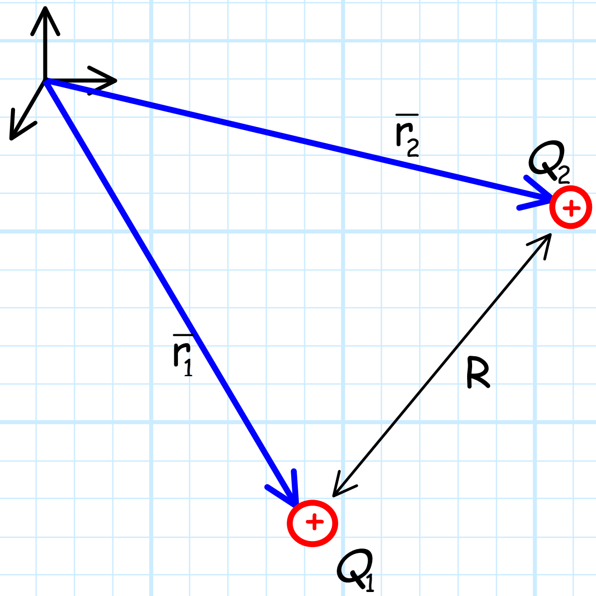
Coulomb's Law of Force

Consider **two** point charges, Q_1 and Q_2, located at positions $\bar{r}_1$ and $\bar{r}_2$, respectively.



We will find that **each** charge has a **force F** (with magnitude and direction) exerted on it.

This force is **dependent** on both the **sign** (+ or -) and the **magnitude** of charges Q_1 and Q_2, as well as the **distance** R between the charges.

Charles Coulomb determined this relationship in the 18th century! We call his result **Coulomb's Law**:

$$\mathbf{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R^2}\,\hat{a}_{21} \quad [N]$$

This force $\mathbf{F}_1$ is the force exerted **on** charge Q_1. Likewise, the force exerted **on** charge Q_2 is equal to:

$$\mathbf{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2\,Q_1}{R^2}\,\hat{a}_{12} \quad [N]$$

In these formula, the value ε_0 is a **constant** that describes the **permittivity of free space** (i.e., a vacuum).

$$\begin{aligned}\varepsilon_0 &\doteq \text{permittivity of free space} \\ &= 8.854 \times 10^{-12} \quad \left[\frac{C^2}{Nm^2} = \frac{farads}{m}\right]\end{aligned}$$

Note the **only difference** between the equations for forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are the **unit vectors** $\hat{a}_{21}$ and $\hat{a}_{12}$.

* Unit vector $\hat{a}_{21}$ points **from** the location of Q_2 (i.e., $\bar{r}_2$) **to** the location of charge Q_1 (i.e., $\bar{r}_1$).

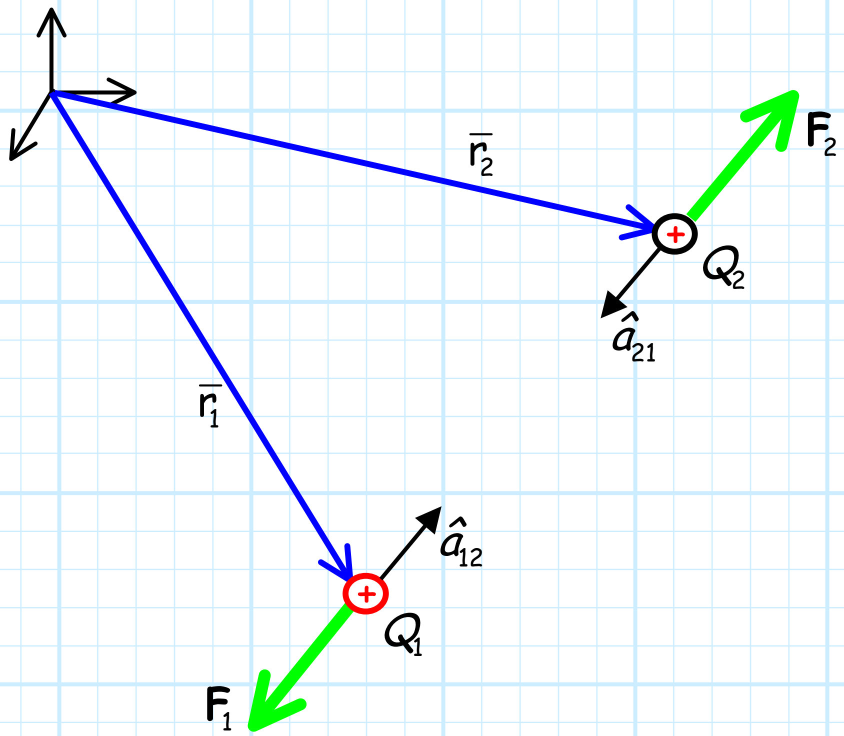
* Likewise, unit vector $\hat{a}_{12}$ points **from** the location of Q_1 (i.e., $\bar{r}_1$) **to** the location of charge Q_2 (i.e., $\bar{r}_2$).

Note therefore, that these unit vectors point in **opposite** directions, a result we express mathematically as $\hat{a}_{21} = -\hat{a}_{12}$.

Therefore we find:

$$\begin{aligned}\mathbf{F}_1 &= \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R^2}\ \hat{a}_{21} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R^2}\ \left(-\hat{a}_{12}\right) \\ &= -\left(\frac{1}{4\pi\varepsilon_0}\frac{Q_2 Q_1}{R^2}\ \hat{a}_{12}\right) \\ &= -\mathbf{F}_2\end{aligned}$$

Look! Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ have **equal magnitude**, but point in **opposite directions** !

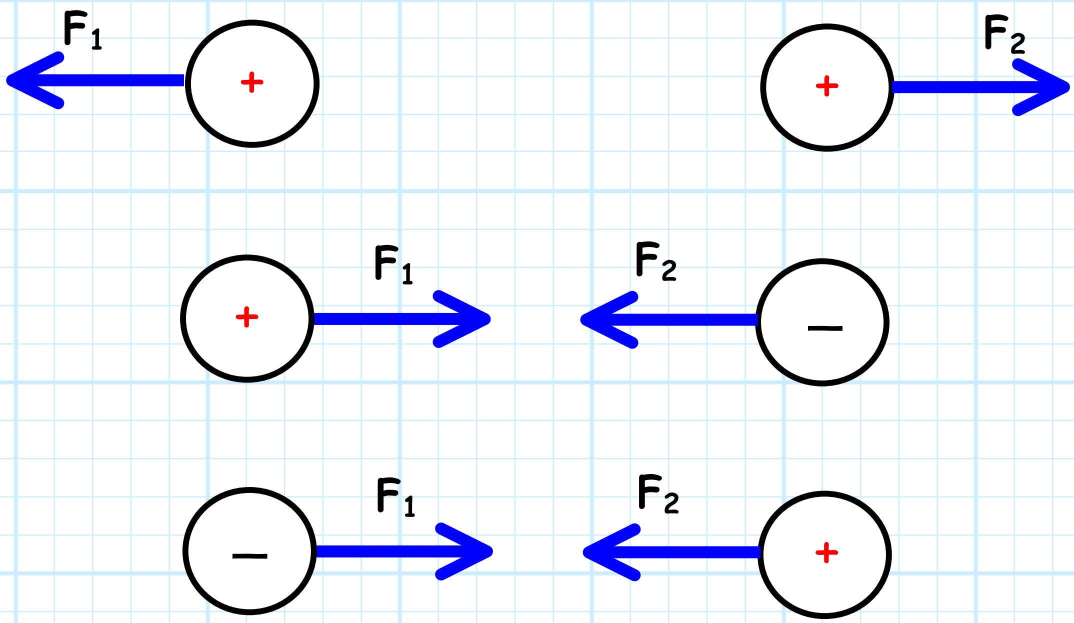


Note in the case shown above, **both** charges were **positive**.

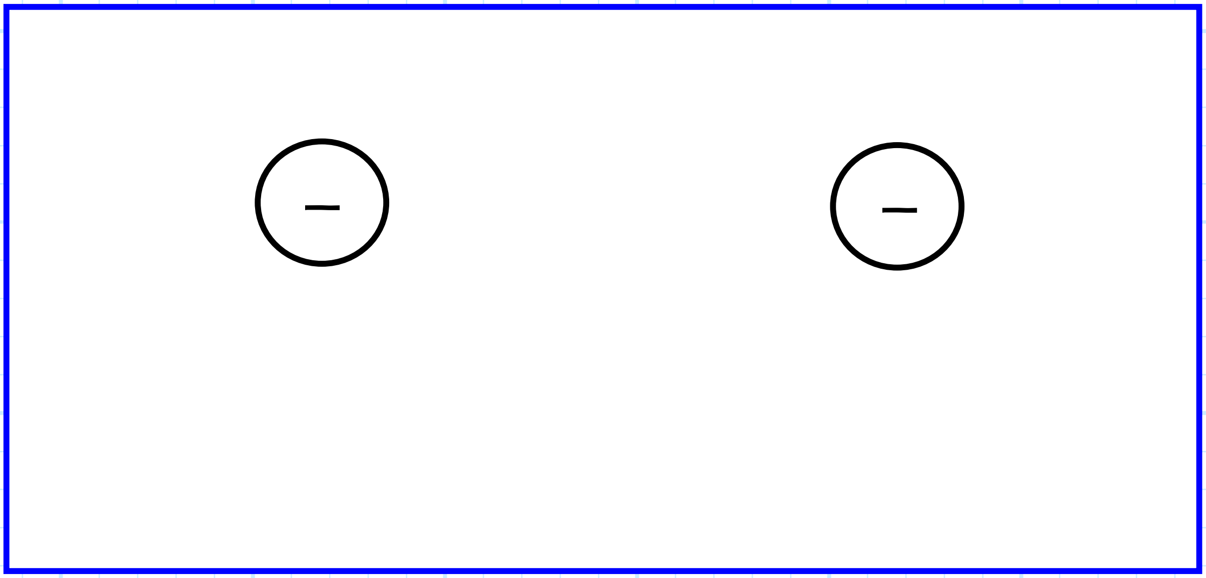
Q: *What happens when* ***one*** *of the charges is* ***negative****?*

A: Look at Coulomb's Law ! If one charge is positive, and the other is negative, then the **product** $Q_1 Q_2$ is **negative**. The resulting force vectors are therefore negative—they point in the **opposite** direction of the previous (i.e., both positive) case!

Therefore, we find that:



What about **this** case ?



We come to the **important** conclusion that:

1) charges of **opposite** sign **attract**.

2) charges with the **same** sign **repel**.

Charles-Augustin de Coulomb (1736-1806), a military civil engineer, retired from the French army because of ill health after years in the West Indies. Forced from Paris by the disturbances of the revolution, he began working at his family estate and discovered that the torsion characteristics of long fibers made them ideal for the sensitive measurement of **magnetic** and **electric** forces. He was familiar with Newton's **inverse-square law** and in the period 1785-1791 he succeeded in showing that **electrostatic** forces obey the **same** rule. (from *www.ee.umd.edu/~taylor/frame1.htm*)