Channel Resistance for Small V_{DS}

Recall voltage v_{DS} will be **directly proportional** to i_D, provided that:

1. A conducting channel has been **induced**.

2. The value of v_{DS} **is small**.

Note for this situation, the MOSFET will be in **triode** region.

Recall also that as we **increase** the value of v_{DS}, the conducting channel will begin to **pinch off**—the current will **no longer** be directly proportional to v_{DS} .

Specifically, we have previously determined that there are **two phenomena** at work as we **increase** v_{DS} while in the **triode** region:

1. *Increasing v_{DS} will increase the potential difference across the conducting channel, an effect that works to proportionally increase the drain current i_D*

2. *Increasing v_{DS} will decrease the conductivity of the induced channel, an effect that works to decrease the drain current i_D.*

Q: *That's quite a* ***coincidence****! There are* ***two*** *physical phenomena at work as we increase* v_{DS}*, and there are* ***two*** *terms in the triode drain current equation!*

$$i_D = K\left[2\left(v_{GS} - V_t\right)v_{DS} - v_{DS}^2\right]$$
$$= 2K\left(v_{GS} - V_t\right)v_{DS} - K\,v_{DS}^2$$

A: This is **no** coincidence! **Each** term of the triode current equation effectively describes **one** of these two physical phenomena.

We can thus **separate** the triode drain current equation into **two components**:

$$i_D = i_{D1} + i_{D2}$$

where:

$$i_{D1} = 2K\left(v_{GS} - V_t\right)v_{DS}$$

and:

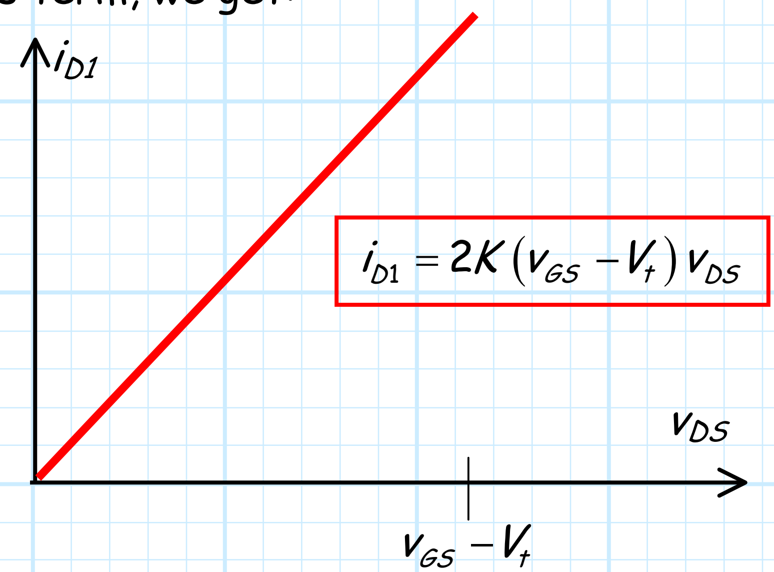
$$i_{D2} = -K\,v_{DS}^2$$

Let's look at each term **individually**.

$$i_{D1} = 2K(v_{GS} - V_t)v_{DS}$$

We first note that this term is **directly proportional** to v_{DS}— if v_{DS} increases 10%, the value of this term will increase 10%. Note that this is true **regardless** of the magnitude of v_{DS}!

Plotting this term, we get:



It is evident that this term describes the **first** of our phenomenon:

1. *Increasing v_{DS} will increase the potential difference across the conducting channel, an effect that works to proportionally increase the drain current i_D.*

In other words, this first term would accurately describe the relationship between i_D and v_{DS} **if** the MOSFET induced channel behaved like a **resistor**!

But of course, it does **not** behave like a resistor! The **second** term i_{D2} describes this very **nonresistor-like** behavior.

$$i_{D2} = -K\, v_{DS}^2$$

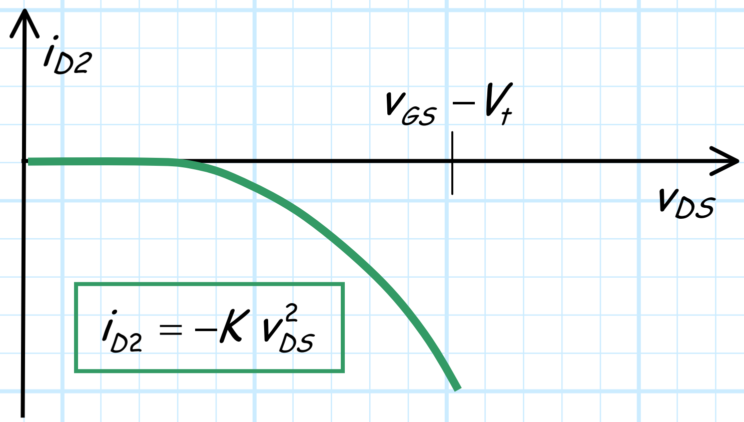
Q: *My Gosh! It is apparent that i_{D2} is **not** directly proportional to v_{DS}, but instead proportional to v_{DS} **squared**!!*

*Moreover, the minus sign out front means that as v_{DS} increases, i_{D2} will actually **decrease**! This behavior is **nothing** like a resistor—what the heck is going on here??*

A: This **second term** i_{D2} essentially describes the result of the **second** phenomena:

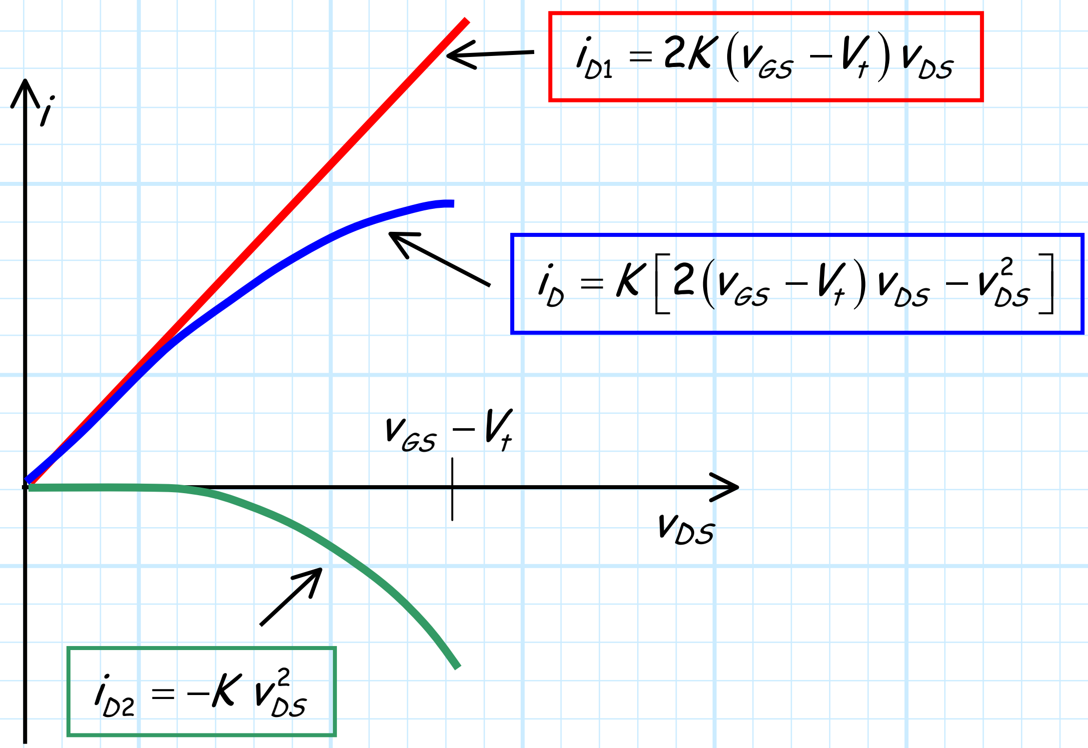
> **2.** *Increasing v_{DS} will decrease the conductivity of the induced channel, an effect that works to decrease the drain current i_D.*

Plotting this term, we get:



A **very** unresistor-like behavior !

Now let's **add** the two terms i_{D1} and i_{D2} together to get the **total triode drain current** i_D :



It is apparent that the second term i_{D2} works to **reduce** the total drain current from its "**resistor-like**" value i_{D1}. This of course is physically due to the **reduction in channel conductivity** as v_{DS} increases.

Q: *But look! It appears to me that for **small** values of v_{DS}, the term i_{D2} is **very small**, and thus $i_D \approx i_{D1}$ (**when** v_{DS} is small)!*

A: Absolutely **true**! Recall this is **consistent** with our earlier discussion about the induced channel—the channel conductivity begins to significantly **degrade** only when v_{DS} becomes **sufficiently large**!

Thus, we can conclude:

$$
\begin{aligned}
i_D &\approx i_{D1} \\
&= 2K\left(v_{GS} - V_t\right)v_{DS} \\
&= k'\left(\frac{W}{L}\right)\left(v_{GS} - V_t\right)v_{DS} \qquad \text{for small } v_{DS}
\end{aligned}
$$

Moreover, we can (for small v_{DS}) approximate the induced channel as a resistor r_{DS} of value $r_{DS} = v_{DS} / i_{DS}$:

$$r_{DS} \doteq \frac{v_{DS}}{i_D}$$

$$= \frac{v_{DS}}{2K\left(v_{GS} - V_t\right)v_{DS}}$$

$$= \frac{1}{2K\left(v_{GS} - V_t\right)} \quad \text{for small } v_{DS}$$

Q: *I've just about had it with this* "for small v_{DS}" *nonsense! Just* ***how small is*** *small? How can we know* ***numerically*** *when this approximation is valid?*

A: Well, we **can** say that this approximation is valid when i_{D2} is much smaller than i_{D1} (i.e., i_{D2} is **insignificant**). **Mathematically**, we can state this as:

$$\left|i_{D2}\right| \ll \left|i_{D1}\right|$$

$$K\,v_{DS}^2 \ll 2K\left(v_{GS} - V_t\right)v_{DS}$$

$$v_{DS} \ll 2\left(v_{GS} - V_t\right)$$

Thus, we can **approximate** the induced channel as a **resistor** r_{DS} when v_{DS} is **much less** than the **twice the excess gate voltage**:

$$
\begin{aligned}
i_D &\approx i_{D1} \\
&= 2K\left(v_{GS} - V_t\right)v_{DS} \\
&= k'\left(\frac{W}{L}\right)\left(v_{GS} - V_t\right)v_{DS} \qquad \text{for } v_{DS} \ll 2\left(v_{GS} - V_t\right)
\end{aligned}
$$

and:

$$
\begin{aligned}
r_{DS} &= \frac{1}{2K\left(v_{GS} - V_t\right)} \\
&= \frac{1}{k'\left(W/L\right)\left(v_{GS} - V_t\right)} \qquad \text{for } v_{DS} \ll 2\left(v_{GS} - V_t\right)
\end{aligned}
$$

Q: *There you go* ***again!*** *The statement* $v_{DS} \ll 2\left(v_{GS} - V_t\right)$ *is only* ***slightly*** *more helpful than the statement "*when v_{DS} is small*". Precisely how* ***much*** *smaller than* ***twice the excess gate voltage*** *must* v_{DS} *be in order for our approximation to be* ***accurate****?*

A: We cannot say **precisely** how much smaller v_{DS} needs to be in relation to $2(v_{GS}-V_t)$ unless we state **precisely** how **accurate** we require our approximation to be!

For example, if we want the **error** associated with the approximation $i_D \approx i_{D1} = 2K\left(v_{GS} - V_t\right)v_{DS}$ to be **less than 10%**, we find that we require the voltage v_{DS} to be **less than 1/10** the value $2(v_{GS}-V_t)$.

In other words, **if**:

$$v_{DS} < \frac{2\left(v_{GS} - V_t\right)}{10} = \frac{v_{GS} - V_t}{5},$$

we find then that i_{D2} is less than 10% of i_{D1}:

$$i_{D2} < \frac{i_{D1}}{10}.$$

This **10% error criteria** is a **typical** "rule-of thumb" for many approximations in electronics. However, this does **not** mean that it is the "correct" criteria for determining the validity of this (or other) approximation.

For some applications, we might require **better** accuracy. For **example**, if we require less than **5% error**, we would find that $v_{DS} < \left(v_{GS} - V_t\right)/10$.

However, **using the 10% error criteria**, we arrive at the conclusion that:

$$
\begin{aligned}
i_D &\approx i_{D1} \\
&= 2K\left(v_{GS} - V_t\right) v_{DS} \\
&= k'\left(\frac{W}{L}\right)\left(v_{GS} - V_t\right) v_{DS} \quad \text{for} \quad v_{DS} < \left(v_{GS} - V_t\right)/5
\end{aligned}
$$

and:

$$
\begin{aligned}
r_{DS} &= \frac{1}{2K\left(v_{GS} - V_t\right)} \\
&= \frac{1}{k'\left(W/L\right)\left(v_{GS} - V_t\right)} \qquad \text{for } v_{DS} < \left(v_{GS} - V_t\right)/5
\end{aligned}
$$

We find that we should use these approximations when we can—it can make our **circuit analysis much easier**!

*See, the thing is, you should use these approximations whenever they are **valid**. They often make your **circuit analysis** task **much** simpler.*