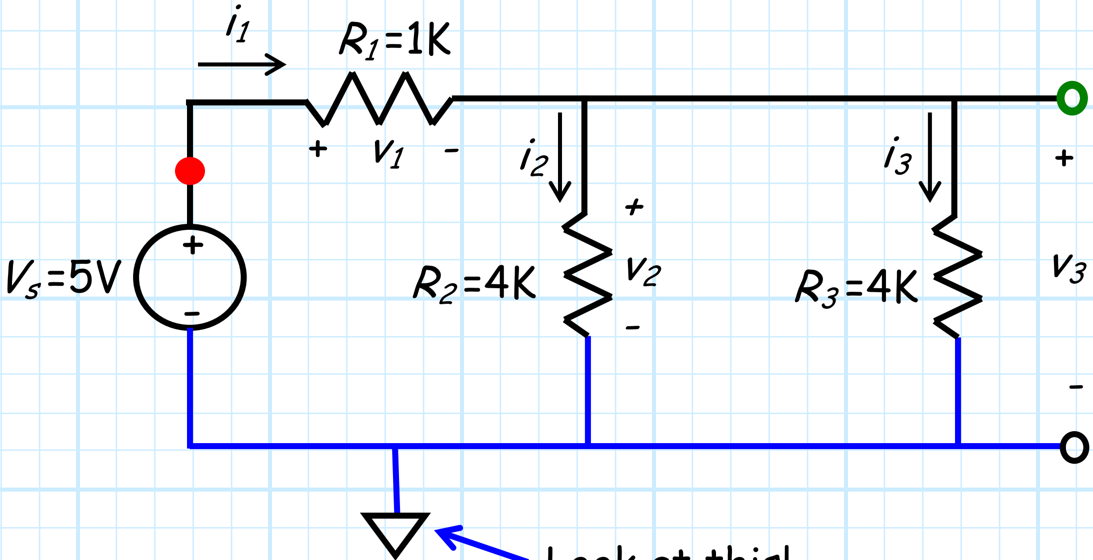
Now, if we **add** the statement:

> *"Point B is at an electric potential of **zero volts** (with respect to ground)."*

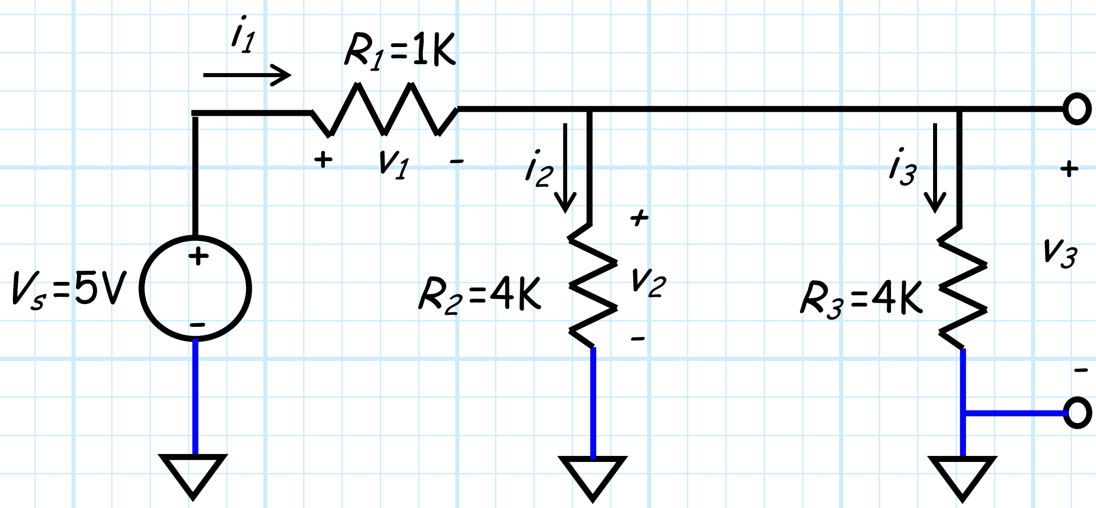
We can conclude:

> *"Point R is at an electric potential of* ***5 Volts*** *(with respect to ground)."*
>
> *"Point G is at an electric potential of* ***v_3 Volts*** *(with respect to ground)."*

Note that all the points within the circuit that reside at ground potential form a rather **large node**:

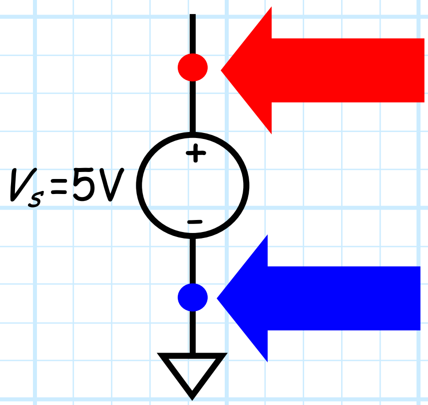


Standard **electronic** notation **simplifies** the schematic by placing the ground symbol at **each device terminal**:



Note that **all** terminals connected to ground are likewise connected to **each other**!

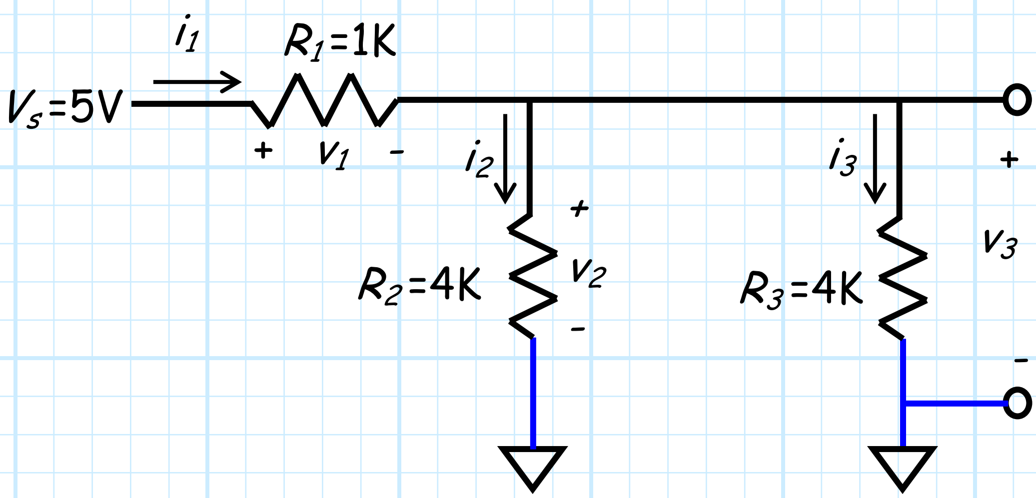
Now, in the case where **one** terminal of a device is connected to **ground** potential, the electric potential (with respect to ground) of the **other** terminal is easily determined:



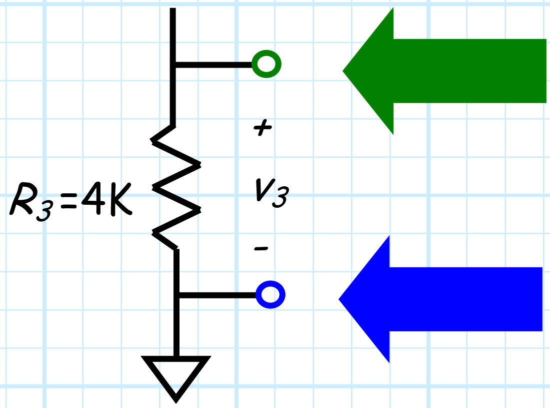
The electric potential at **this** point in the circuit is **5 volts greater** than ground (i.e., 5 volts).

This point is at **ground** potential (i.e., zero volts).

For this example, it is apparent that the voltage source simply **enforces** the condition that the + terminal is at **5.0 Volts with respect to ground**. Thus, we often **simplify** our electronic circuit schematics as:



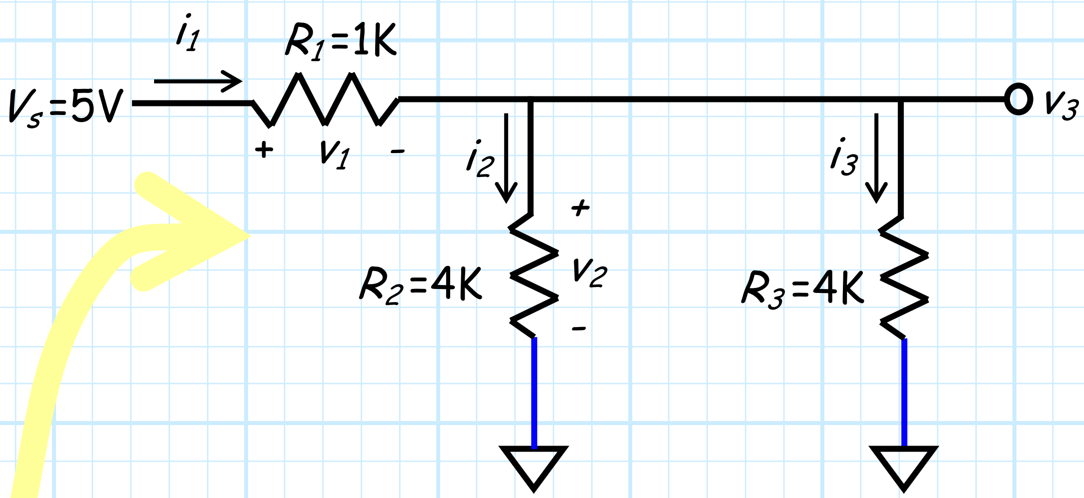
Finally, we find that:



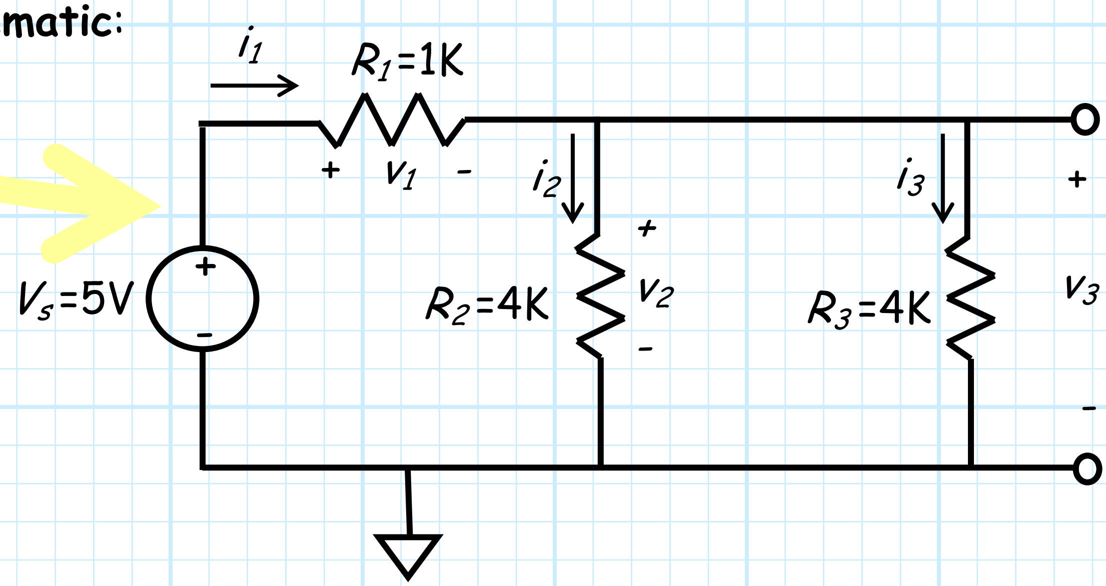
The electric potential at **this** point in the circuit is v_2 **volts greater** than ground potential (i.e., v_3).

This point is at ground potential (i.e. **zero** volts).

Thus, we can **simplify** our circuit further as:

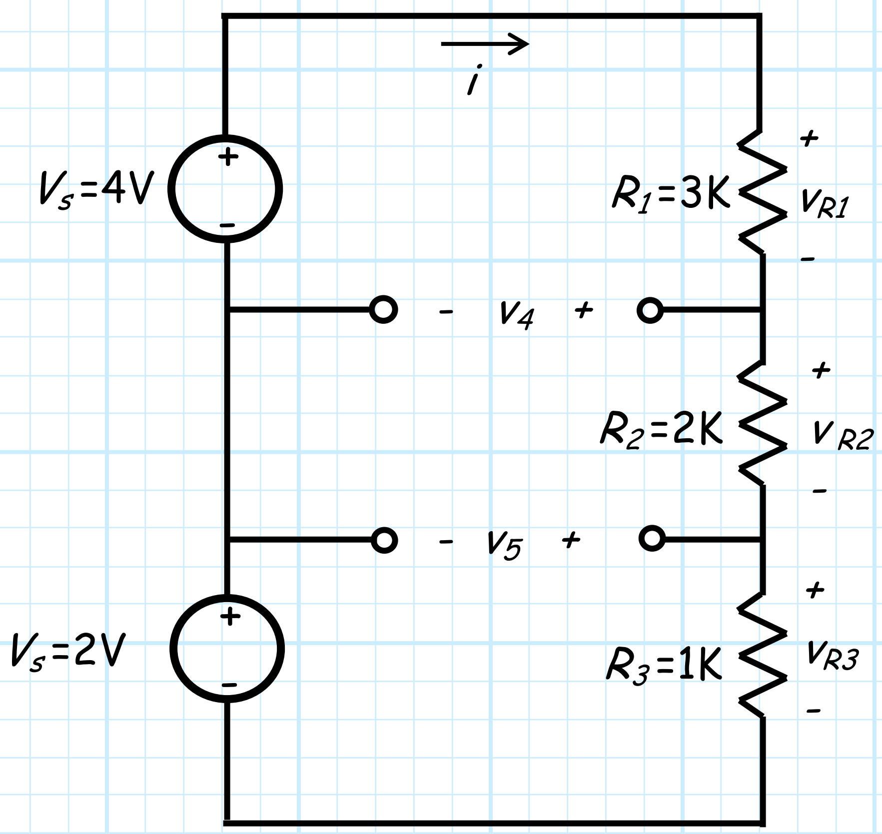


This circuit schematic is **precisely the same** as our **original schematic**:

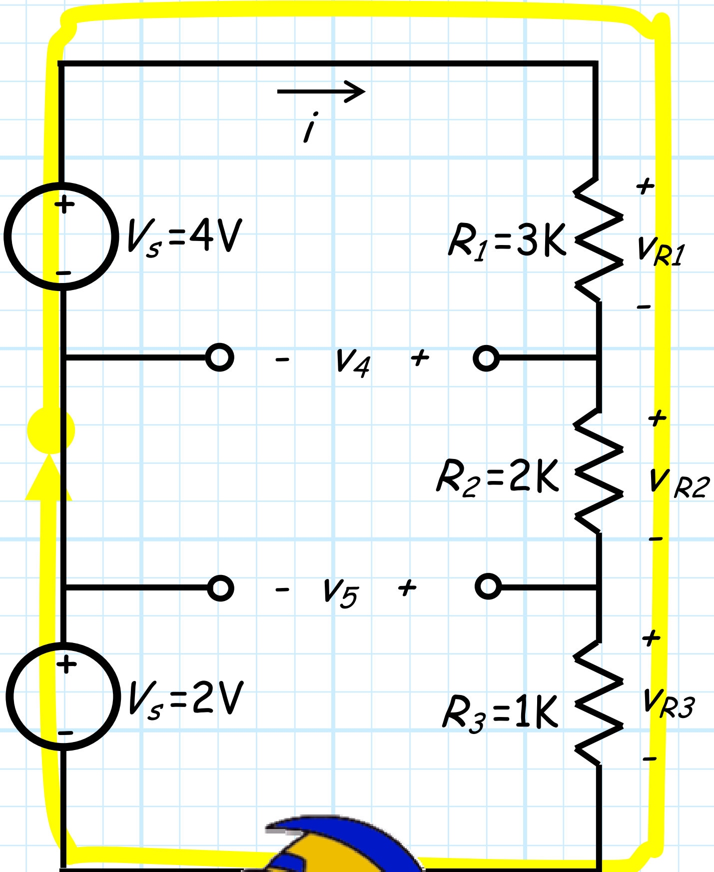


KVL and Electronic Circuit Notation

Consider **this** circuit:



We can apply **Kirchoff's Voltage Law** (KVL) to relate the **voltages** in this circuit in **any number** of ways.



For example, the KVL around this loop is:

$$-4 + v_{R1} + v_{R2} + v_{R3} - 2 = 0$$

We could multiply both sides of the equation by **-1** and likewise get a **valid** equation:

$$4 - v_{R1} - v_{R2} - v_{R3} + 2 = 0$$

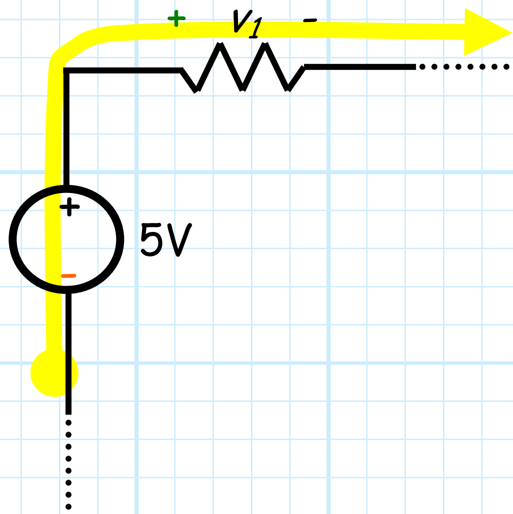
Q: *But which equation is correct? Which one do we use? Which one is **the** KVL result?*

A: Each result is **equally** valid; **both** will provide the same correct answers.

Essentially, the **first** KVL equation is constructed using the **convention** that we **add** the circuit element voltage if we first encounter a **plus** (+) sign as we move along the loop, and **subtract** the circuit element voltage if we first encounter a **minus** (-) sign as we move along the loop.

For example:

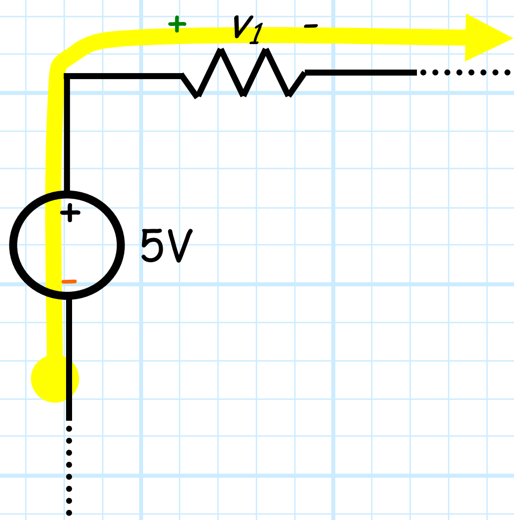
$$-5 + v_1 + \cdots$$



But, we **could** also use the convention that we **subtract** the circuit element voltage if we first encounter a **plus** (+) sign as we move along the loop, and **add** the circuit element voltage if we first encounter a **minus** (-) sign as we move along the loop!

For example:

$$+5 - v_1 + \cdots$$



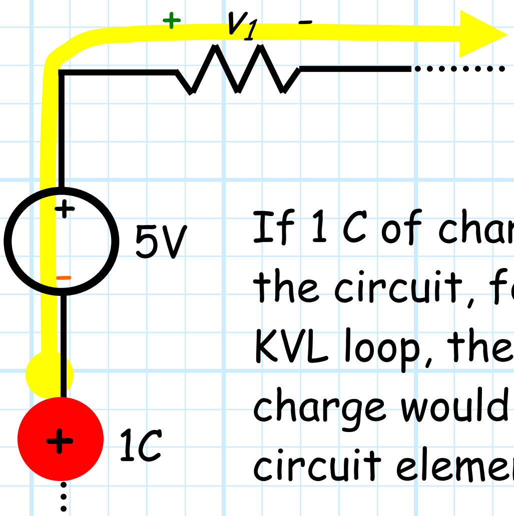
This convention would provide us with the **second** of the two KVL equations for our original circuit:

$$4 - v_{R1} - v_{R2} - v_{R2} + 2 = 0$$

Q: *Huh?! What kind of **sense** does this convention make? We **subtract** when encountering a + ? We **add** when encountering a - ??*

A: Actually, this **second** convention is more **logical** than the first if we consider the **physical** meaning of voltage!

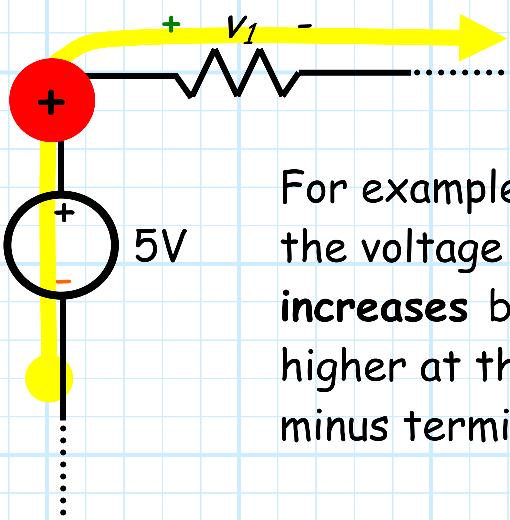
Remember, "the voltage" is simply a measure of **potential energy**—the potential energy of **1 Coulomb** of charge.



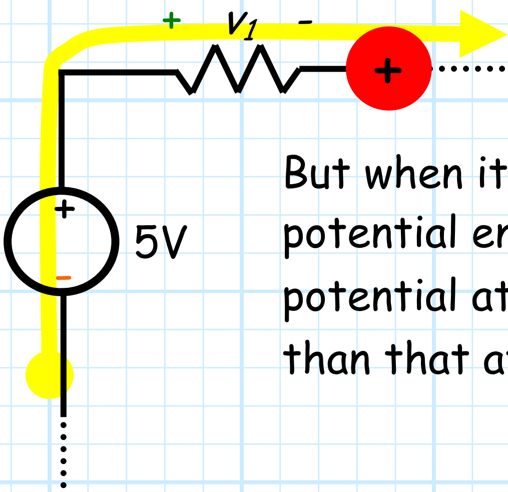
If 1 C of charge were to be **transported** around the circuit, following the **path** defined by our KVL loop, then the potential energy of this charge would **change** as is moved through each circuit element.

In other words, its potential energy would go **up**, or it would go **down**.

The **second** convention describes this increase/decrease!

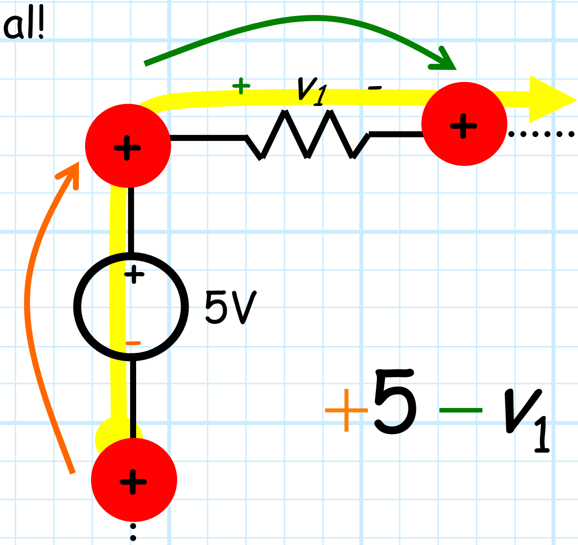


For example as our 1C charge moves through the voltage source, its potential energy is **increases** by 5 Joules (the potential is 5 V higher at the + terminal than it was at the minus terminal)!



But when it moves through the resistor, its potential energy **drops** by v_1 Joules (the potential at the minus terminal is v_1 Volts less than that at the plus terminal).

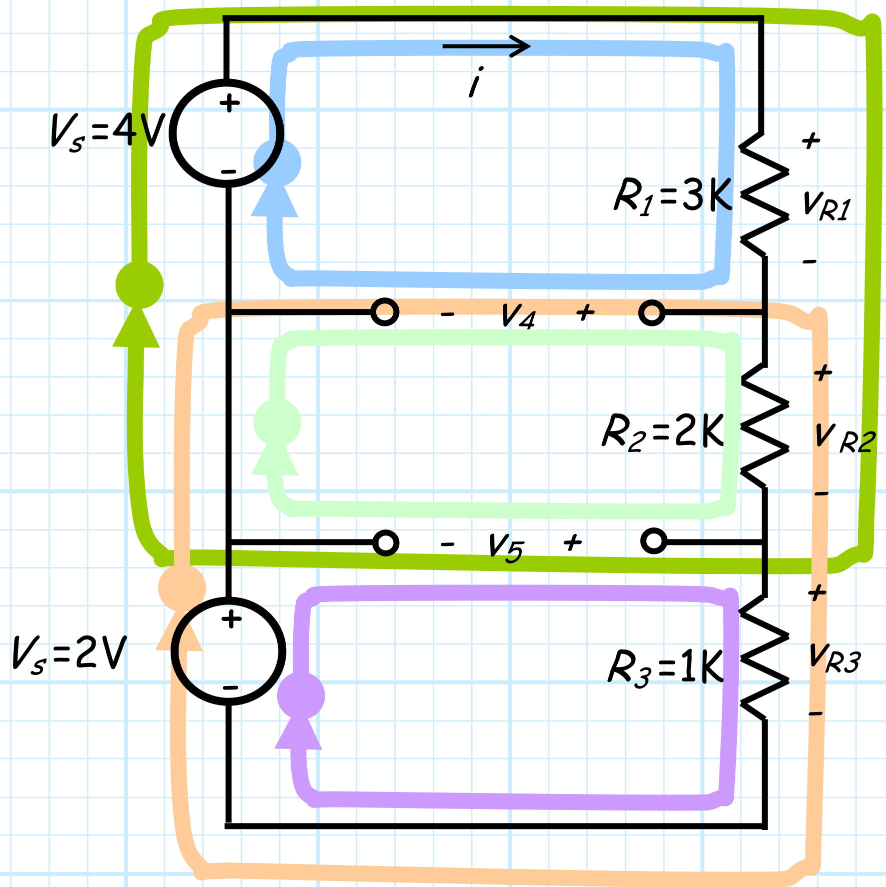
Thus, the second convention is a more accurate "**accounting**" of the change in potential!



$$+5 - v_1 + \cdots$$

This convention is the one typically used for **electronic circuits**. You of course will get the correct answer **either way**, but the second convention allows us to easily determine the **absolute potential** (i.e., with respect to **ground**) at each individual point in a circuit.

To see this, let's return to our original circuit:



The **KVL** from these loops are thus:

$$+4 - v_{R1} - v_4 = 0$$

$$+v_4 - v_{R2} - v_5 = 0$$

$$+2 + v_5 - v_{R3} = 0$$

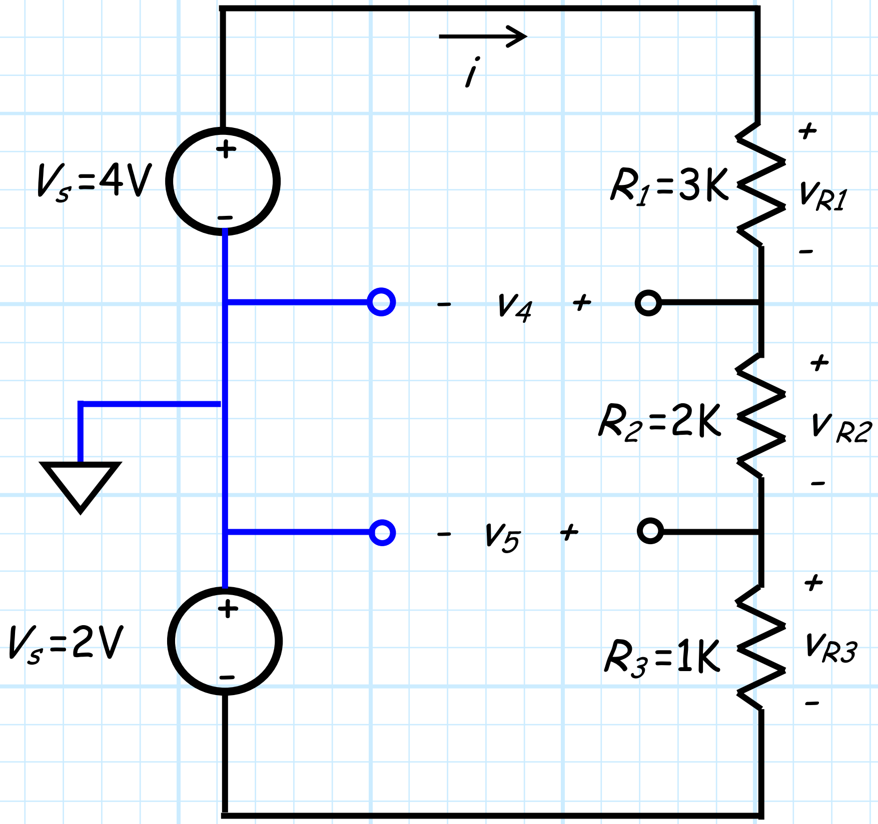
$$+4 - v_{R1} - v_{R2} - v_5 = 0$$

$$+v_4 - v_{R2} - v_{R3} + 2 = 0$$

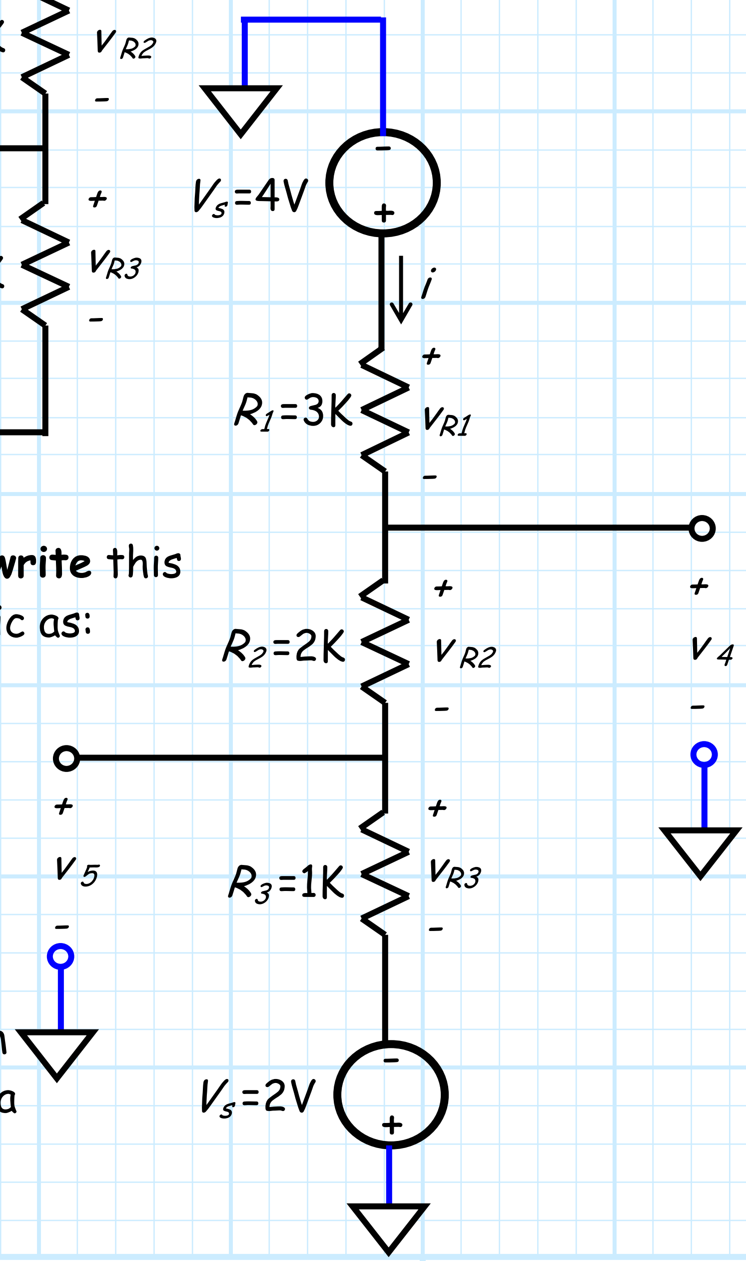
Q: *I don't see how this new convention helps us determine the **"absolute" potenial** at each point in the circuit?*

A: That's because we have not defined a **ground potential**!

Let's do that **now**:



We can thus **rewrite** this circuit schematic as:

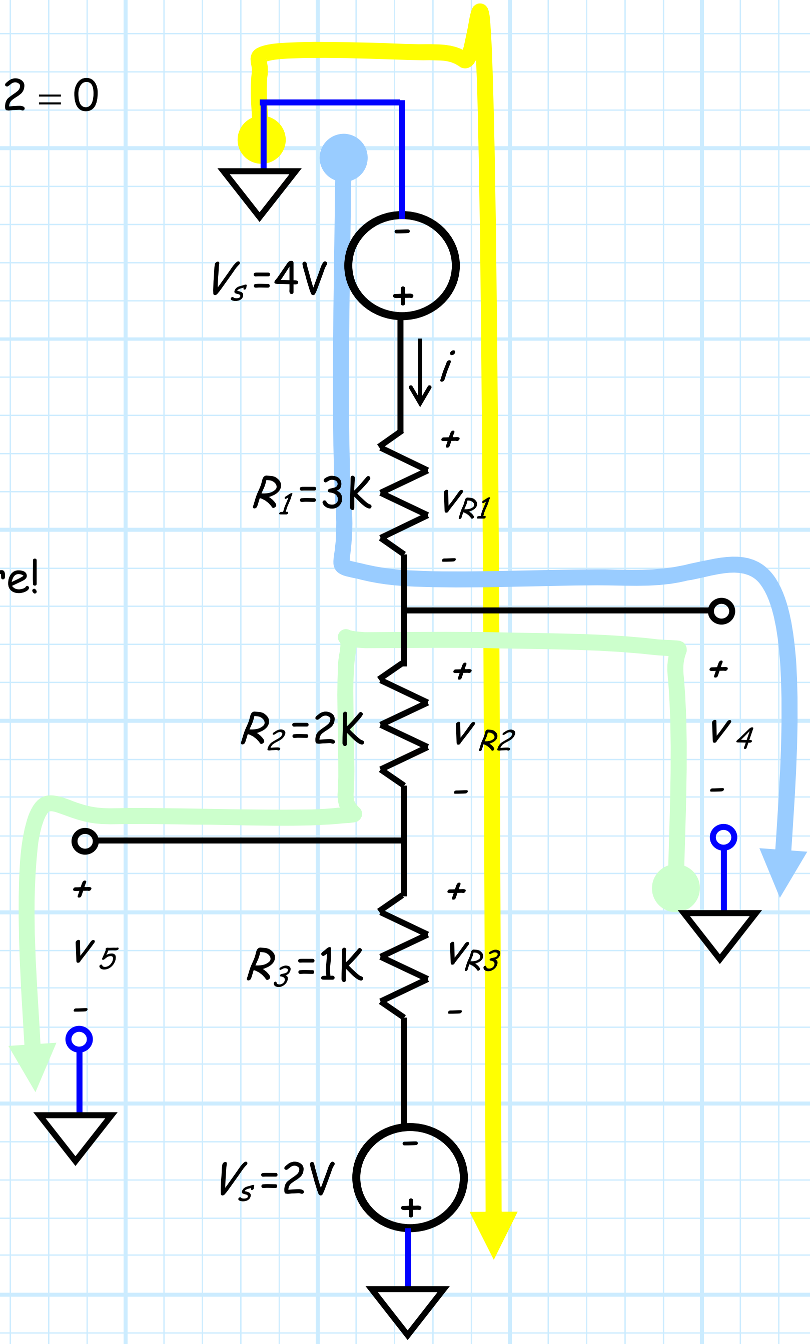


Remember that **all** ground terminals are connected to **each other**, so we can perform KVL by starting and ending at a ground node:

$$+4 - v_{R1} - v_{R2} - v_{R3} + 2 = 0$$

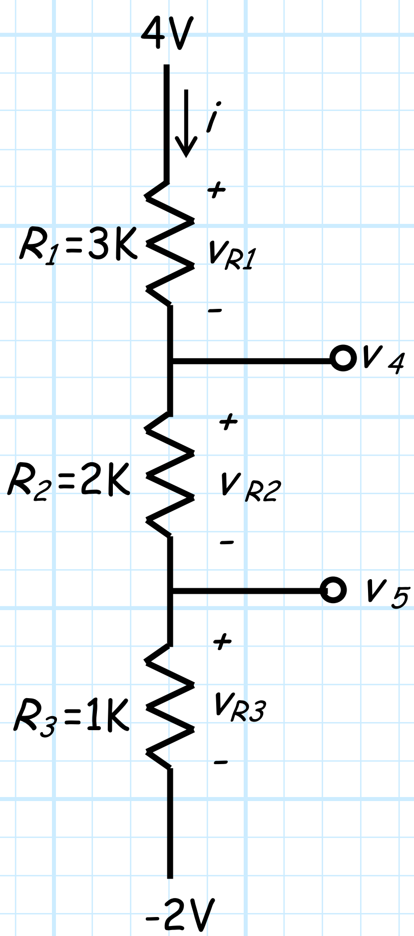
$$+4 - v_{R1} - v_4 = 0$$

$$+v_4 - v_{R2} - v_5 = 0$$

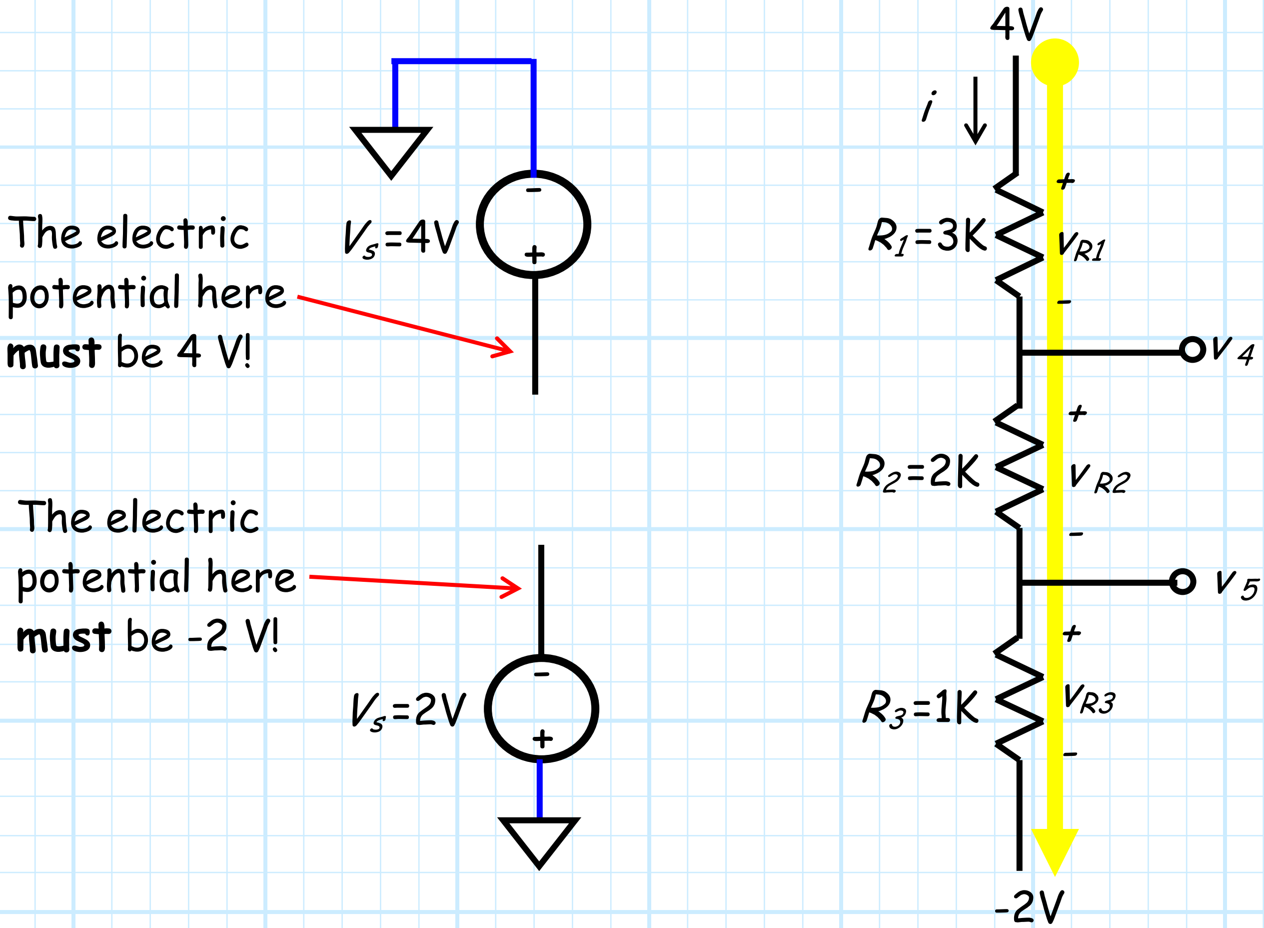


The **same** results as before!

Now, we can **further** simplify the schematic:



Note that we were able to **replace the voltage sources** with a direct, simple statement about the electric potential at two points within the circuit.



Note the KCL equation we determined earlier:

$$+4 - v_{R1} - v_{R2} - v_{R3} + 2 = 0$$

Let's **subtract 2.0** from both sides:

$$+4 - v_{R1} - v_{R2} - v_{R3} = -2$$

This is the **same** equation as before—a valid result from KVL.

Yet, this result has a very interesting **interpretation!**

The value 4.0 V is the **initial electric potential**—the potential at **beginning node** of the "loop" .

The values v_{R1}, v_{R2}, and v_{R3} describe the voltage **drop** as we move through each resistor. The potential is thus **decreased** by these values, and thus they are **subtracted** from the initial potential of 4.0.

When we reach the bottom of the circuit, the potential at that point **wrtg (with respect to ground)** must be equal to:

$$+4 - v_{R1} - v_{R2} - v_{R3}$$

But we **also** know that the potential at the **"bottom"** of the circuit is equal to -2.0 V! Thus we conclude:

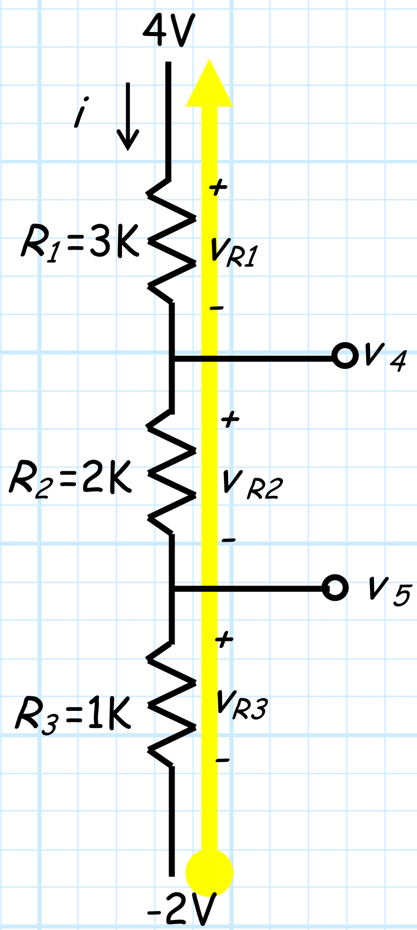
$$+4 - v_{R1} - v_{R2} - v_{R3} = -2$$

Our **KVL** equation!

In general, we can move through a circuit written with or electronic circuit notation with this "law":

*The electric potential at the **initial** node (wrtg), minus(plus) the **voltage drop**(increase) of each circuit element encountered, will be equal to the electric potential at the **final node** (wrtg).*

For **example**, let's analyze our circuit in the opposite direction!



Here, the electric potential at the **first** node is -2.0 volts (wrtg) and the potential at the **last** is 4.0.

Note as we move through the resistors, we find that the potential **increases** by v_R:

$$-2 + v_{R3} + v_{R2} + v_{R1} = 4$$

Note this is the effectively the **same** equation as before:

$$+4 - v_{R1} - v_{R2} - v_{R3} = -2$$

Both equations accurately state KVL, and either will the same correct answer!

Now, we can use our new found knowledge to come to ***these correct conclusions****, see if these results make sense to* ***you****!*

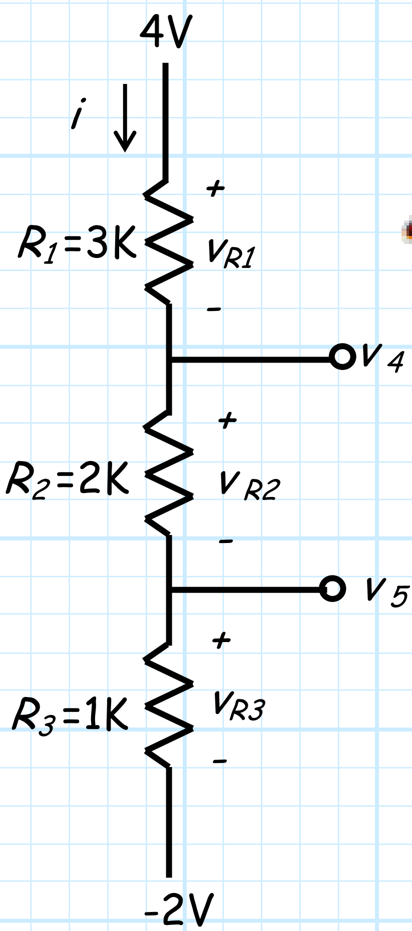
$$4 - v_{R1} = v_4$$

$$v_4 - v_{R2} = v_5$$

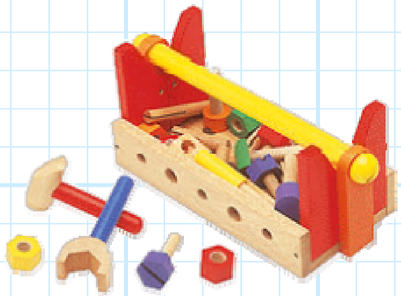
$$-2 + v_{R3} = v_5$$

$$4 - v_{R1} - v_{R2} = v_5$$

$$-2 + v_{R3} + v_{R2} = v_4$$



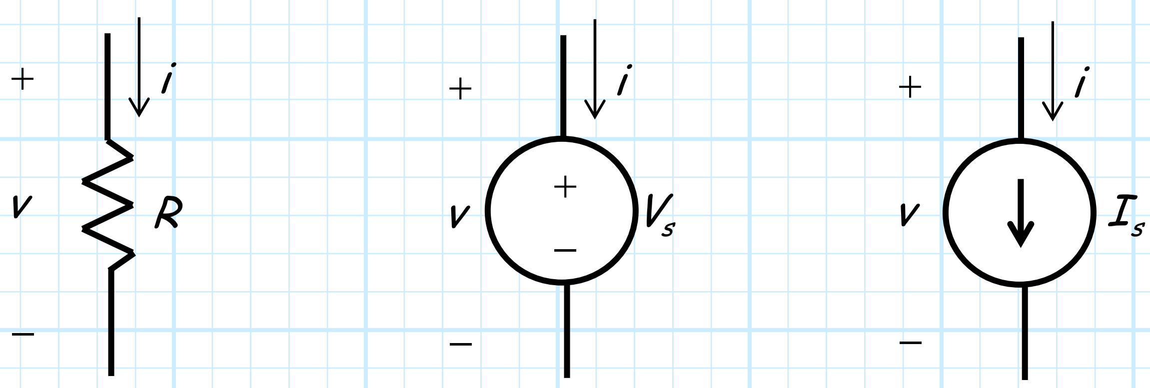
Analysis of Electronic Circuits



In EECS 211 you acquired the **tools** necessary for circuit analysis. Fortunately, all those tools are **still applicable** and useful when analyzing electronic circuits!

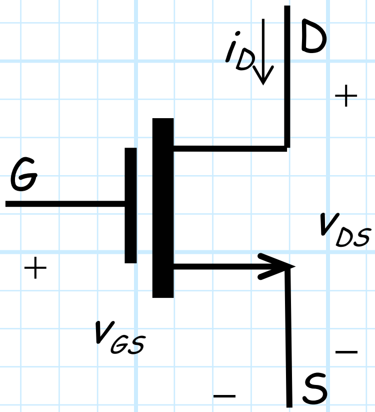
Ohm's Law, KVL and KCL are all still valid, **but** (isn't there always a **but**?) the **complicating** factor in electronic circuit analysis is the **new devices** we will introduce in EECS 312.

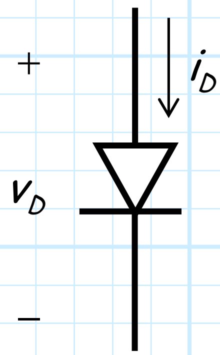
In EECS 211 you learned about devices such as voltage sources, current sources, and resistors. These devices all had very simple **device equations**:



$v = i\,R$ $\qquad$ $v = V_s$ $\qquad$ $i = I_s$

But (that word again!), in EECS 312 we will learn about electronic devices such as **diodes** and **transistors**. The device equations for these new circuit elements will be quite a bit more **complicated**!

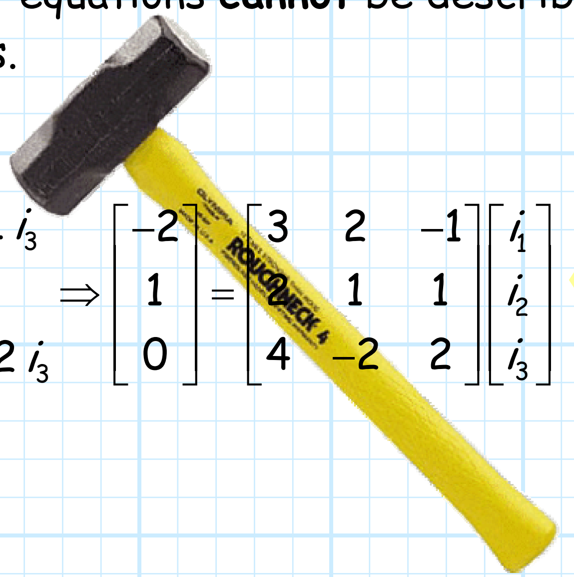




$$i_D = K\left[2\left(v_{GS} - V_t\right)v_{DS} - v_{DS}^2\right]$$

$$i_D = I_S\left(e^{v_D / nV_T} - 1\right)$$

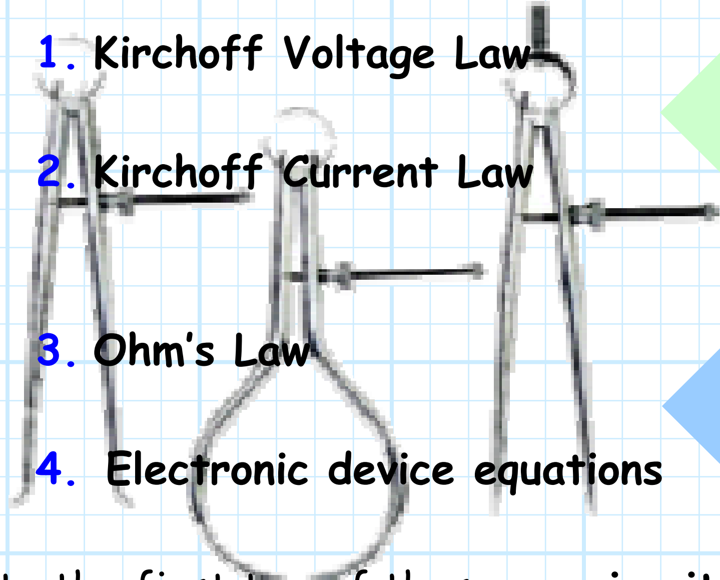
As a result, we often find that both node and mesh analysis tools are a bit clumsy when analyzing electronic circuits. This is because electronic devices are **non-linear**, and so the resulting circuit equations **cannot** be described by as set of **linear** equations.



$$\begin{aligned} -2 &= 3\,i_1 + 2\,i_2 - 1\,i_3 \\ 1 &= 2\,i_1 + 1\,i_2 \\ 0 &= 4\,i_1 - 2\,i_2 + 2\,i_3 \end{aligned} \quad\Rightarrow\quad \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 1 & 1 \\ 4 & -2 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

Not from an electronic circuit!

Instead, we find that electronic circuits are more effectively analyzed by a more **precise** and **subtle** application of:



1. **Kirchoff Voltage Law**
2. **Kirchoff Current Law**

← Circuit Equations

3. **Ohm's Law**
4. **Electronic device equations**

← Device Equations

Note the first two of these are **circuit laws**—they either relate every **voltage** of the circuit to every other **voltage** of the circuit (KVL), or relate every **current** in the circuit to every other **current** in the circuit.

$$I_1 + I_2 + I_3 = 0 \qquad V_1 + V_2 + V_3 = 0$$

The last two items of our list are **device equations**—they relate the **voltage(s)** of a specific device to the **current(s)** of that same device. **Ohm's Law** of course describes the current-voltage behavior of a resistor (but **only** the behavior of a resistor!).

$$V_2 = I_2 \, R_2$$

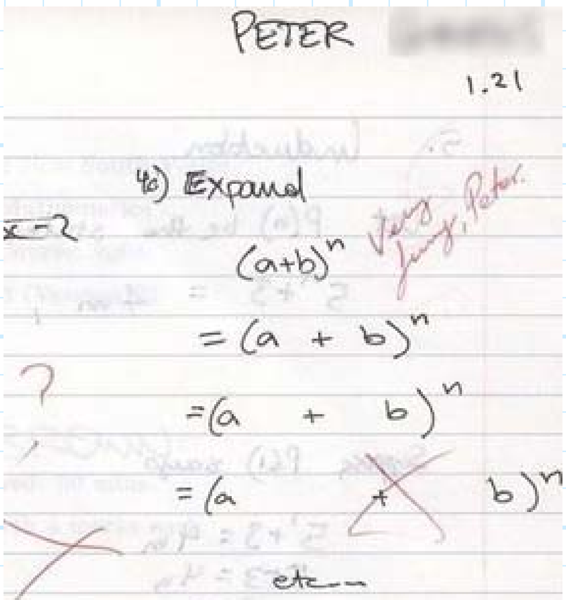
So, if **you**:

1. mathematically state the relationship between all the **currents** in the circuit (using **KCL**), and:

2. mathematically state the relationship between all the **voltages** of the circuit (using **KVL**), and:

3. mathematically state the **current-voltage** relationship of each **device** in the circuit, then:

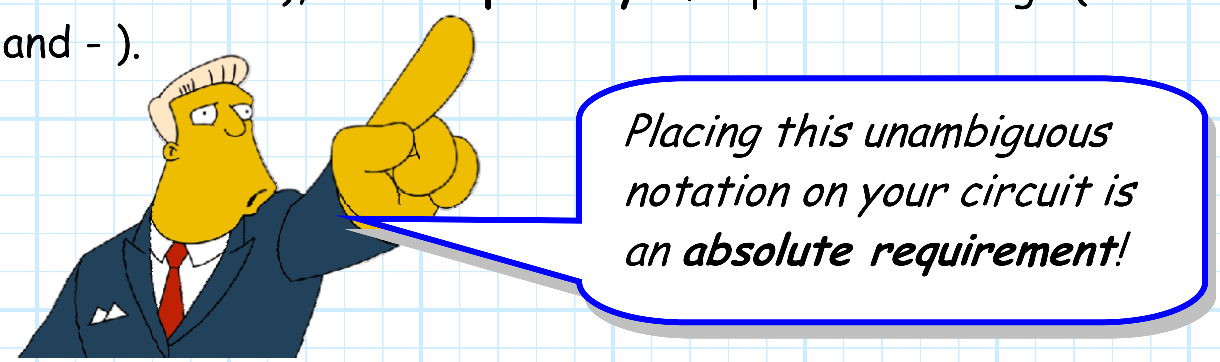
you have mathematically **described** your circuit—**completely!**



At this point you will find that the number of unknown currents and voltages will **equal** the number of equations, and your circuit analysis simply becomes an **algebra problem!**

But be careful! In order to get the correct answer from your analysis, you must unambiguously define each and every voltage and current variable in your circuit!!!!!!!!!

We do this by defining the **direction** of a positive current (with and arrow), and the **polarity** of a positive voltage (with a + and -).



Q: *An absolute requirement in order to **achieve** what?*

A: An absolute requirement in order to:

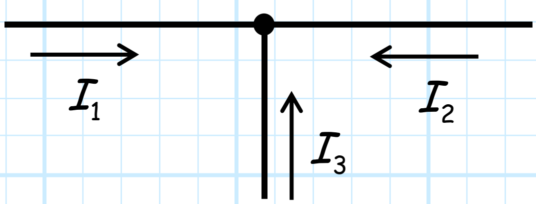
1. determine the **correct answers**.

2. receive **full credit** on exams/homework.

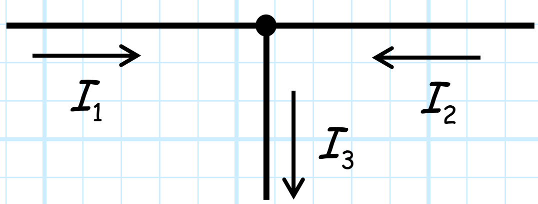
Q: *But **why** must I unambiguously define each current and voltage variable in order to determine the correct answers?*

A: The mathematical expressions (descriptions) of the circuit provided by **KVL**, **KCL** and all device equations are **directly dependent** on the **polarity** and **direction** of each voltage and current definition!

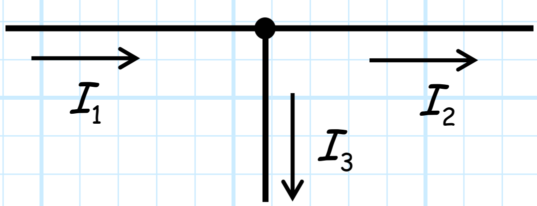
For **example**, consider a three current node, with currents I_1, I_2, I_3. We can of course use KCL to relate these values, but the resulting mathematical expression depends on how we define the direction of these currents:



$$I_1 + I_2 + I_3 = 0$$



$$I_1 + I_2 - I_3 = 0$$
$$\Rightarrow \quad I_1 + I_2 = I_3$$



$$I_1 - I_2 - I_3 = 0$$
$$\Rightarrow \quad I_1 = I_2 + I_3$$

Q: *But that's the problem! How do I **know** which direction the current is flowing in **before** I analyze the circuit?? What if I put the arrow in the **wrong** direction?*

A: Remember, there is **no way** to incorrectly orient the current arrows of voltage polarity for KCL and KVL. If the current or voltage is **opposite** that of your convention, then the numeric result will simply be **negative**.

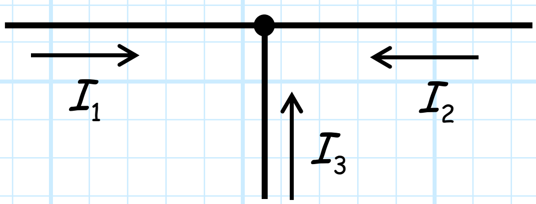
For example, say that in a 3-wire node there is:

3 mA flowing **toward** the node in wire 1

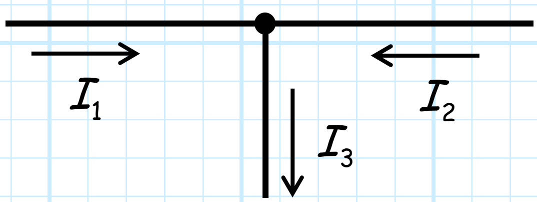
2 mA flowing **toward** the node in wire 2

5 mA flowing **away** from the node in wire 3

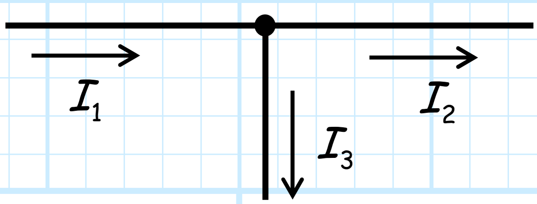
Depending on how you define the currents, the numerical answers for I_1, I_2 and I_3 will all be different, but there physical interpretation will all be the same!



$$I_1 = +3mA,\ I_2 = +2mA,\ I_3 = -5mA$$



$$I_1 = +3mA,\ I_2 = +2mA,\ I_3 = +5mA$$



$$I_1 = +3mA,\ I_2 = -2mA,\ I_3 = +5mA$$

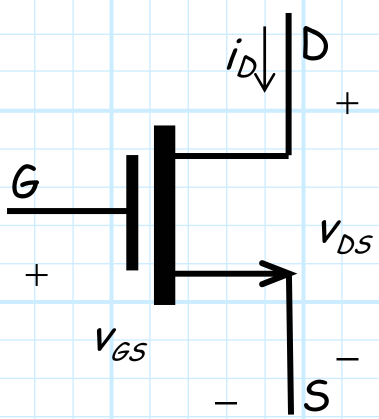
Remember, a **negative** value of current (or voltage) means that the current is flowing in the **opposite** direction (or polarity) of that denoted in the circuit.

So, without current arrows and voltage polarities, there is no way to **physically interpret** positive or negative values!

Now we know that with respect to **KCL** or **KVL**, the current/voltage conventions are **arbitrary** (it up to you to decide!).

However, we will find that the voltage/current conventions of electronic **devices** are **not** generally arbitrary, but instead have **required** orientations.

Q: *Why is that?*



$$v_D = K\left[2\left(v_{GS} - V_t\right)v_{DS} - v_{DS}^2\right]$$

A: The conventions are coupled to electronic device **equations**—these equations are only accurate when using the specific voltage/current **conventions**!

Thus, you must know **both** the device equation and the current/voltage convention for each electronic device. Furthermore, you **must correctly label** and uses these current/voltage conventions in all circuits that contain these devices!

Volts, Milli-Amps, and Kilo-Ohms

Let's determine the **voltage** across a **7 $K\Omega$** resistor if a current of **2 mA** is flowing through it:

$$v = (0.002)(7000) = 1.4\ V$$

Or the **resistance** of a resistor if a current of **2 mA** results in a voltage drop of **20 V**:

$$R = \frac{20}{0.002} = 1000\ \Omega$$

Or the **current** through a **2 $K\Omega$** resistor if the voltage drop across it is **4.0 V**:

$$i = \frac{4}{2000} = 0.2\ mA$$

There's just one big **problem** with this analysis, and that problem is:

The **correct** answers are 14 Volts, 10 KΩ, and 2.0 mA.

The problem of course is all those **decimal places**! It is **easy** to get incorrect answers when resistances are in the **kilo-ohms** (or higher) and the currents are in the **milli-amps** (or smaller).

Unfortunately, that's **exactly** the situation that we have to deal with in electronic circuits!

Frequently, we find that in electronic circuits:

1. **Voltages** are in the range of 0.1 to 50 Volts.

2. **Currents** are in the range of 0.1 to 100 mA.

3. **Resistances** are in the range of 0.1 $K\Omega$ to 50.0 $K\Omega$.

Fortunately, there is an **easy solution** to this problem.

> In **electronic** circuits, the standard unit of voltage is **volts**, the standard unit of current is **milli-amps**, and the standard unit of resistance is **kilo-ohms**.

This works well for **Ohm's Law**, because the product of current **in milli-amps** and resistance **in** $K\Omega$ is voltage in **volts**:

$$v[V] = i[mA] \times R[K\Omega]$$

And so:

$$i[mA] = \frac{v[V]}{R[K\Omega]}$$

$$R[K\Omega] = \frac{v[V]}{i[mA]}$$

The trick then is **not** to numerically express currents in **Amps**, or resistances in **Ohms**, but instead to **leave** the values in ***mA*** and ***K***Ω !!!

For example, let's recompute our **earlier examples** in this way:

The voltage across a **7** $K\Omega$ resistor if a current of **2 mA** is flowing through it:

$$v = 2(7) = 14\ V$$

Or the resistance of a resistor if a current of **2 mA** results in a voltage drop of **20 V**:

$$R = \frac{20}{2} = 10\ K\Omega$$

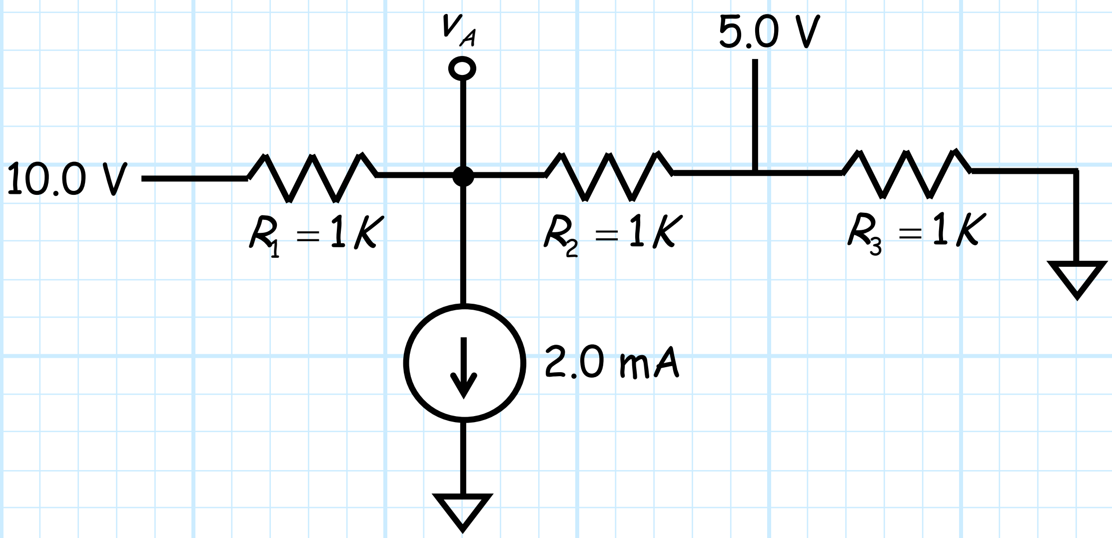
Or the current through a **2** $K\Omega$ resistor if the voltage drop across it is **4.0 V**:

$$i = \frac{4}{2} = 2.0 \; mA$$

Not that these are all **obviously** the correct answers!!!!

Example: Circuit Analysis using Electronic Circuit Notation

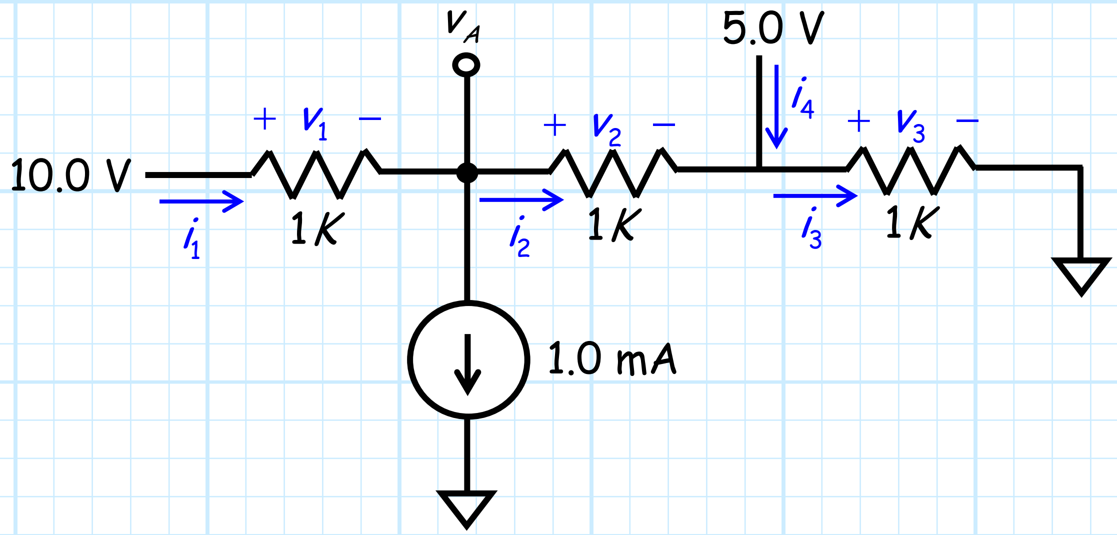
Consider the circuit below:



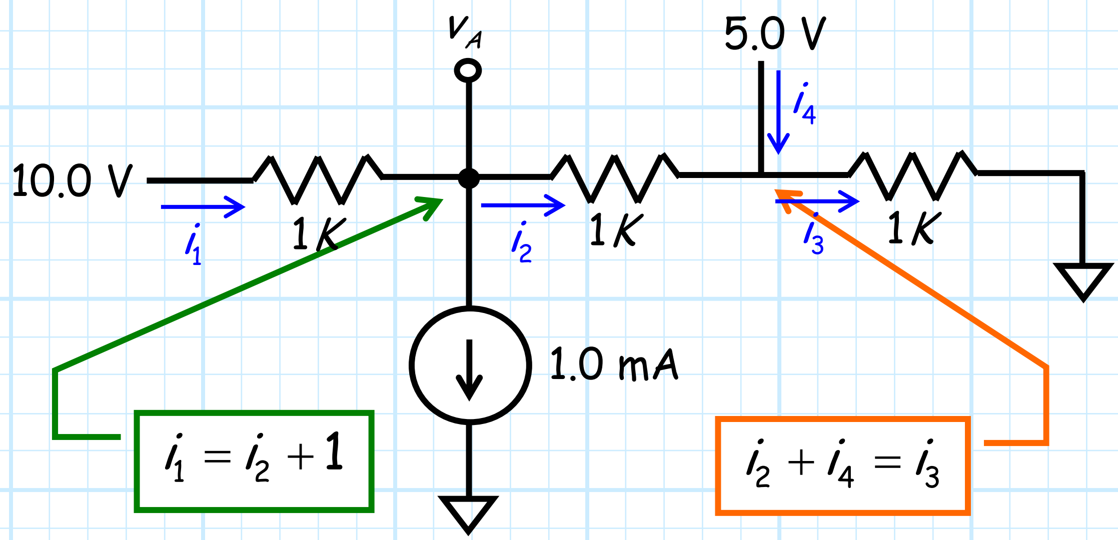
Determine the voltage v_A, **and** the current through each of the three resistors.

Solution

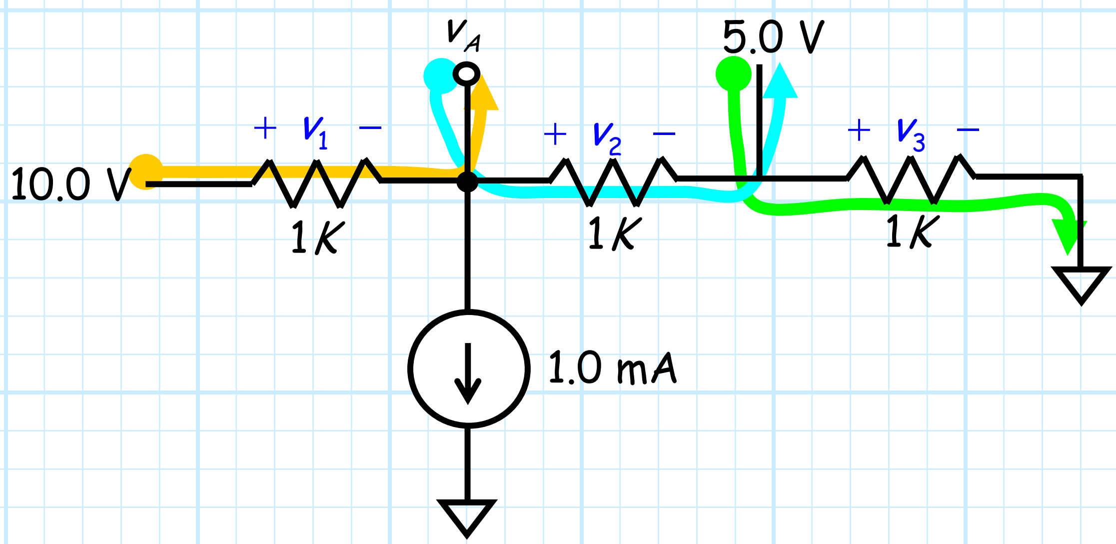
Our first task is to **unambiguously label** the currents and voltages of this circuit:



Now lets relate **all the currents** using **KCL**:



And relate **all the voltages** using **KVL**:



$$10 - v_1 = v_A \quad \Rightarrow \quad v_1 = 10 - v_A$$

$$v_A - v_2 = 5 \quad \Rightarrow \quad v_2 = v_A - 5$$

$$5 - v_3 = 0 \quad \Rightarrow \quad v_3 = \underline{\underline{5.0 \ V}}$$

And finally, a **device equation** for each resistor:

$$i_1 = \frac{v_1}{R_1} \qquad i_2 = \frac{v_2}{R_2} \qquad i_3 = \frac{v_3}{R_3}$$

The equations above provide a **complete mathematical description** of the circuit.

Note there are **eight** unknown variables ($i_1, i_2, i_3, i_4, v_1, v_2, v_3, v_A$), and we have constructed a total of eight equations!

Thus, we simply need to **solve** these **8 equations** for the **8 unknown** values. First, we insert the **KVL** results into our **device equations**:

$$i_1 = \frac{v_1}{R_1} = \frac{10 - v_A}{1} = 10 - v_A$$

$$i_2 = \frac{v_2}{R_2} = \frac{v_A - 5}{1} = v_A - 5$$

$$i_3 = \frac{v_3}{R_3} = \frac{5}{1} = \underline{\underline{5.0\ mA}}$$

And now insert these results into our **KCL** equations:

$$i_1 = i_2 + 1$$
$$10 - v_A = (v_A - 5) + 1$$

and:

$$i_2 + i_4 = i_3$$
$$(v_A - 5) + i_4 = 5$$

Note the **first** KCL equation has a **single** unknown. Solving this equation for v_A:

$$10 - v_A = (v_A - 5) + 1$$

$$\Rightarrow v_A = \frac{10 + 5 - 1}{2} = \frac{14}{2} = \underline{\underline{7.0\ V}}$$

And now solving the **second** KCL equation for i_4:

$$(v_A - 5) + i_4 = 5$$

$$\Rightarrow i_4 = 5 - v_A + 5 = 10 - 7 = \underline{\underline{3.0 \;\; mA}}$$

From these results we can directly determine the **remaining** voltages and currents:

$$v_1 = 10 - v_A = 10 - 7 = \underline{\underline{3.0 \;\; V}}$$

$$v_2 = v_A - 5 = 7 - 5 = \underline{\underline{2.0 \;\; V}}$$

$$i_1 = 10 - v_A = 10 - 7 = \underline{\underline{3.0 \;\; mA}}$$

$$i_2 = v_A - 5 = 7 - 5 = \underline{\underline{2.0 \;\; mA}}$$