DC and AC Impedance of Reactive Elements

Now that we are considering **time-varying** signals, we need to consider circuits that include **reactive** elements—specifically, **inductors** and **capacitors**.

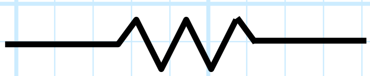
First, we will assume that all circuit sources are **sinusoidal**, with **frequency** ω:

$$\begin{aligned} v_S(t) &= \mathrm{Re}\left\{A_v\, e^{-j(\omega t-\varphi)}\right\} \\ &= A_v \cos(\omega t-\varphi) \end{aligned}$$

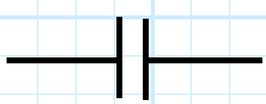
Note here that **IF** $\omega \neq 0$, the signal above is purely an ***AC*** signal (**no** DC component!).

However, **IF** $\omega = 0$, then $v_S(t) = A_v \cos(0) = A_v$ — a **DC** signal!

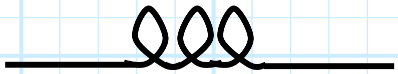
Now, recall from EECS 211 the **complex impedances** of our basic circuit elements:



$$Z_R = R$$



$$Z_C = \frac{1}{j\omega C}$$



$$Z_L = j\omega L$$

For a **DC** signal ($\omega = 0$), we find that:

$$Z_R = R$$

$$Z_C = \lim_{\omega \to 0} \frac{1}{j\omega C} = \infty$$

$$Z_L = j(0)L = 0$$

Thus, at **DC** we know that:

* a **capacitor** acts as an **open** circuit (I_C =0).

* an **inductor** acts as a **short** circuit (V_L = 0).

Now, let's consider **two** important cases:

1. A **capacitor** whose capacitance C is **unfathomably large**.

2. An **inductor** whose inductance L is **unfathomably large**.

1. The Unfathomably Large Capacitor

In this case, we consider a capacitor whose capacitance is **finite**, but **very, very, very** large.

For **DC** signals ($\omega = 0$), this device acts **still** acts like an **open** circuit.

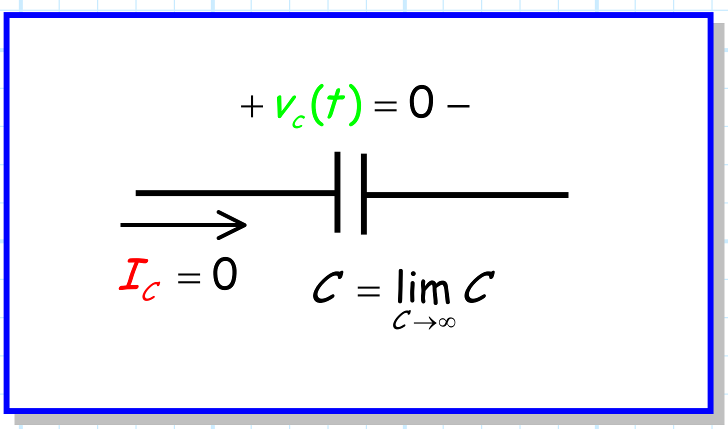
However, now consider the **AC signal case**, where $\omega \neq 0$. The **impedance** of an unfathomably large capacitor is:

$$Z_C = \lim_{C \to \infty} \frac{1}{j\omega C} = 0$$

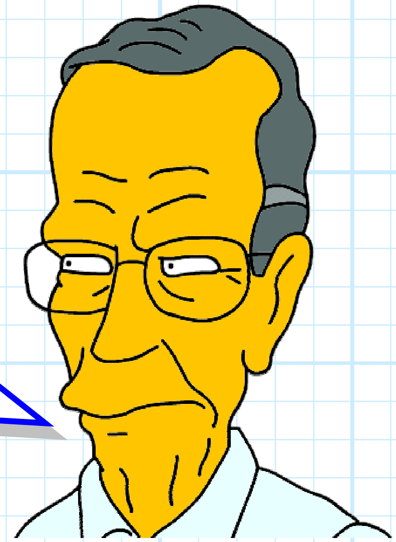
Zero impedance!

→ An unfathomably large capacitor acts like an ***AC* short**.

Quite a trick! The unfathomably large capacitance acts like an **open** to **DC** signals, but likewise acts like a **short** to ***AC*** signals!



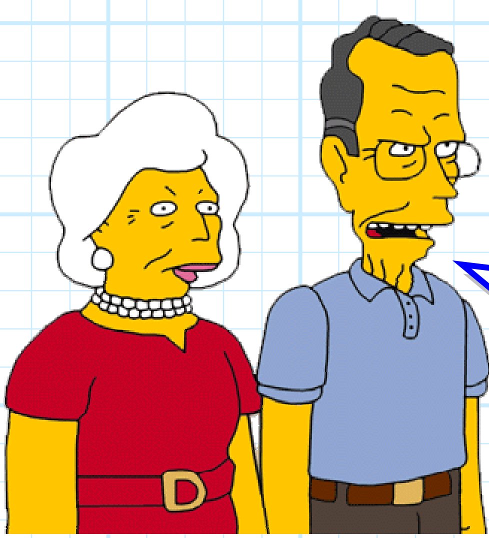
Q: *I fail to see the* ***relevance*** *of this analysis at this juncture. After all,* ***unfathomably*** *large capacitors do* ***not*** *exist, and are* ***impossible*** *to make (being unfathomable and all).*



A: True enough! However, we can make **very big** (but **fathomably** large) capacitors. Big capacitors will not act as a **perfect** AC short circuit, but **will** exhibit an impedance of **very small** magnitude (e.g., a few Ohms), provided that the AC signal frequency is sufficiently large.

In this way, a **very large** capacitor acts as an **approximate AC short**, and as a **perfect DC open**.

We call these large capacitors **DC blocking capacitors**, as they allow **no DC current** to flow through them, while allowing AC current to flow **nearly unimpeded**!



Q: *But you just said this is true "provided that the AC signal frequency is* ***sufficiently large.****" Just* ***how*** *large does the signal frequency ω need to be?*

A: Say we desire the AC impedance of our capacitor to have a magnitude of **less than ten Ohms**:

$$|Z_C| < 10$$

Rearranging, we find that this will occur **if** the frequency ω is:

$$\begin{aligned} 10 &> |Z_C| \\ 10 &> \frac{1}{\omega C} \\ \omega &> \frac{1}{10C} \end{aligned}$$

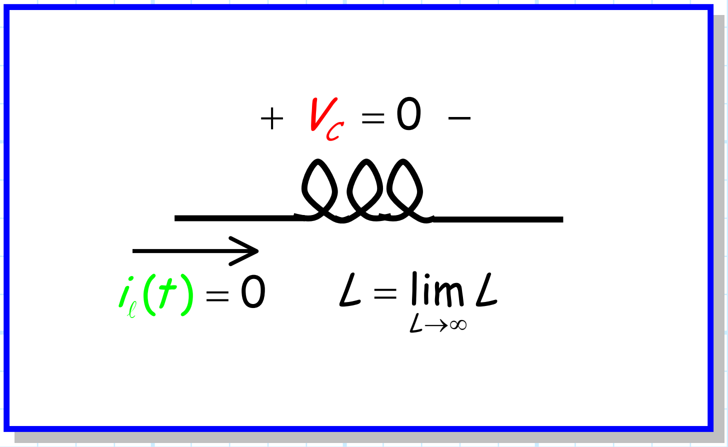
For **example**, a 50 μF capacitor will exhibit an impedance whose magnitude is less than 10 Ohms for all AC signal frequencies above **320 Hz**. Likewise, **almost** all AC signals in modern electronics will operate in a spectrum much higher than 320 Hz. Thus, a 50 μF blocking capacitor will **approximately** act as an AC short and (precisely) act as a DC open.

2. The Unfathomably Large Inductor

Similarly, we can consider an **unfathomably large inductor**. In addition to a **DC** impedance of **zero** (a DC short), we find for the **AC** case (where $\omega \neq 0$):

$$Z_L = \lim_{L \to \infty} j\omega L = \infty$$

In other words, an unfathomably large inductor acts like an **AC open circuit!**



As before, an unfathomably large inductor is **impossible** to build. However, a **very large** inductor will typically exhibit a **very large AC impedance** for all but the lowest of signal frequencies ω.

We call these large inductors "AC chokes" (also known RF chokes), as they act as a **perfect short** to **DC** signals, yet so effectively impede AC signals (with sufficiently high frequency) that they act **approximately** as an **AC open circuit**.

For example, if we desire an **AC choke** with an impedance magnitude greater than 100 kΩ, we find that:

$$|Z_L| > 10^5$$
$$\omega L > 10^5$$
$$\omega > \frac{10^5}{L}$$

Thus, an AC choke of 50 mH would exhibit an impedance magnitude of greater than 100 kΩ for all signal frequencies greater than **320 kHz**. Note that this is still a fairly low signal frequency for **many** modern electronic applications, and thus this inductor would be an adequate AC choke.

Note however, that building and AC choke for **audio** signals (20 Hz to 20 kHz) is typically **very** difficult!