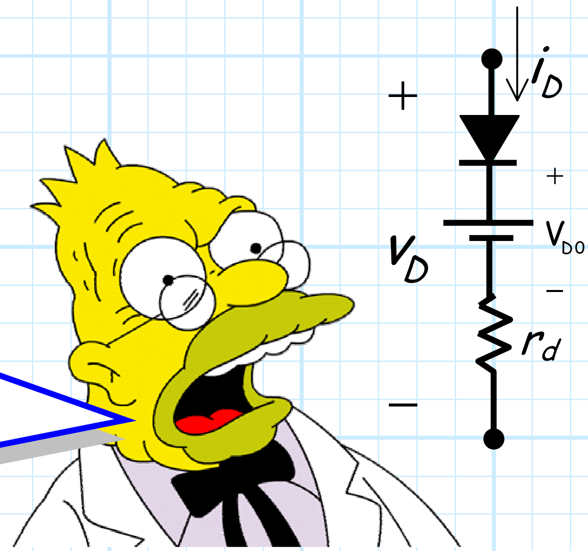
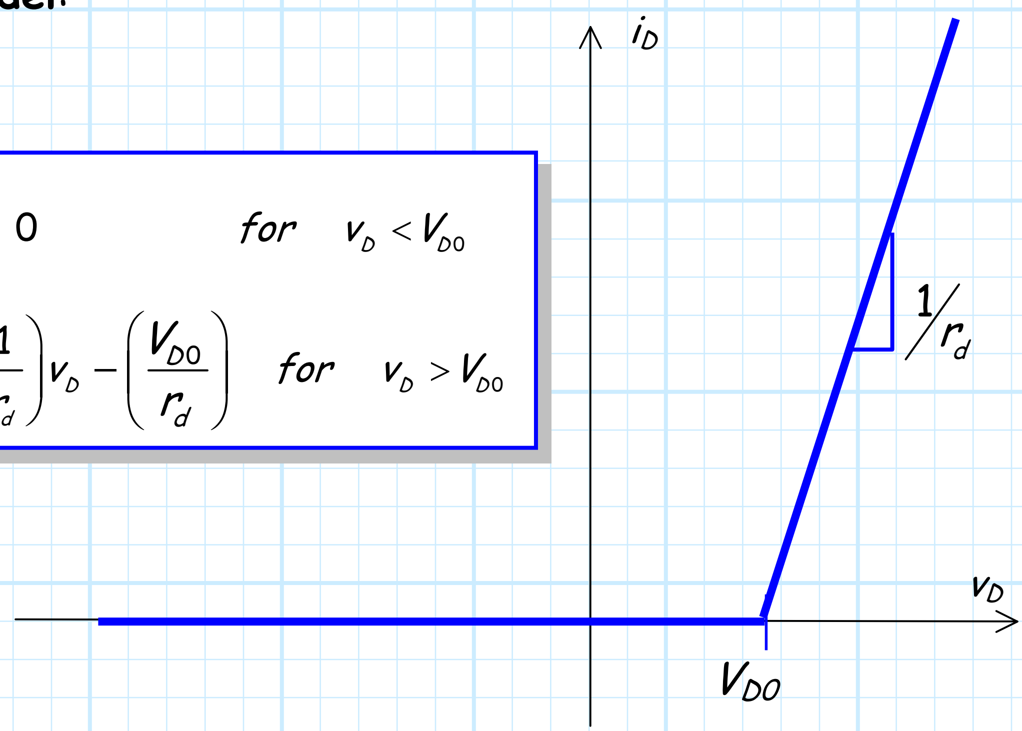
# Constructing the PWL Junction Diode Model

**Q:** *Wait a minute! How the heck are we supposed to use the **PWL model** to analyze junction diode circuits? **You** have yet to tell us the numeric **values** of voltage source $V_{D0}$ and resistor $r_d$ !*




**A:** That's right! The reason is that the **proper** values of voltage source $V_{D0}$ and resistor $r_d$ are up to **you** to determine! To see why, consider the current voltage relationship of the **PWL model**:

$$i_D = \begin{cases} 0 & \text{for} \quad v_D < V_{D0} \\ \left(\dfrac{1}{r_d}\right) v_D - \left(\dfrac{V_{D0}}{r_d}\right) & \text{for} \quad v_D > V_{D0} \end{cases}$$



Note that when the **ideal** diode in the PWL model is forward biased, the current-voltage relationship is simply the equation of a **line**!

$$i_D = \left(\frac{1}{r_d}\right) v_D - \left(\frac{V_{D0}}{r_d}\right)$$

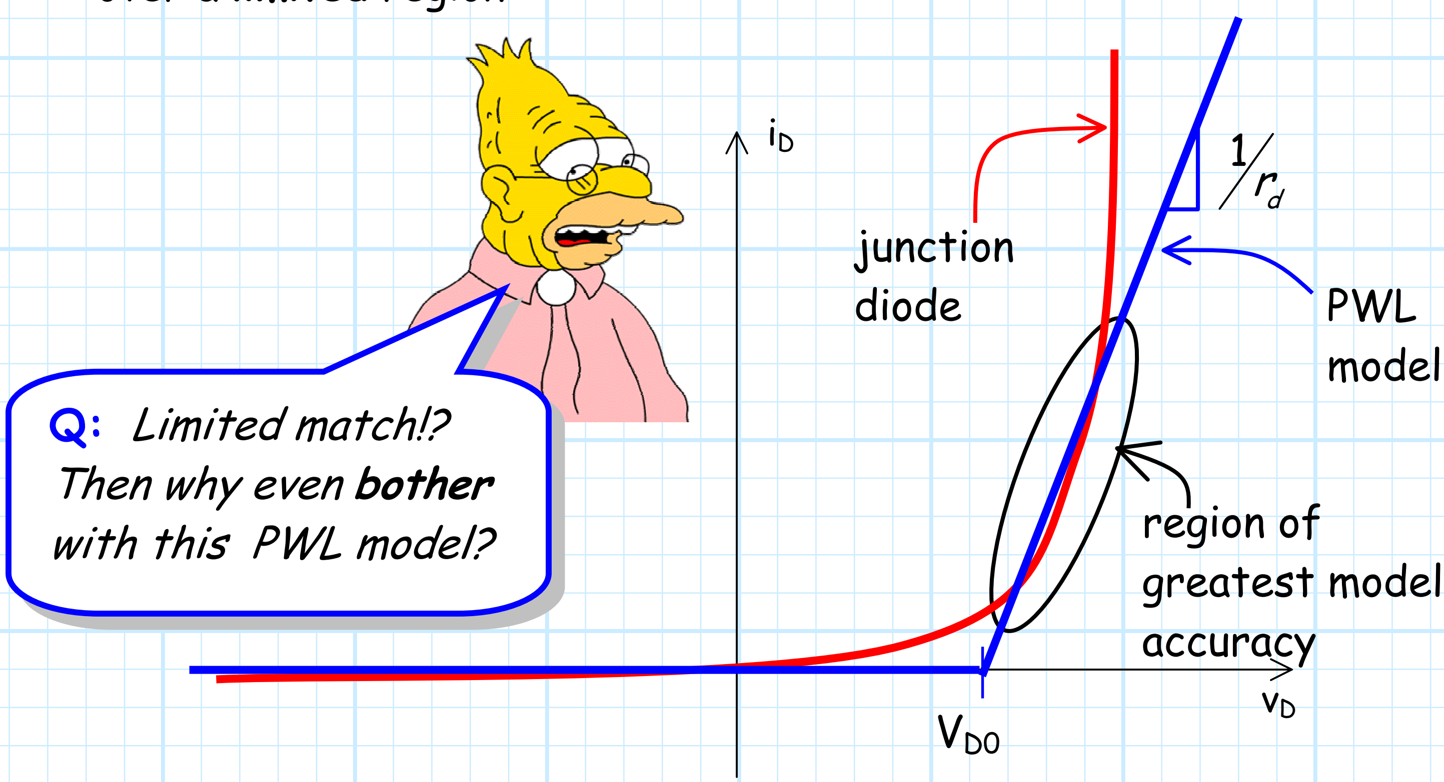
$$\boxed{y} = \boxed{m}\ \boxed{x} + \boxed{b}$$

**Compare** the above to the forward biased junction diode approximation:

$$i_D = I_s e^{v_D / nV_T}$$

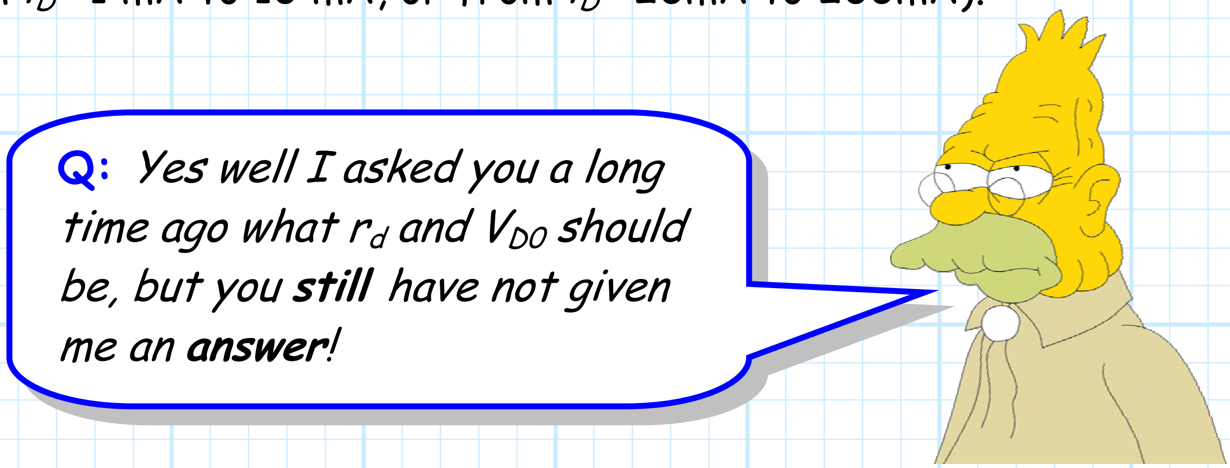
An **exponential** equation!

An exponential function and the equation of a line are **very** different—the two functions can approximately "match" only over a **limited** region:



**A:** Remember, the PWL model is **more accurate** than our two **alternatives**—the ideal diode model and the CVD model.

At the very least, the PWL model (**unlike** the two alternatives) shows an **increasing** voltage $v_D$ with **increasing** $i_D$. Moreover, **if** we select the values of $V_{D0}$ and $r_d$ properly, the PWL can **very** accurately "match" the actual (exponential) junction diode curve over a **decade** or more of current (e.g., accurate from $i_D$= 1 mA to 10 mA, or from $i_D$= 20mA to 200mA).




**A:** OK. We now know that the values of $r_d$ and $V_{D0}$ specify a **line**. We also know there are **4** potential ways to **specify** a line:

1. Specify **two points** on the line.
2. Specify one **point** on the line, as well as its **slope** $m$.
3. Specify one **point** on the line, as well as its ***y*-intercept** $b$.
4. Specify both its **slope** and its ***y*-intercept** $b$.

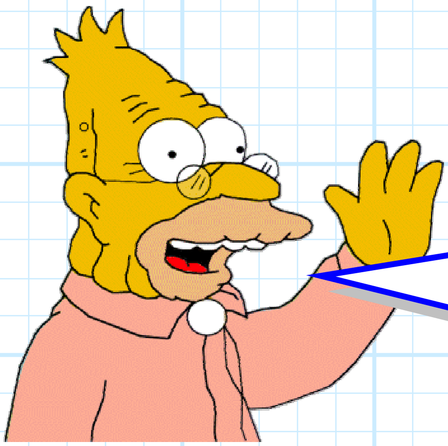
We will find that the **first two** methods are the most useful. Let's address them one at a time.

## 1. Specify two points on the line

The **obvious** question here is: **Which** two points ?

Hopefully it is **equally** obvious that the two points should be points lying on the **junction diode** exponential curve (after all, it is this curve that we are **attempting to approximate**!).

Typically, we pick **two current values** separated by about a **decade** (i.e., 10 times). For **example**, we might select $i_{D1}$=10 mA and $i_{D2}$=100 mA. We will find that the resulting PWL model will be **fairly accurate** over this region.



**Q:** *I've got a **question**! How do we find the corresponding **voltage** values $v_{D1}$ and $v_{D2}$ for these two currents?*

**A:** Remember, we are selecting two points **on** the exponential junction diode curve. Thus, we can use the **junction diode equation** to determine the corresponding voltages:

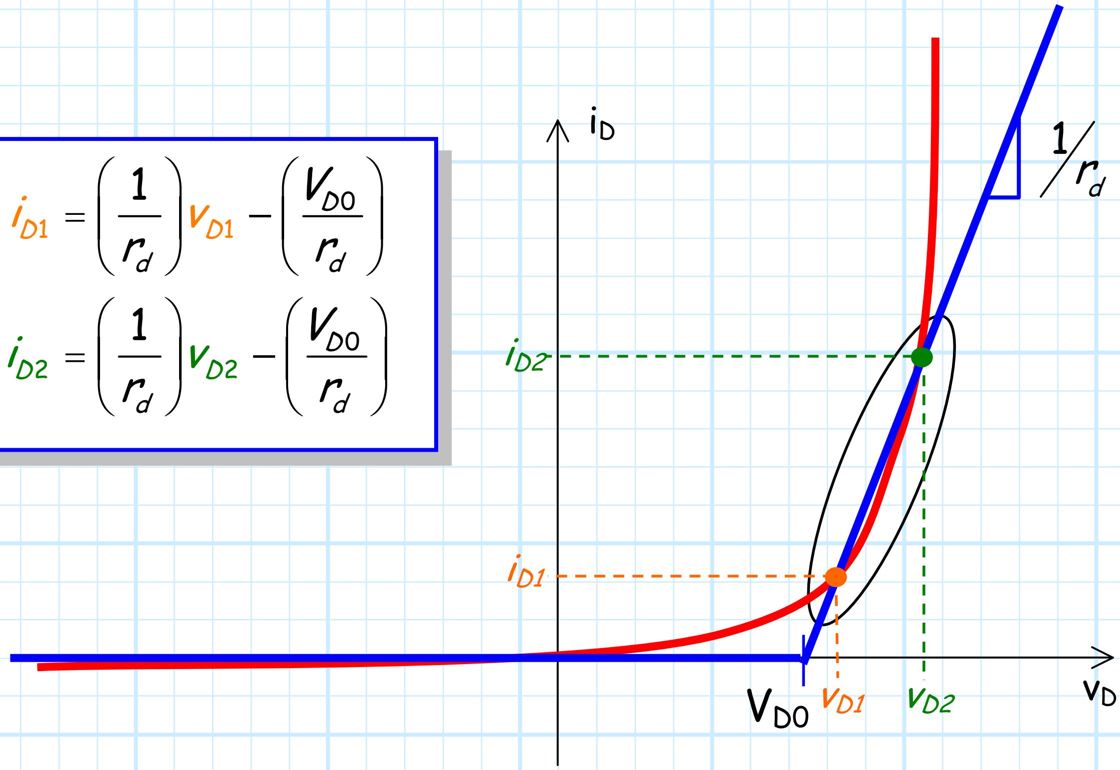
$$v_{D1} = nV_T \ ln\left[\frac{i_{D1}}{I_s}\right]$$

$$v_{D2} = nV_T \ ln\left[\frac{i_{D2}}{I_s}\right]$$

Now, the rest is simply **Middle School mathematics**. If our PWL "line" intersects these two points, then:

$$i_{D1} = \left(\frac{1}{r_d}\right) v_{D1} - \left(\frac{V_{D0}}{r_d}\right)$$

$$i_{D2} = \left(\frac{1}{r_d}\right) v_{D2} - \left(\frac{V_{D0}}{r_d}\right)$$




Thus, we can solve the above **two equations** to determine the **two unknown** values of $V_{D0}$ and $r_d$, such that our PWL "line" will intersect the two specified points on the junction diode curve:

$$m = \frac{1}{r_d} = \frac{i_{D2} - i_{D1}}{v_{D2} - v_{D1}} \qquad \therefore \qquad r_d = \frac{v_{D2} - v_{D1}}{i_{D2} - i_{D1}}$$

And then we use our PWL "line" equation to find $r_d$:

$$V_{D0} = v_{D1} - i_{D1}\, r_d \qquad or \qquad V_{D0} = v_{D2} - i_{D2}\, r_d$$

(note these two equations are **KVL**!).

## 2. Specify one point and the slope

Now let's examine **another** way of constructing our PWL model. We first specify just **one** point that the PWL "line" must intersect. Let's denote this point as ($I_D$, $V_D$) and call this point our **bias point**.

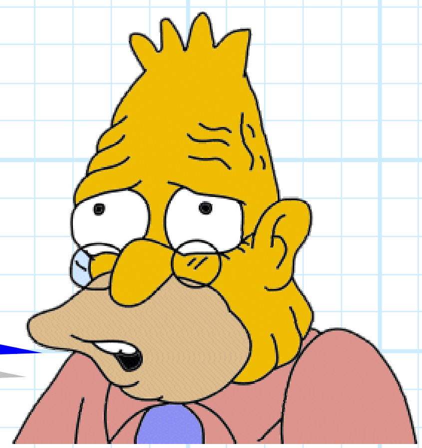
Of course, we want our bias point to **lie on** the exponential junction diode curve, i.e.:

$$I_D = I_s e^{V_D / nV_T} \quad \text{or equivalently} \quad V_D = nV_T \ln\left[\frac{I_D}{I_s}\right]$$

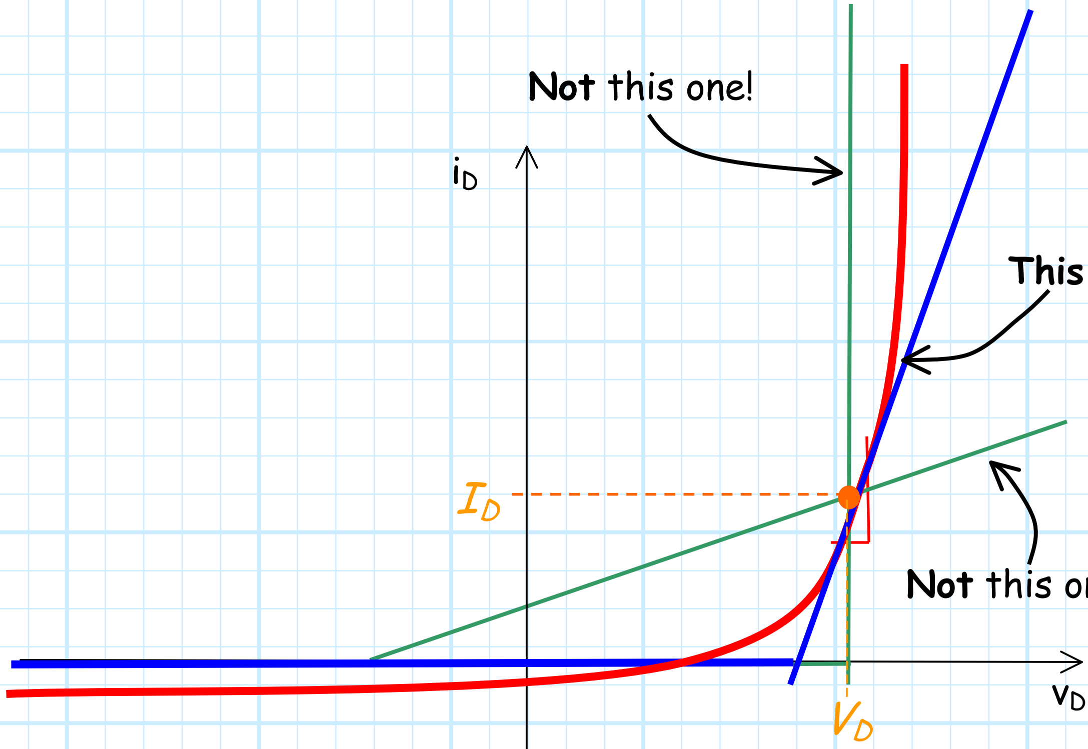
Now, **instead** of specifying a **second** intersection point, we merely **specify directly** the PWL line **slope** (i.e., directly specify the value of $r_d$!):

$$m = \frac{1}{r_d}$$

**Q:** *But I have **no idea** what the value of this slope should be!?!*



**A: Think** about it. Of all possible PWL models that intersect the bias point, the one that is most accurate is the one that has a slope **equal** to the slope of the exponential junction diode curve (that is, **at** the bias point)!




**Q:** *What! Just how is it possible to* ***determine*** *the slope of the junction diode curve at the bias point?!?*

**A:** Easy! We simply take the **first derivative** of the junction diode equation:

$$\frac{d\,i_D}{d\,v_D} = \frac{d}{d\,v_D}\left(I_s e^{v_D / nV_T}\right)$$

$$= \frac{I_s e^{v_D / nV_T}}{nV_T}$$

**Q:** *Of course! This* ***equation*** *is the slope of the junction diode curve at the bias point!*

**A:** Actually **no**. The above equation is **not** the slope of the junction diode curve at the bias point. This equation provides the slope of the curve **as a function diode voltage** $v_D$. The slope of the junction diode curve is in fact different at **every** point on the junction diode curve.

In fact, as the equation above clearly states, the slope of the junction diode curve **exponential increases** with increasing $v_D$!

**Q:** *Yikes! So **what** is the derivate equation **good** for?*

**A:** Remember, we are interested in the value of the slope of the curve at **one** particular point—the **bias point**. Thus, we simply **evaluate** the derivative function at that point. The result is a numeric value of the slope **at our bias point**!

$$m = \frac{d}{d\,v_D}\left( I_s e^{v_D / nV_T} \right)\Bigg|_{v_D = V_D}$$

$$= \frac{I_s e^{v_D / nV_T}}{nV_T}\Bigg|_{v_D = V_D}$$

$$= \frac{I_s e^{V_D / nV_T}}{nV_T}$$

Note the **numerator** of this result! We recognize this numerator as simply the value of the **bias current** $I_D$:

$$I_D = I_s e^{V_D / nV_T}$$

Therefore, we find that the **slope** at the bias point is:

$$m = \frac{I_s e^{V_D / nV_T}}{nV_T} = \frac{I_D}{nV_T}$$

Now, we want the slope of our **PWL model** line to be **equal** to the slope of the **junction diode curve** at our bias point. Therefore, we desire:

$$\frac{1}{r_d} = m = \frac{I_D}{nV_T}$$

Thus, **rearranging** this equation, we find that the PWL model **resistor value** should be:

$$r_d = \frac{nV_T}{I_D}$$

We likewise can rearrange the PWL "line" equation to determine the value of the **model voltage source** $V_{D0}$:

$$V_{D0} = V_D - I_D\, r_d \qquad \textbf{(KVL !)}$$

Now, combining the previous two equations, we find:

$$\begin{aligned} V_{D0} &= V_D - I_D\, r_d \\ &= V_D - I_D \left( \frac{nV_T}{I_D} \right) \\ &= V_D - nV_T \end{aligned}$$

So, let's **recap** what we have learned about constructing a PWL model using this particular approach.

**1.** We first select a single **bias point** ($I_D$, $V_D$), a point that lies on the junction diode curve, i.e.:

$$I_D = I_s e^{V_D / nV_T}$$

**2.** Using the current and voltage values of this bias point, we can then determine **directly** the PWL model **resistor value**:

$$\boxed{r_d = \frac{nV_T}{I_D}}$$

**3.** We can also directly determine the **value** of the model **voltage source**:

$$\boxed{V_{D0} = V_D - nV_T}$$

***This*** *method for constructing a* ***PWL model*** *produces a very* ***precise*** *match over a relatively small region of the junction diode curve.*

*We will find that this is* ***very useful*** *for many practical diode circuit problems and analysis!*

*This PWL model produced by this last method (as described by the equations of the previous page) is called the junction diode* ***small-signal model****.*

*We will use the* ***small-signal model*** *again—make sure that* ***you*** *know* ***what*** *it is and* ***how*** *we construct it!*