**Coulomb's Law of Force**

Consider **two** point charges, $Q_1$ and $Q_2$, located at positions $\vec{r}_1$ and $\vec{r}_2$, respectively.

We will find that **each** charge has a **force** $F$ (with magnitude and direction) exerted on it.

This force is **dependent** on both the **sign** (+ or -) and the **magnitude** of charges $Q_1$ and $Q_2$, as well as the **distance** $R$ between the charges.

**Charles Coulomb** determined this relationship in the 18$^{th}$ century! We call his result **Coulomb's Law**:

$$ F = \frac{1}{4\pi \varepsilon_0} \frac{Q_1 Q_2}{R^2} \hat{a}_{21} \quad [\text{N}] $$

This force $F_1$ is the force exerted on charge $Q_1$. Likewise, the force exerted on charge $Q_2$ is equal to:
\[
F_2 = \frac{1}{4\pi\varepsilon_0} \frac{Q_2 Q_1}{R^2} \hat{a}_{12} \quad [N]
\]

In these formula, the value \(\varepsilon_0\) is a **constant** that describes the **permittivity of free space** (i.e., a vacuum).

\[
\varepsilon_0 \doteq \text{permittivity of free space} = 8.854 \times 10^{-12} \left[ \frac{C^2}{Nm^2} = \frac{\text{farads}}{m} \right]
\]

Note the **only difference** between the equations for forces \(F_1\) and \(F_2\) are the **unit vectors** \(\hat{a}_{21}\) and \(\hat{a}_{12}\).

* Unit vector \(\hat{a}_{21}\) points **from** the location of \(Q_2\) (i.e., \(\vec{r}_2\)) **to** the location of charge \(Q_1\) (i.e., \(\vec{r}_1\)).

* Likewise, unit vector \(\hat{a}_{12}\) points **from** the location of \(Q_1\) (i.e., \(\vec{r}_1\)) **to** the location of charge \(Q_2\) (i.e., \(\vec{r}_2\)).

Note therefore, that these unit vectors point in **opposite** directions, a result we express mathematically as \(\hat{a}_{21} = -\hat{a}_{12}\).
Therefore we find:

\[
F_1 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R^2} \hat{a}_{21}
\]

\[
= \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R^2} (\hat{a}_{12})
\]

\[
= -\left( \frac{1}{4\pi\varepsilon_0} \frac{Q_2 Q_1}{R^2} \hat{a}_{12} \right)
\]

\[
= -F_2
\]

**Look!** Forces \( F_1 \) and \( F_2 \) have **equal magnitude**, but point in **opposite directions**!
Note in the case shown above, both charges were positive.

**Q:** What happens when one of the charges is negative?

**A:** Look at Coulomb's Law! If one charge is positive, and the other is negative, then the product $Q_1 Q_2$ is negative. The resulting force vectors are therefore negative—they point in the opposite direction of the previous (i.e., both positive) case!

Therefore, we find that:

![Diagram showing force vectors for different charge combinations](attachment:image.png)
What about this case?

We come to the important conclusion that:

1) charges of **opposite** sign attract.

2) charges with the **same** sign repel.

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Charles-Augustin de Coulomb (1736-1806), a military civil engineer, retired from the French army because of ill health after years in the West Indies. Forced from Paris by the disturbances of the revolution, he began working at his family estate and discovered that the torsion characteristics of long fibers made them ideal for the sensitive measurement of **magnetic** and **electric** forces. He was familiar with Newton’s **inverse-square law** and in the period 1785-1791 he succeeded in showing that **electrostatic** forces obey the **same** rule. (from [www.ee.umd.edu/~taylor/frame1.htm](http://www.ee.umd.edu/~taylor/frame1.htm))