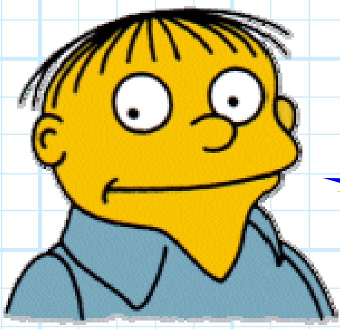
Introduction: Analysis of Electronic Circuits

Reading Assignment: *KVL and KCL text from EECS 211*

Just like EECS 211, the majority of problems (hw and exam) in EECS 312 will be **circuit analysis** problems. Thus, a key to doing well in 312 is to thoroughly **know the material from 211**!!

So, before we get started with 312, let's **review** 211 and see how it applies to **applies to electronic circuits**.



Q: *I **aced** EECS 211 last semester; can I just **skip** this "review"??*

A: Even if you did extremely well in 211, you will want to pay attention to this review. You will see that the concepts of 211 are applied a little **differently** when we analyze **electronic** circuits.

Both the conventions and the approach used for analyzing electronic circuits will **perhaps** be unfamiliar to you at first— I thus imagine that everyone (I hope) will find this review to be **helpful**.

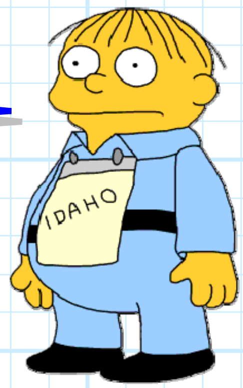
ELECTRONIC CIRCUIT NOTATION

KVL AND ELECTRONIC CIRCUIT NOTATION

ANALYSIS OF ELECTRONIC CIRCUITS

Even the **quantities** of current and resistance are a **little** different for electronic circuits!

Q: *You mean we don't use* ***Amperes*** *and* ***Ohms****??*



A: Not exactly!

VOLTS, MILLI-AMPS, KILO-OHMS

Now let's try an **example**!

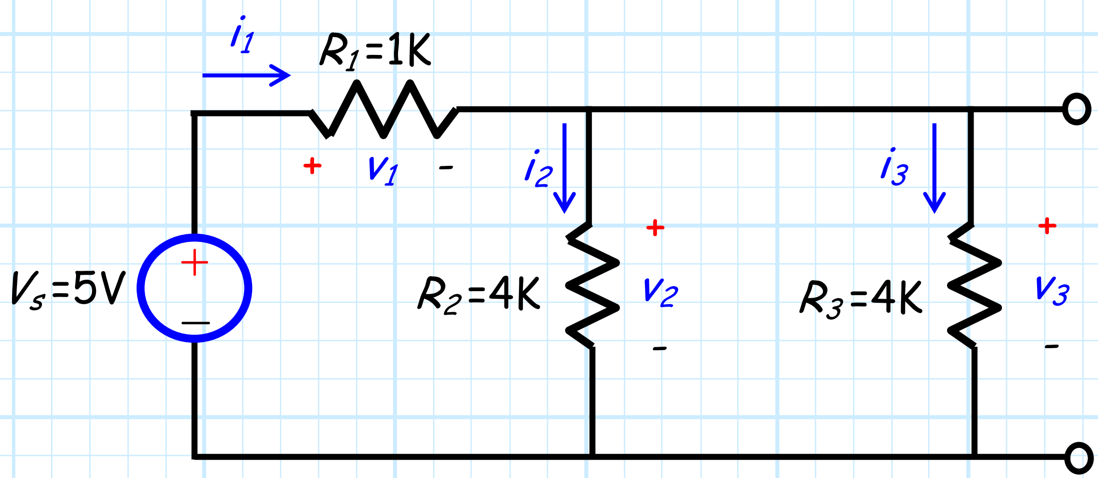
EXAMPLE: CIRCUIT ANALYSIS USING ELECTRONIC CIRCUIT NOTATION

Electronic Circuit Notation

The standard **electronic circuit notation** may be a little **different** than what you used in EECS 211.

The **electronic** circuit notation has a few **"shorthand"** standards that can **simplify** circuit schematics!

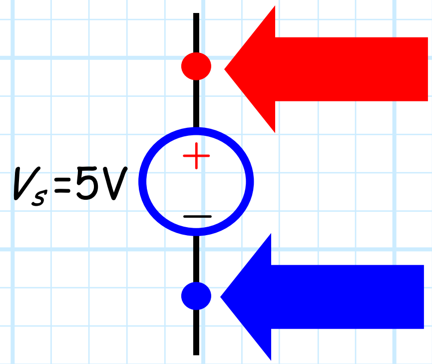
Consider the circuit below:



Note the voltage values in this circuit (i.e., V_s, v_1, v_2, v_3) provide values of potential **difference** between two points in the circuit.

It's the voltage *across* the device!

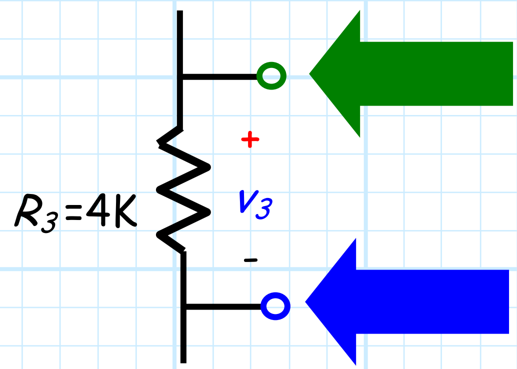
For example, from the **voltage source** we can conclude:



The electric potential at **this** point in the circuit is **5 volts greater** than:

the electric potential at **this** point in the circuit.

Or the **resistor voltage** v_3 means:



The electric potential at **this** point in the circuit is **v_3 volts greater** than:

the electric potential at **this** point in the circuit.

But remember, v_3 could be a **negative** value! Thus, the values of voltages are **comparative**—they tell us the **difference** in electric potential between two points with in the circuit.

KVL of tall buildings

As an **analogy**, Say John, Sally, and Joe work in a very **tall building**. Our circuit voltages are little like saying:

"John is 5 floors above Joe"

"Sally is 2 floors above Joe"

From this **comparative** information we can deduce that John is **3 floors** above Sally.

What we **cannot** determine is on **what floor** John, Sally, or Joe are actually located.

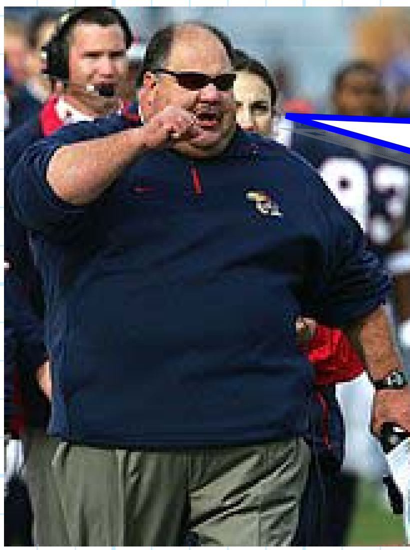
They could be located at the **highest** floors of the building, or at the **lowest** (or anywhere in between).

On what floor is Sally?

Similarly, we **cannot** deduce from the values V_s, v_1, v_2, v_3 the electric potential at each point in the circuit, only the **relative** values—relative to other points in the circuit. E.G.:

"*Point R has an electric potential 5V higher than point B*"

"*Point G has an electric potential v_3 higher than point B*"



Q: *So how do we determine **the** value of electric potential at a specific point in a circuit?*

A: Recall that electric potential at some point is equal to the **potential energy** possessed by 1 Coulomb of charge if located **at** that point.

Thus to determine the "**absolute**" (as opposed to relative) value of the electric potential, we first must determine **where** that electric potential is **zero**.

Everything is relative to ground

The problem is similar to that of the **potential energy** possessed by 1.0 *kg* of mass in a **gravitational field**.

We ask ourselves: **Where** does this potential energy equal **zero**?

The answer of course is when the mass is located **on the ground**!

But this answer is a bit **subjective**; is the "ground":

A. where the carpet is located?

B. where the sidewalk is located?

C. The basement floor?

D. Sea level?

E. The center of the Earth?

The answer is: it can be **any** of these things!

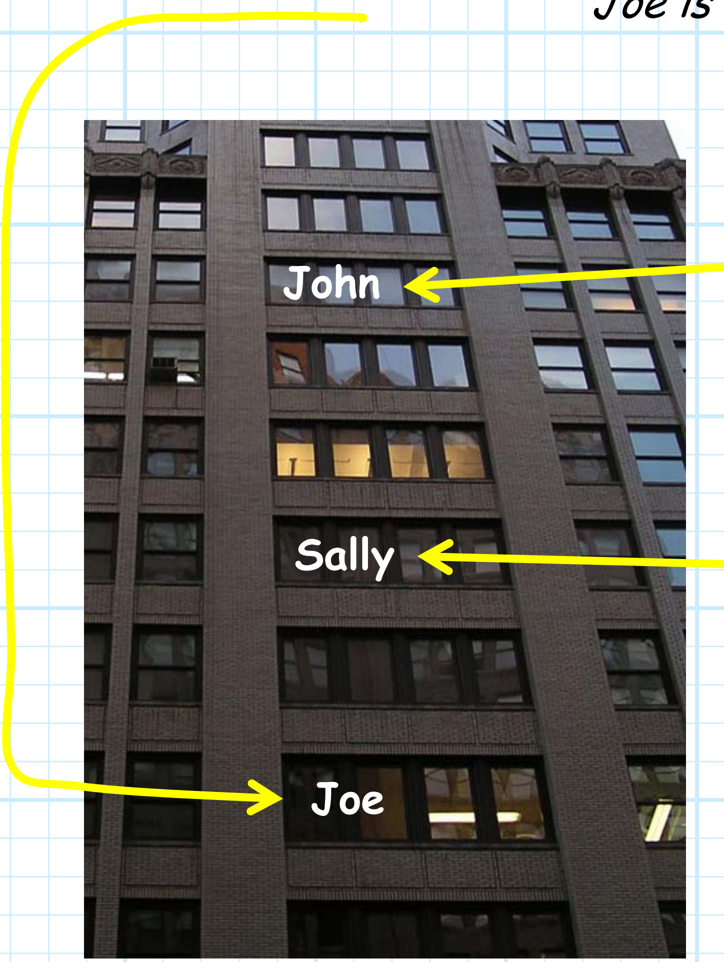
We can rather **arbitrarily** set some point as the location of ground.

The potential energy is therefore described in **reference** to this ground point.

We can now determine where they are—with respect to ground

For tall buildings, the ground floor is usually defined as the floor containing the **front door.** Now, having **defined** a ground reference, if we add to our earlier statements:

"Joe is ***32 floors*** *above ground"*



We can deduce:

*"John is 5 floors above Joe—****therefore*** *John is on the* ***37^{th} floor****"*

*"Sally is 2 floors above Joe—****therefore*** *Sally is on the* ***34^{th} floor****"*

Answer his question, or he will force you to do push-ups

Q: *So, can we **define** a ground potential for our **circuit**?*

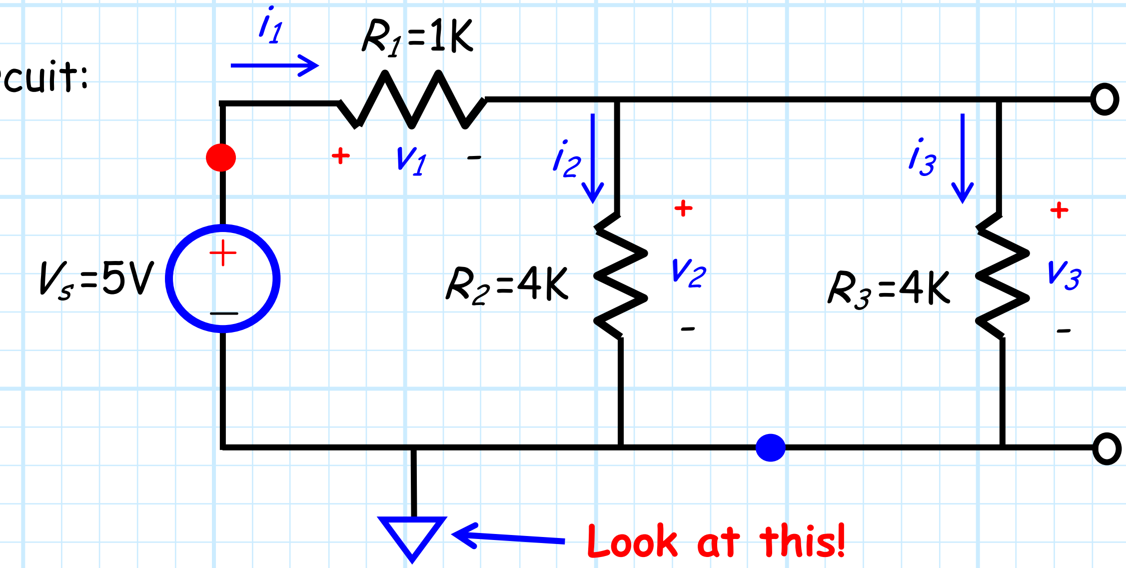
A: Absolutely!

We just **pick** a point on the circuit and **call** it the **ground potential**.

We can then reference the electric potential at every point in the circuit with **respect** to this ground potential!

Ground Potential

Consider **now** the circuit:



Note we have added an **"upside-down triangle"** to the circuit—this denotes the location we define as our **ground potential**!

Now, if we **add** the statement:

*"Point B is at an electric potential of **zero volts** (with respect to ground)."*

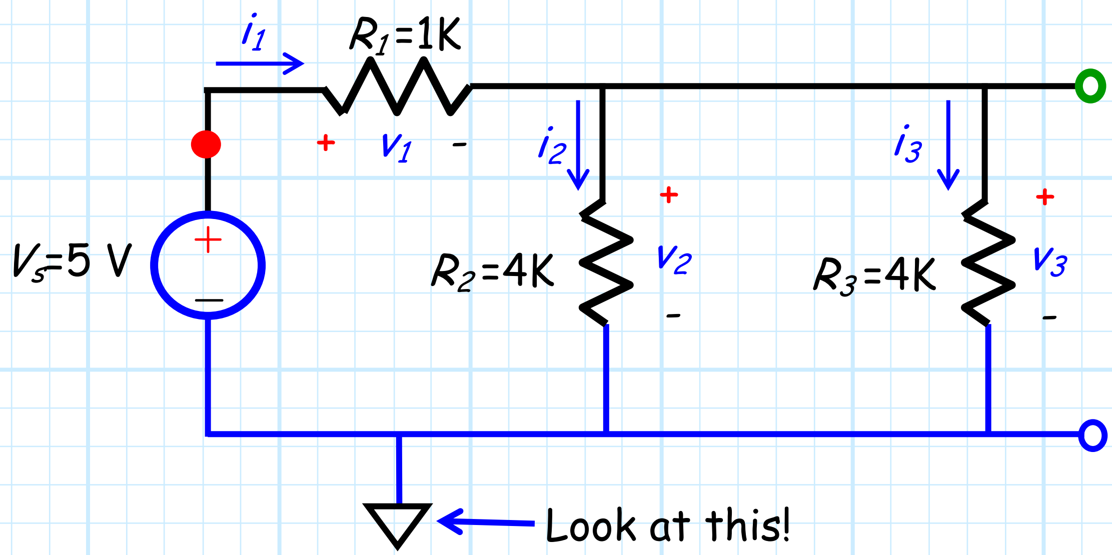
We can conclude:

*"Point R is at an electric potential of **5 Volts** (with respect to ground)."*

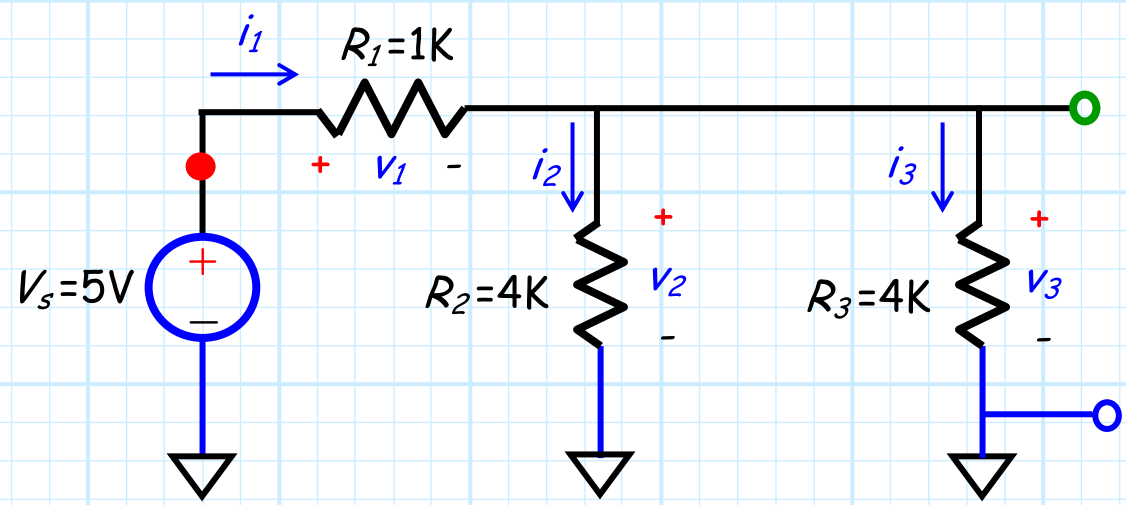
*"Point G is at an electric potential of **v_3 Volts** (with respect to ground)."*

All grounds are connected

Note that all the points within the circuit that reside at ground potential form a rather **large node**:



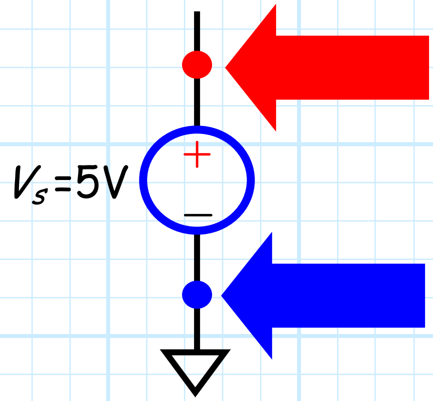
Standard **electronic** notation **simplifies** the schematic by placing the ground symbol at **each device terminal**:



Note that **all** terminals connected to ground are likewise connected to **each other**!

We need not know the particulars

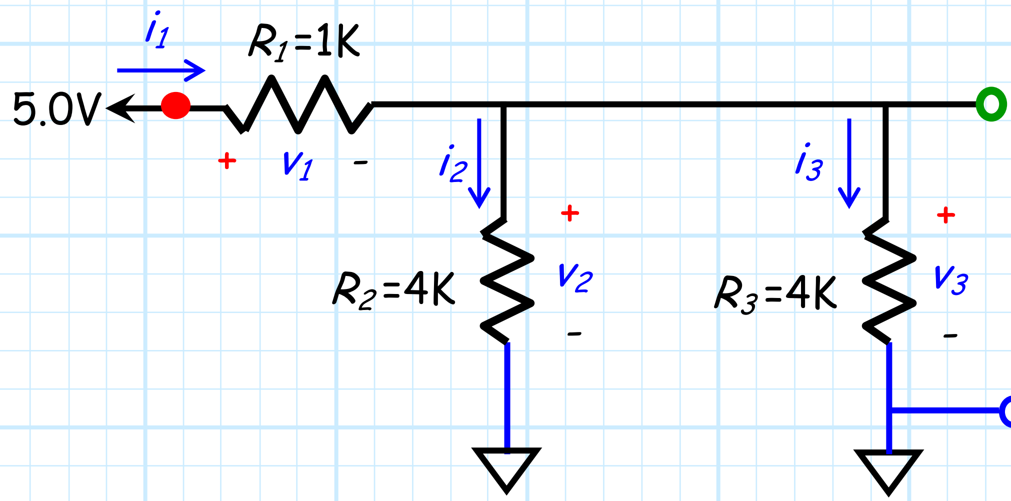
Now, in the case where **one** terminal of a device is connected to **ground** potential, the electric potential (with respect to ground) of the **other** terminal is easily determined:



The electric potential at **this** point in the circuit is **5 volts greater** than ground (i.e., 5 volts).

This point is at **ground** potential (i.e., zero volts).

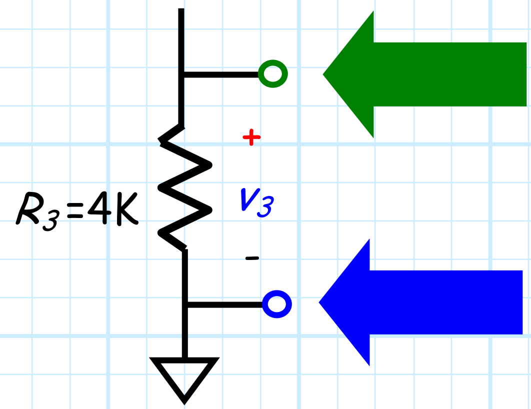
For this example, it is apparent that the voltage source simply **enforces** the condition that the + terminal is at **5.0 Volts with respect to ground**.



Thus, we often **simplify** our electronic circuit schematics as:

We can express the voltage at each node—with respect to ground

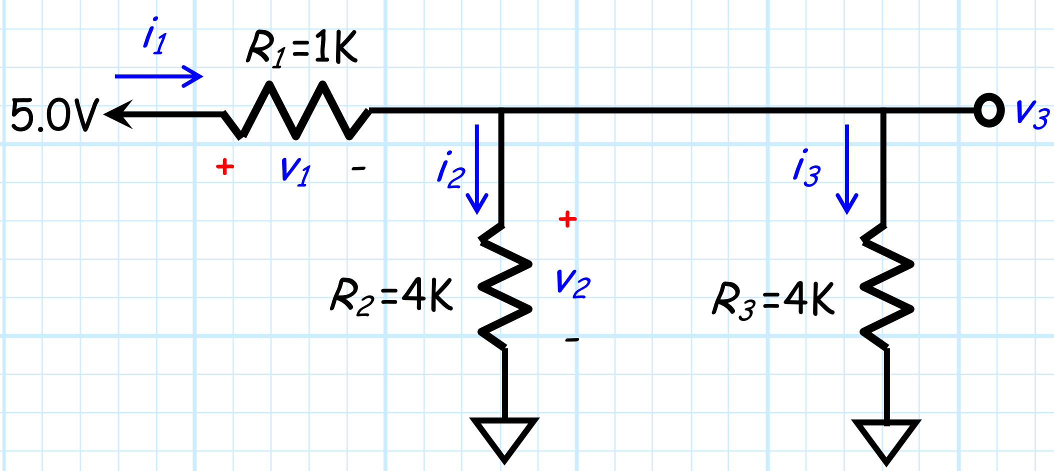
Finally, we find that:



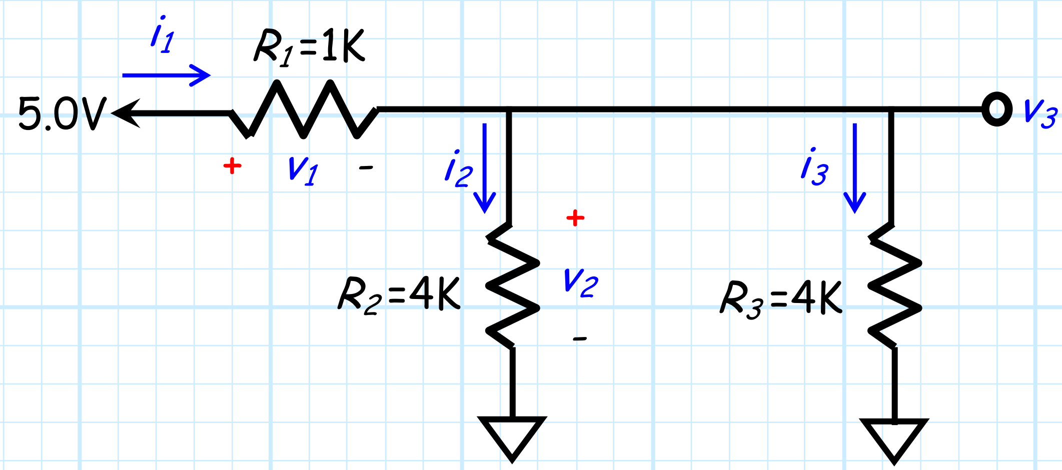
The electric potential at **this** point in the circuit is ***v_2* volts greater** than ground potential (i.e., v_3).

This point is at ground potential (i.e. **zero** volts).

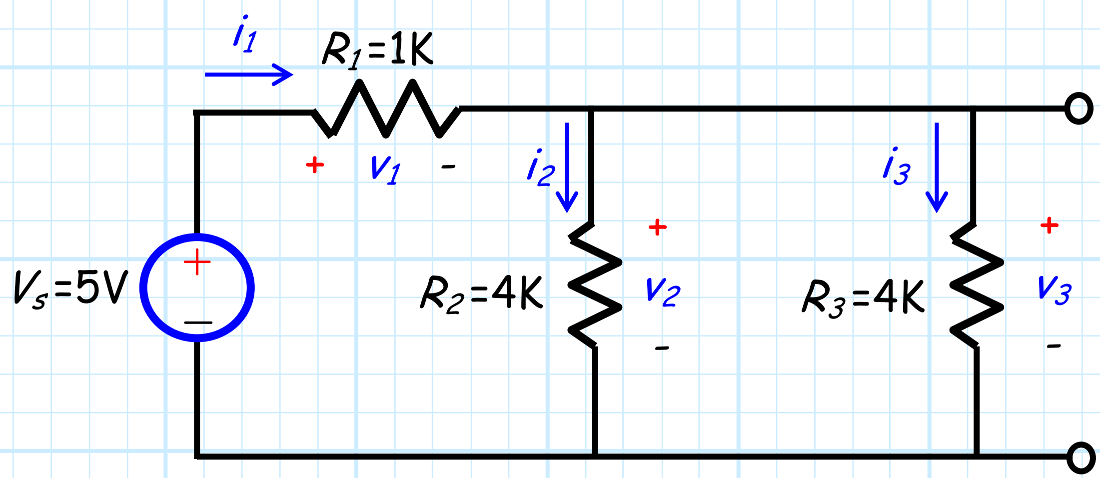
Thus, we can **simplify** our circuit further as:



These two schematics are exactly the same!

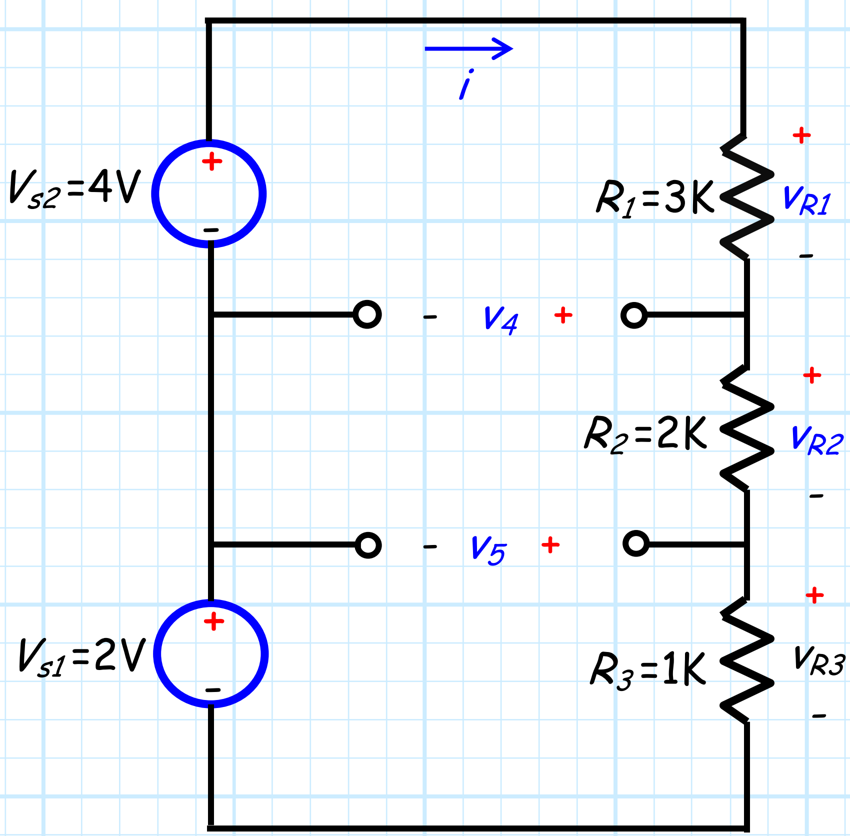


This circuit schematic is **precisely the same** as our **original schematic**:



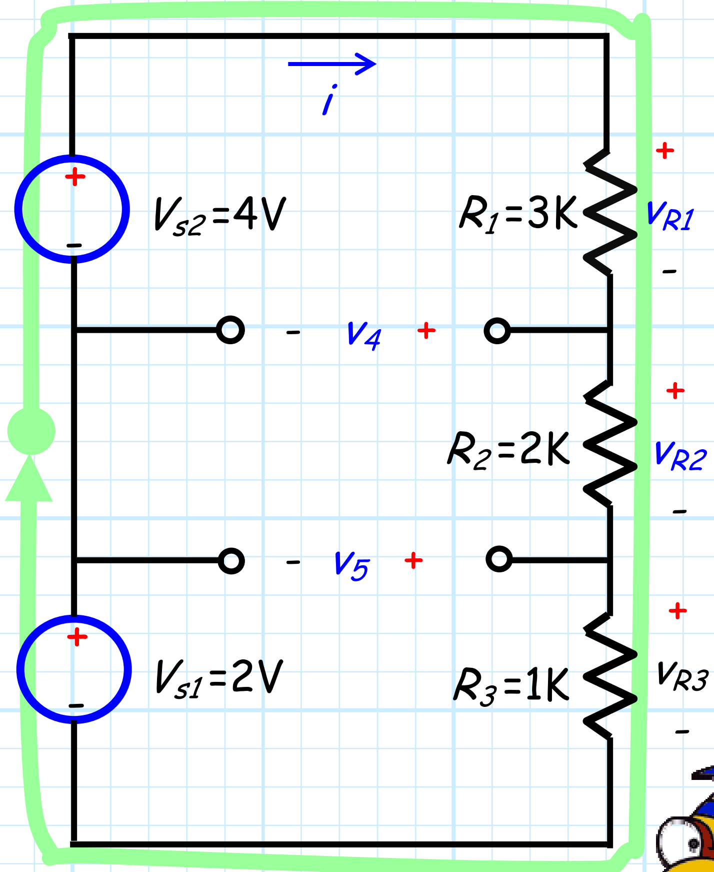
KVL and Electronic Circuit Notation

Consider **this** circuit:



We can apply **Kirchoff's Voltage Law** (KVL) to relate the **voltages** in this circuit in **any number** of ways.

Two equally valid equations



For **example**, the KVL around **this** loop is:

$$-4 + v_{R1} + v_{R2} + v_{R3} - 2 = 0$$

We could multiply both sides of the equation by **-1** and **likewise** get a **valid** equation:

$$4 - v_{R1} - v_{R2} - v_{R3} + 2 = 0$$

Q: *But which equation is correct? Which one do we use? Which one is **the** KVL result?*

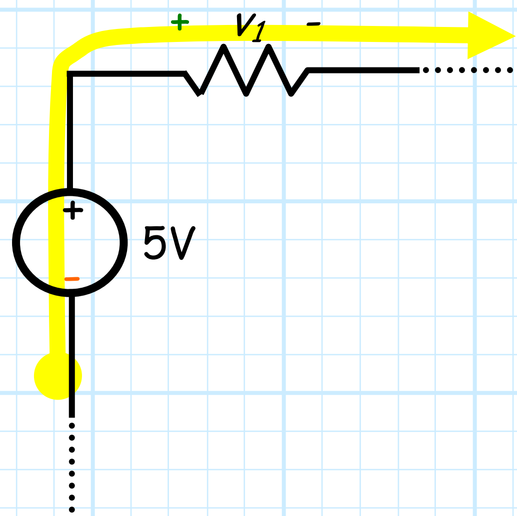
A: Each result is **equally** valid; **both** will provide the same correct answers.

A new convention for KVL

Essentially, the **first** KVL equation is constructed using the **convention** that we **add** the circuit element voltage if we first encounter a **plus** (+) sign as we move along the loop, and **subtract** the circuit element voltage if we first encounter a **minus** (-) sign as we move along the loop.

For example:

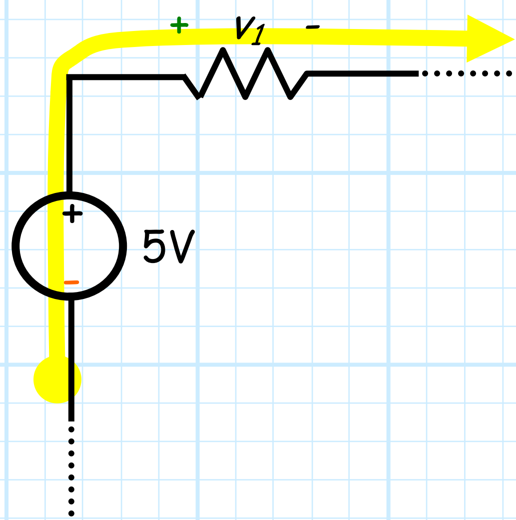
$$-5 + v_1 + \cdots$$



But, we **could** also use the convention that we **subtract** the circuit element voltage if we first encounter a **plus** (+) sign as we move along the loop, and **add** the circuit element voltage if we first encounter a **minus** (-) sign as we move along the loop!

An example

For example:



$$+5 - v_1 + \cdots$$

This convention would provide us with the **second** of the two KVL equations for our original circuit:

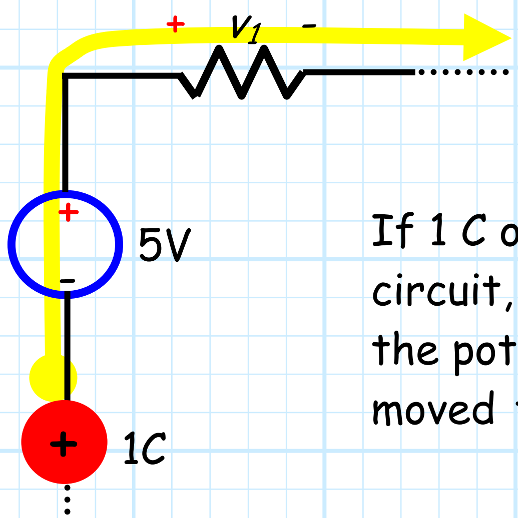
$$4 - v_{R1} - v_{R2} - v_{R2} + 2 = 0$$

Q: *Huh?! What kind of **sense** does this convention make? We **subtract** when encountering a + ? We **add** when encountering a - ??*

A: Actually, this **second** convention is more **logical** than the first if we consider the **physical** meaning of voltage!

Let's keep track of potential energy

Remember, "the voltage" is simply a measure of **potential energy**—the potential energy of **1 Coulomb** of charge.

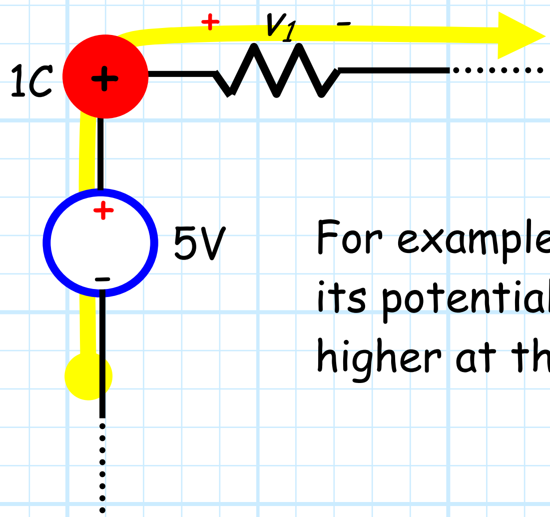


If 1 C of charge were to be **transported** around the circuit, following the **path** defined by our KVL loop, then the potential energy of this charge would **change** as is moved through each circuit element.

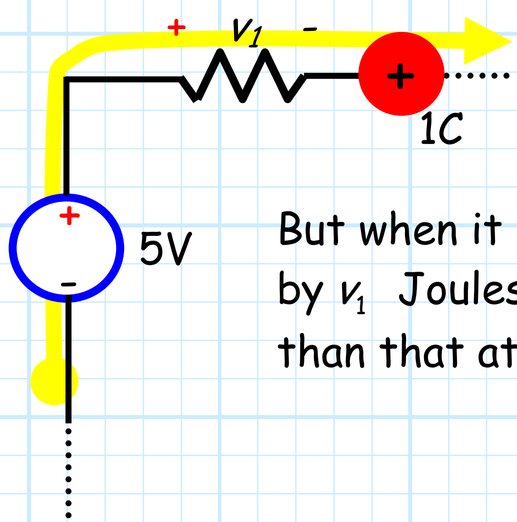
In other words, its potential energy would go **up**, or it would go **down**.

→ The **second** convention describes this increase/decrease!

Make this make sense to you



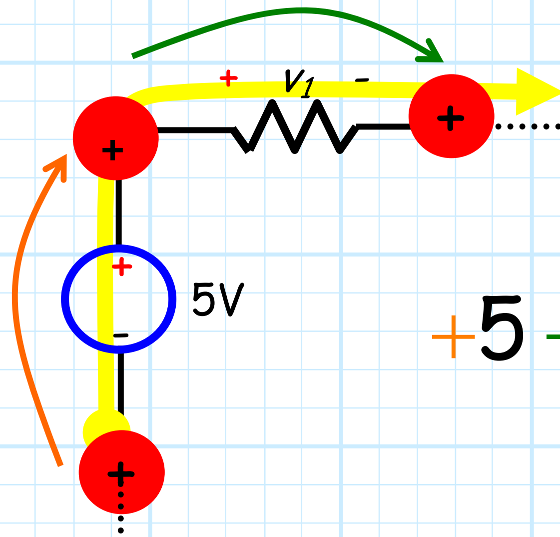
For example, as our 1C charge moves through the voltage source, its potential energy is **increases** by 5 Joules (the potential is 5 V higher at the + terminal than it was at the minus terminal)!



But when it moves through the resistor, its potential energy **drops** by v_1 Joules (the potential at the minus terminal is v_1 Volts less than that at the plus terminal).

Your parents always wanted you to be an accountant

Thus, the second convention is a more accurate "**accounting**" of the change in potential!



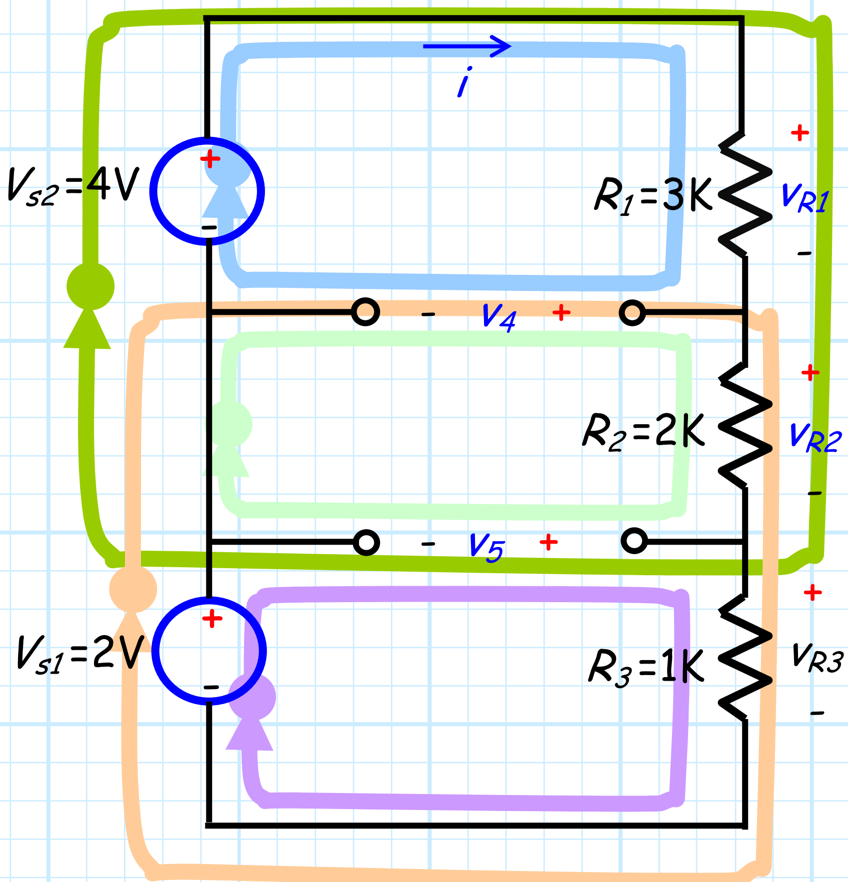
$$+5 - v_1 + \cdots$$

This convention is the one typically used for **electronic circuits**.

You of course will get the correct answer **either way**, but the second convention allows us to easily determine the **absolute potential** (i.e., with respect to **ground**) at each individual point in a circuit.

Using our new convention

To see this, let's return to our **original circuit**:



The **KVL** from these loops are thus:

$$+4 - v_{R1} - v_4 = 0$$

$$+v_4 - v_{R2} - v_5 = 0$$

$$+2 + v_5 - v_{R3} = 0$$

$$+4 - v_{R1} - v_{R2} - v_5 = 0$$

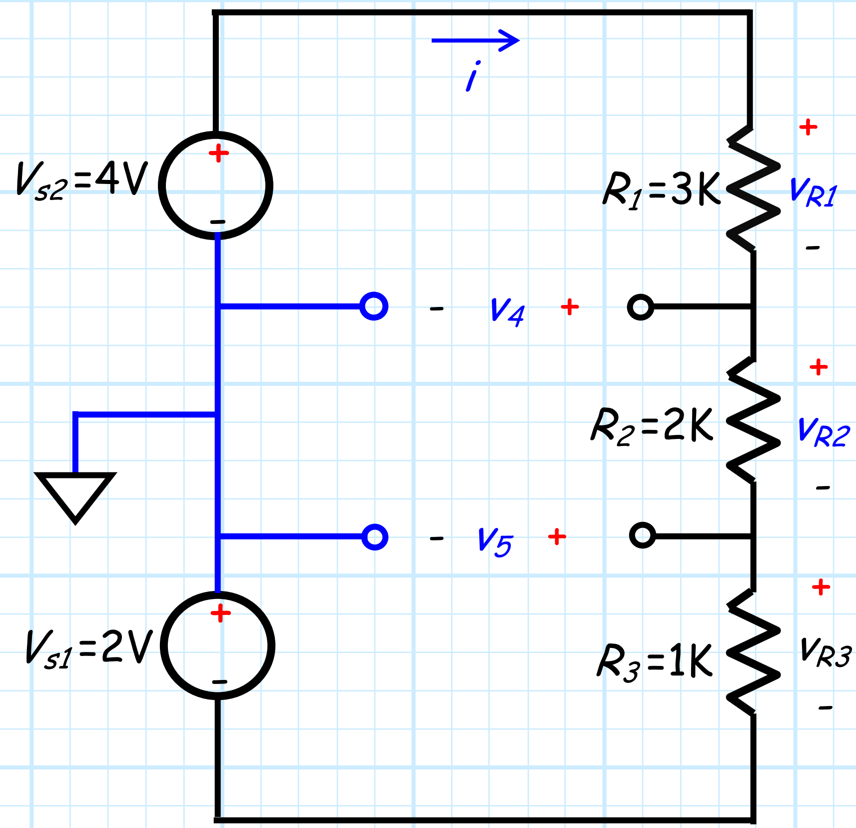
$$+v_4 - v_{R2} - v_{R3} + 2 = 0$$

We need to define ground

Q: *I don't see how this new convention helps us determine the **"absolute" potenial** at each point in the circuit?*

A: That's because we have not defined a **ground potential**!

Let's do that **now**:



See? We get the same results!

We can thus **rewrite** this circuit schematic as:

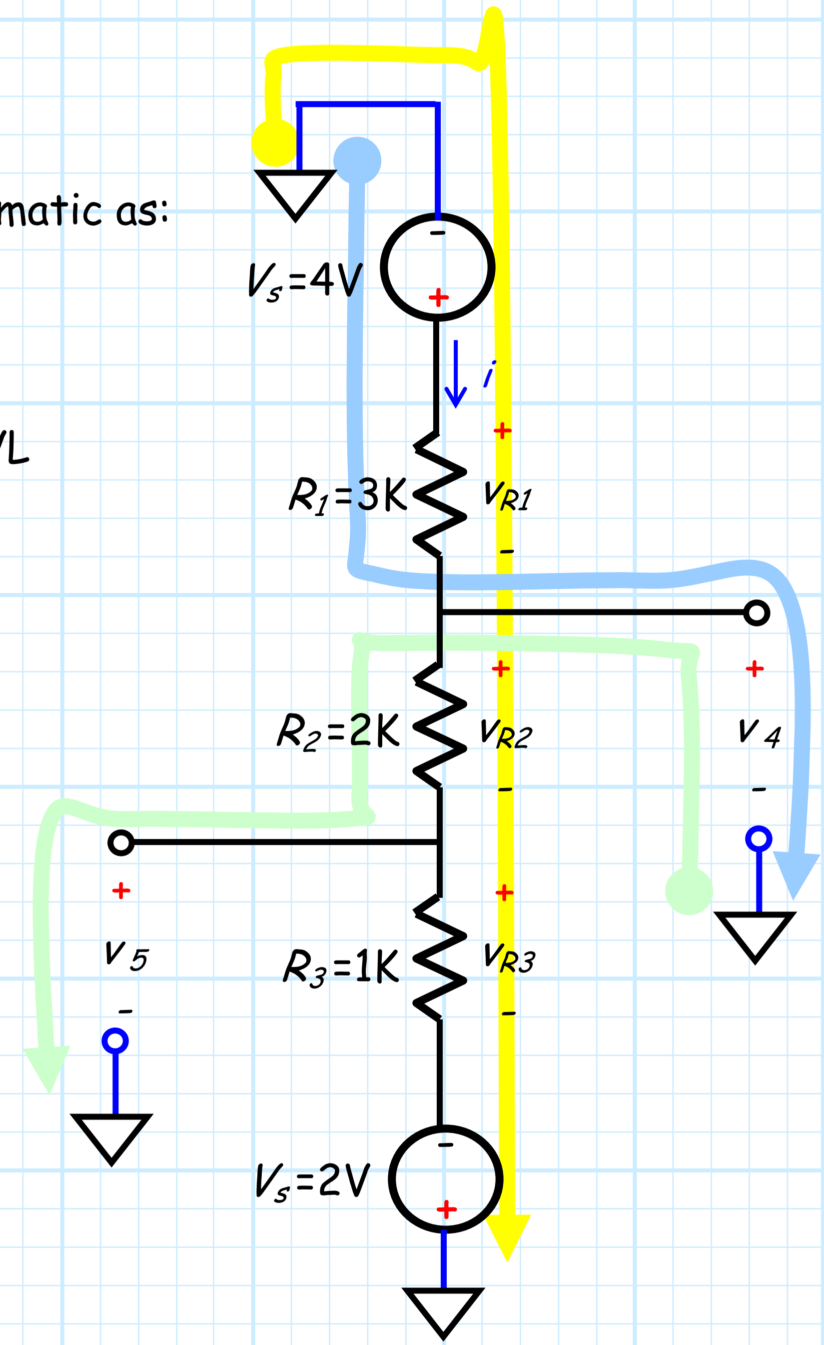
Remember that **all** ground terminals are connected to **each other**, so we can perform KVL by starting and ending at a ground node:

$$+4 - v_{R1} - v_{R2} - v_{R3} + 2 = 0$$

$$+4 - v_{R1} - v_4 = 0$$

$$+v_4 - v_{R2} - v_5 = 0$$

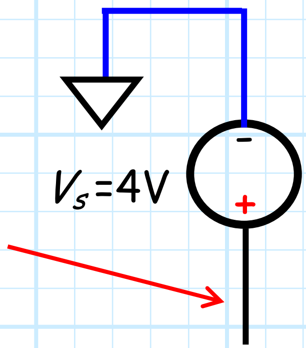
The **same** results as before!



Let's make this really simple

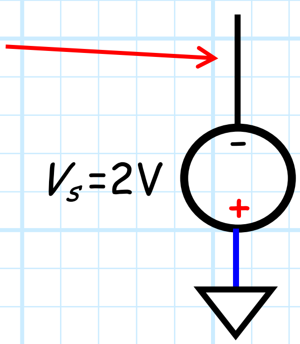
Now, we can **further** simplify the schematic:

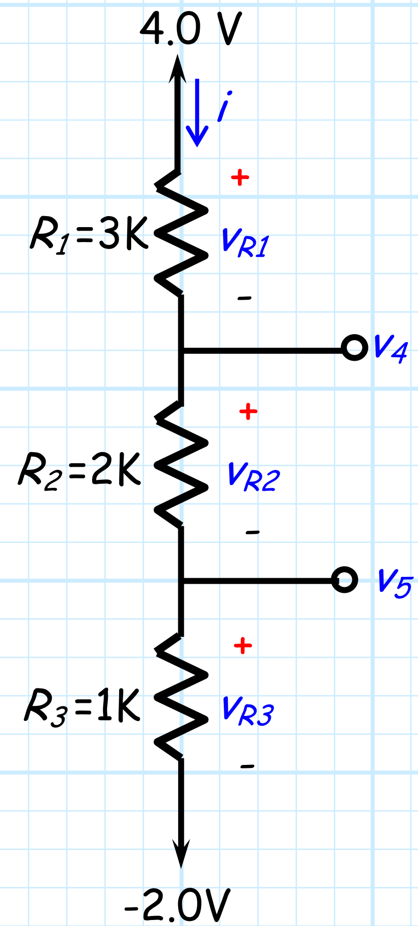
Note that we were able to **replace the voltage sources** with a direct, simple statement about the electric potential at two points within the circuit.



The electric potential here **must** be 4 V!

The electric potential here **must** be -2 V!

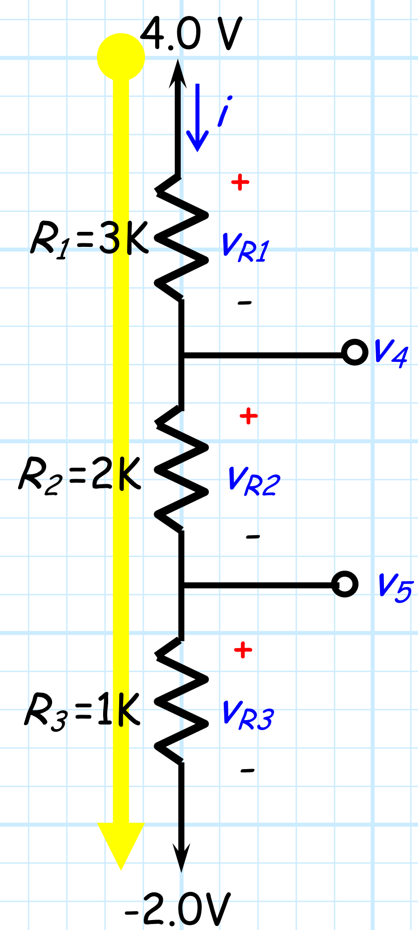




This result makes physical sense

Note the KCL equation we determined earlier:

$$+4 - v_{R1} - v_{R2} - v_{R3} + 2 = 0$$



Let's **subtract 2.0** from both sides:

$$+4 - v_{R1} - v_{R2} - v_{R3} = -2$$

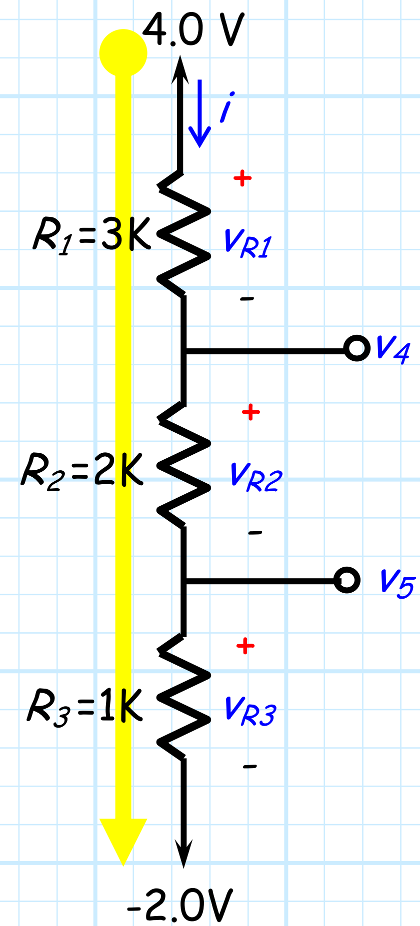
This is the **same** equation as before—a valid result from KVL.

→ Yet, this result has a very interesting **interpretation!**

Must be a parking garage under the building

The value 4.0 V is the **initial electric potential**—the potential at **beginning node** of the "loop" .

The values v_{R1}, v_{R2}, and v_{R3} describe the voltage **drop** as we move through each resistor. The potential is thus **decreased** by these values, and thus they are **subtracted** from the initial potential of 4.0.



When we reach the bottom of the circuit, the potential at that point **wrtg** (**with respect to ground**) must be equal to:

$$+4 - v_{R1} - v_{R2} - v_{R3}$$

But we **also** know that the potential at the **"bottom"** of the circuit is equal to -2.0 V! Thus we conclude:

$$+4 - v_{R1} - v_{R2} - v_{R3} = -2$$

Our **KVL** equation!

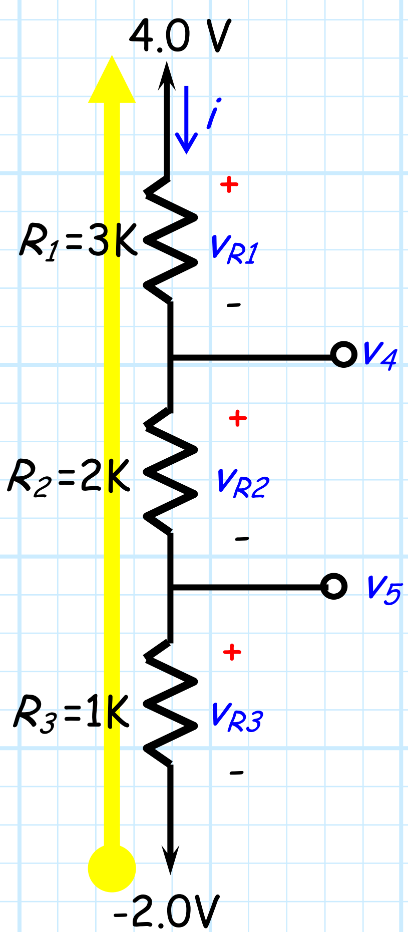
This is not rocket science

In general, we can move through a circuit written with or electronic circuit notation with this "law":

> *The electric potential at the i**nitial** node (wrtg), minus(plus) the **voltage drop**(increase) of each circuit element encountered, will be equal to the electric potential at the **final node** (wrtg).*

Just for fun, let's try this!

For **example**, let's analyze our circuit in the opposite direction!



Here, the electric potential at the **first** node is -2.0 volts (wrtg) and the potential at the **last** is 4.0.

Note as we move through the resistors, we find that the potential **increases** by v_R :

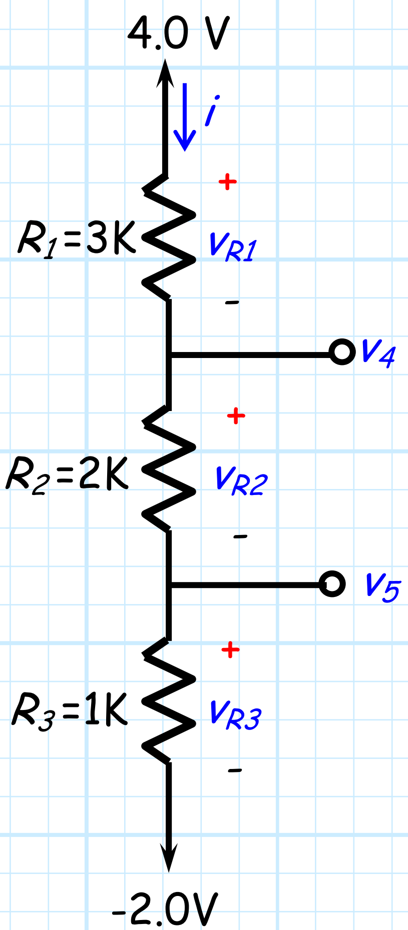
$$-2 + v_{R3} + v_{R2} + v_{R1} = 4$$

Note this is the effectively the **same** equation as before:

$$+4 - v_{R1} - v_{R2} - v_{R3} = -2$$

Both equations accurately state KVL, and either will the same correct answer!

This is the same circuit, and the same KVL equations that we started out with!



Now, we can use our new found knowledge to come to ***these correct conclusions****, see if these results make sense to* ***you****!*

$$4 - v_{R1} = v_4$$

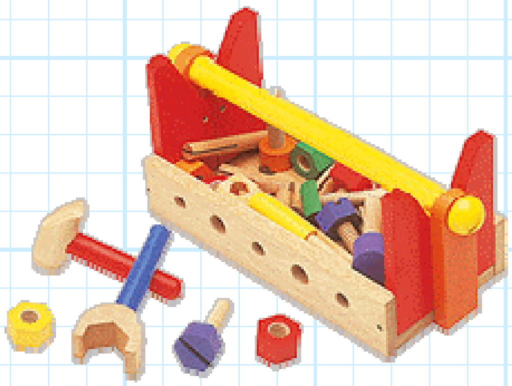
$$v_4 - v_{R2} = v_5$$

$$-2 + v_{R3} = v_5$$

$$4 - v_{R1} - v_{R2} = v_5$$

$$-2 + v_{R3} + v_{R2} = v_4$$

Analysis of Electronic Circuits



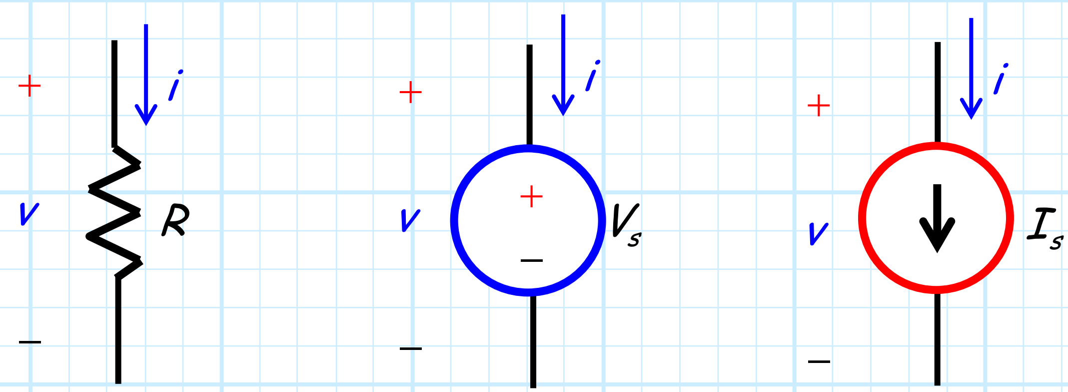
In EECS 211 you acquired the **tools** necessary for circuit analysis.

Fortunately, all those tools are **still applicable** and useful when analyzing electronic circuits!

Ohm's Law, KVL and KCL are all still valid, **but** (isn't there always a **but**?) the **complicating** factor in electronic circuit analysis is the **new devices** we will introduce in EECS 312.

In EECS 211 you learned about devices such as voltage sources, current sources, and resistors.

These devices all had very simple **device equations**:

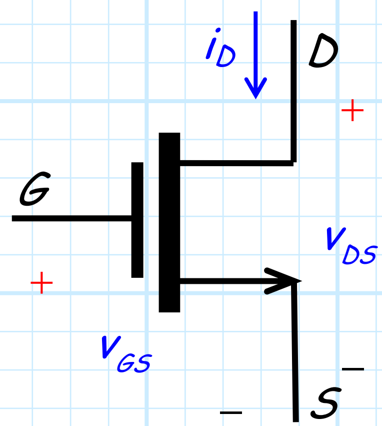


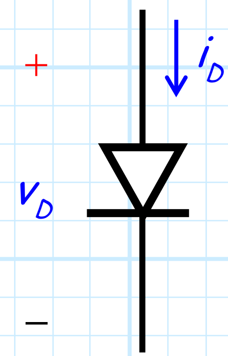
$v = i\,R$ $v = V_s$ $i = I_s$

Two equally valid equations

But (that word again!), in EECS 312 we will learn about electronic devices such as **diodes** and **transistors**.

The device equations for these new circuit elements will be quite a bit more **complicated**!





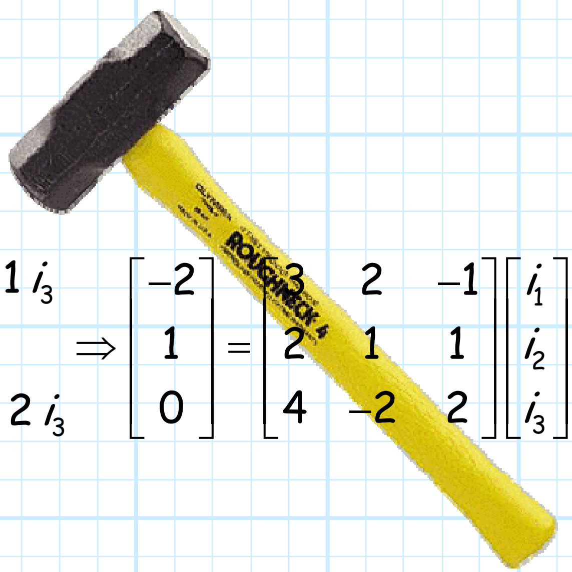
$$i_D = K\left[2\left(v_{GS} - V_t\right)v_{DS} - v_{DS}^2\right]$$

$$i_D = I_S\left(e^{v_D / nV_T} - 1\right)$$

Two equally valid equations

As a result, we often find that both node and mesh analysis tools are a bit clumsy when analyzing electronic circuits.

This is because electronic devices are **non-linear**, and so the resulting circuit equations **cannot** be described by as set of **linear** equations.

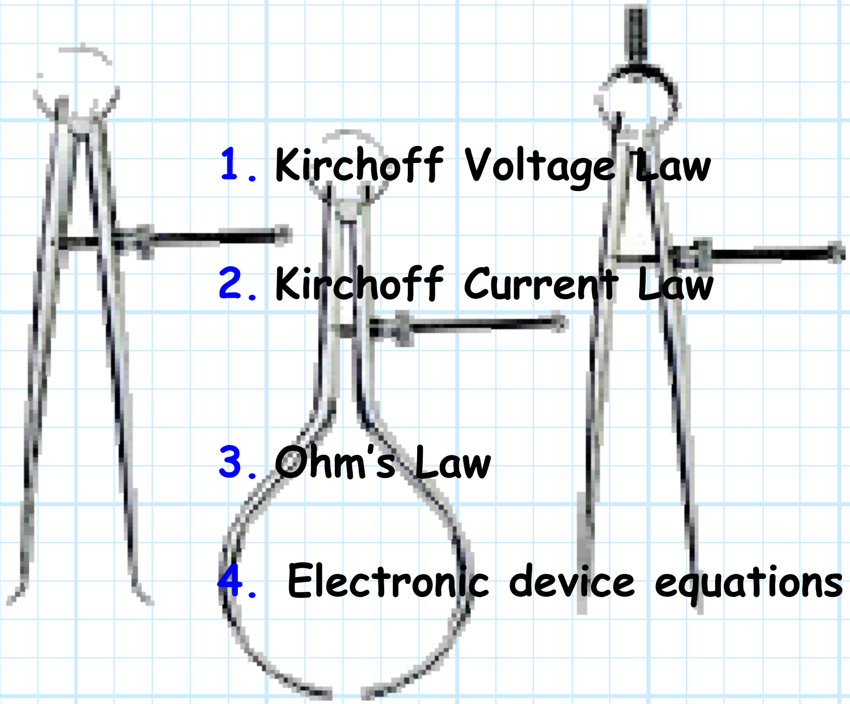


$$\begin{aligned} -2 &= 3\,i_1 + 2\,i_2 - 1\,i_3 \\ 1 &= 2\,i_1 + 1\,i_2 \\ 0 &= 4\,i_1 - 2\,i_2 + 2\,i_3 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 1 & 1 \\ 4 & -2 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

Not from an electronic circuit!

Two equally valid equations

Instead, we find that electronic circuits are more effectively analyzed by a more **precise** and **subtle** application of:



1. Kirchoff Voltage Law
2. Kirchoff Current Law

Circuit Equations

3. Ohm's Law
4. Electronic device equations

Device Equations

Two equally valid equations

Note the first two of these are **circuit laws**—they either relate every **voltage** of the circuit to every other **voltage** of the circuit (KVL), or relate every **current** in the circuit to every other **current** in the circuit.

$$I_1 + I_2 + I_3 = 0 \qquad V_1 + V_2 + V_3 = 0$$

The last two items of our list are **device equations**—they relate the **voltage(s)** of a specific device to the **current(s)** of that same device.

Ohm's Law of course describes the current-voltage behavior of a resistor (but **only** the behavior of a resistor!).

$$V_2 = I_2 \, R_2$$

Two equally valid equations

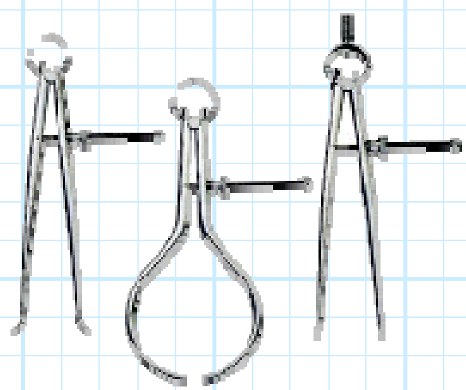
So, if **you**:

1. mathematically state the relationship between all the **currents** in the circuit (using **KCL**), and:

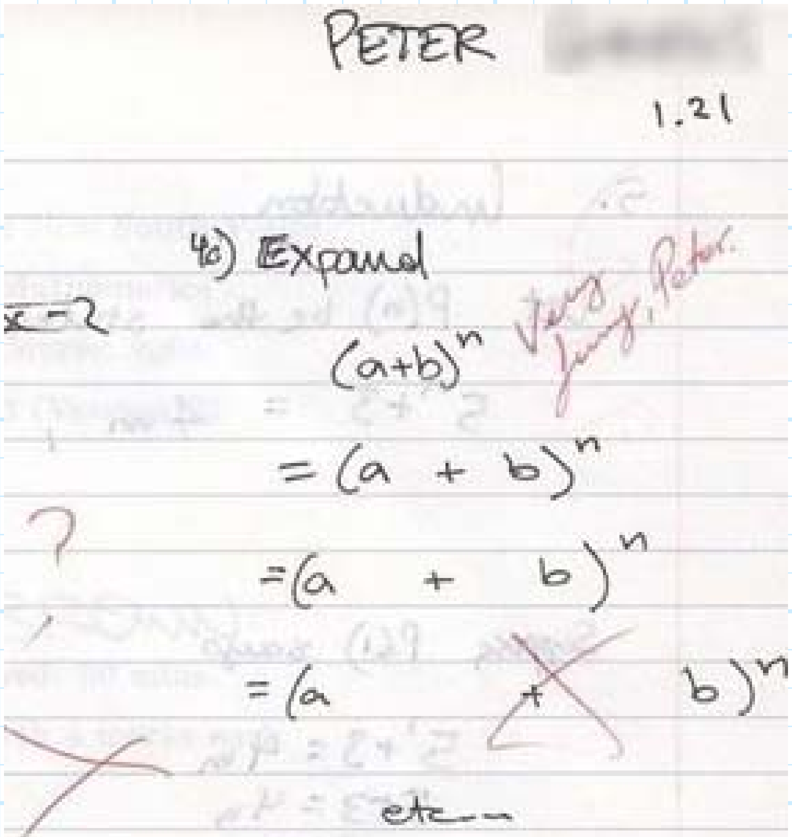
2. mathematically state the relationship between all the **voltages** of the circuit (using **KVL**), and:

3. mathematically state the **current-voltage** relationship of each **device** in the circuit, then:

then you have mathematically **described** your circuit—**completely!**



Two equally valid equations



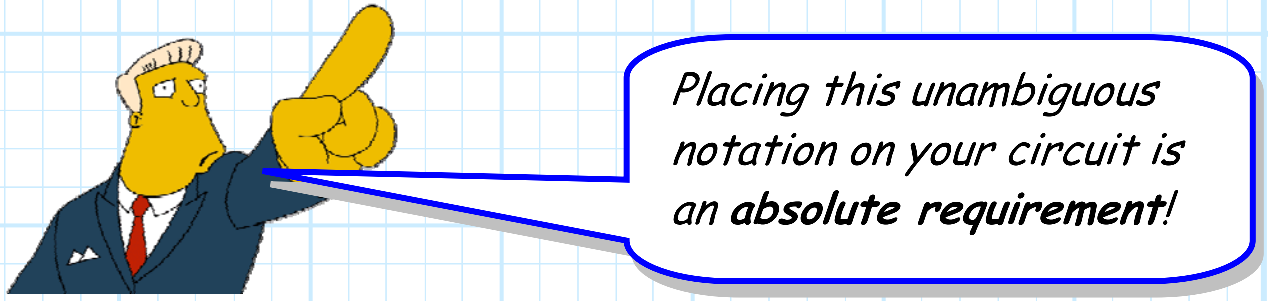
At this point you will find that the number of unknown currents and voltages will **equal** the number of equations, and your circuit analysis simply becomes an **algebra problem!**

But be careful!

In order to get the correct answer from your analysis, you must **unambiguously** define each and **every** voltage and current variable in your circuit!!!!!!!!!

Two equally valid equations

We do this by defining the **direction** of a positive current (with and arrow), and the **polarity** of a positive voltage (with a + and -).



Q: *An absolute requirement in order to **achieve** what?*

A: An **absolute requirement** in order to:

1. determine the **correct answers**.

2. receive **full credit** on exams/homework.

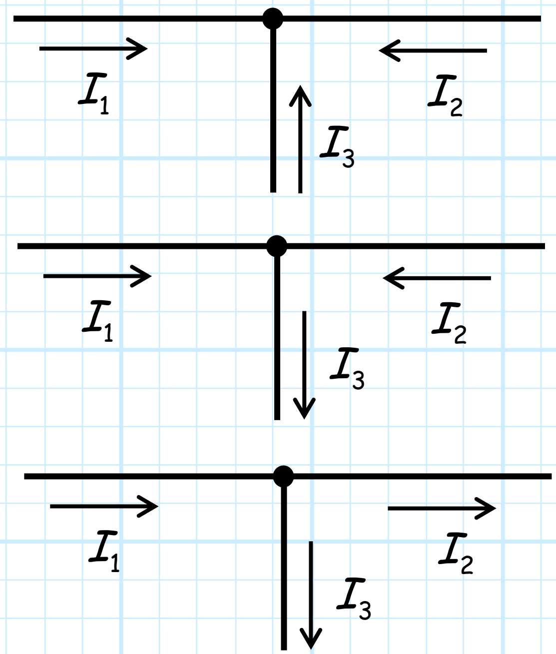
Q: *But **why** must I unambiguously define each current and voltage variable in order to determine the correct answers?*

A: The mathematical expressions (descriptions) of the circuit provided by **KVL**, **KCL** and all device equations are **directly dependent** on the **polarity** and **direction** of each voltage and current definition!

Two equally valid equations

For **example**, consider a three current node, with currents I_1, I_2, I_3.

We can of course use **KCL** to relate these values, but the resulting mathematical expression **depends** on how we **define** the direction of these currents:



$$I_1 + I_2 + I_3 = 0$$

$$I_1 + I_2 - I_3 = 0$$
$$\Rightarrow \quad I_1 + I_2 = I_3$$

$$I_1 - I_2 - I_3 = 0$$
$$\Rightarrow \quad I_1 = I_2 + I_3$$

Q: *But that's the problem! How do I **know** which direction the current is flowing in **before** I analyze the circuit?? What if I put the arrow in the **wrong** direction?*

A: Remember, there is **no way** to incorrectly orient the current arrows of voltage polarity for KCL and KVL.

If the current or voltage is **opposite** that of your convention, then the numeric result will simply be **negative**.

Two equally valid equations

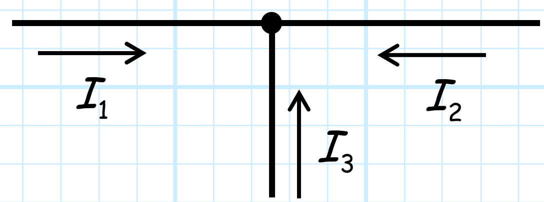
For example, say that in a 3-wire node there is:

3 mA flowing **toward** the node in wire 1

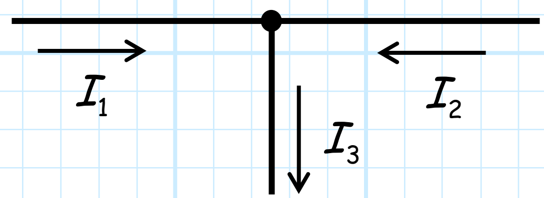
2 mA flowing **toward** the node in wire 2

5 mA flowing **away** from the node in wire 3

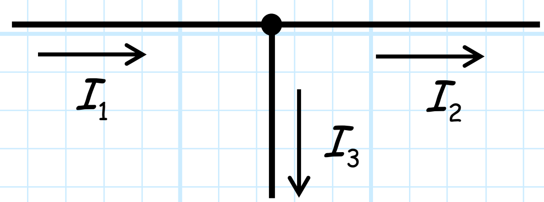
Depending on how you define the currents, the **numerical** answers for I_1, I_2 and I_3 will all be **different**, but there **physical** interpretation will all be the **same**!



$$I_1 = +3mA, I_2 = +2mA, I_3 = -5mA$$



$$I_1 = +3mA, I_2 = +2mA, I_3 = +5mA$$



$$I_1 = +3mA, I_2 = -2mA, I_3 = +5mA$$

Two equally valid equations

Remember, a **negative** value of current (or voltage) means that the current is flowing in the **opposite** direction (or polarity) of that denoted in the circuit.

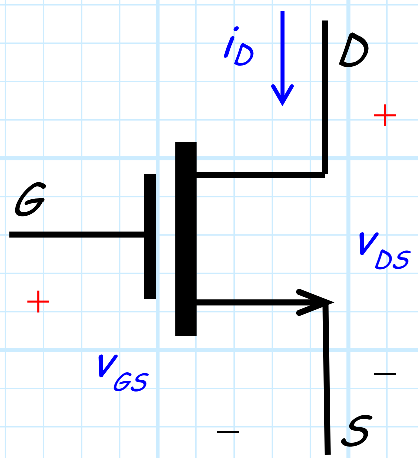
So, without current arrows and voltage polarities, there is no way to **physically interpret** positive or negative values!

Now we know that with respect to **KCL** or **KVL**, the current/voltage conventions are **arbitrary** (it up to you to decide!).

Two equally valid equations

However, we will find that the voltage/current conventions of **electronic devices** are **not** generally arbitrary, but instead have **required** orientations.

Q: *Why is that?*



$$i_D = K\left[2\left(v_{GS} - V_t\right)v_{DS} - v_{DS}^2\right]$$

A: The conventions are coupled to electronic device **equations**—these equations are only accurate when using the specific voltage/current **conventions**!

Thus, you must know **both** the device equation and the current/voltage convention for each electronic device.

Furthermore, you **must correctly label** and uses these current/voltage conventions in all circuits that contain these devices!