Example: NMOS Circuit Analysis

Consider this DC MOSFET circuit:

Let's ASSUME the NMOS device is in saturation.

\[ K = 0.4 \ \text{mA/V}^2 \]

\[ V_t = 2.0 \ \text{V} \]
Thus, we must ENFORCE the condition that:

\[ I_D = K (V_{GS} - V_t)^2 \]

Now we must ANALYZE the circuit.

**Q:** What now? How do we proceed with this analysis?

**A:** It’s certainly not clear. Let’s write the circuit equations and see what happens.

From the Gate-Source loop KVL:

\[ 0.0 - V_{GS} - (1)I_D = -5.0 \]

Therefore, rearranging:

\[ I_D = 5.0 - V_{GS} \]

And from the Drain-Source loop KVL:

\[ 5.0 - (1)I_D - V_{DS} - (1)I_D = -5.0 \]

Therefore, rearranging:

\[ V_{DS} = 10.0 - 2I_D \]
Look! We can equate the NMOS device equation and G-S equation to find $V_{GS}$.

$$I_D = K \left( V_{GS} - V_t \right)^2 = 5.0 - V_{GS}$$

$$\therefore 0 = K V_{GS}^2 + V_{GS} \left( 1 - 2K V_t \right) + \left( K V_t^2 - 5.0 \right)$$

A quadratic equation!

The solutions to this equation are:

$$V_{GS} = 3.76 \ V \quad \text{or} \quad V_{GS} = -2.26 \ V$$

Q: Yikes! Two solutions! Which one is correct?

A: Note we assumed saturation. If the MOSFET is in saturation, we know that:

$$V_{GS} > V_t = 2.0$$

Only one solution of the quadratic satisfies this condition, i.e.:

$$V_{GS} = 3.76 > V_t$$

Thus, we use $V_{GS} = 3.76 \ V$ -- the solution that is consistent with our original assumption.
Inserting this voltage into the Gate-Source KVL equation, we find that the drain current is:

\[ I_D = 5.0 - V_{\text{gs}} \]
\[ = 5.0 - 3.76 \]
\[ = 2.24 \, mA \]

And using the Drain-Source KVL, we find the remaining voltage:

\[ V_{\text{ds}} = 10.0 - 2.0I_D \]
\[ = 10.0 - 2(2.24) \]
\[ = 5.52 \, V \]

Even though we have answers (one current and two voltages), we still are not finished, as we now must CHECK our solution to see if it is consistent with the saturation mode inequalities.

\[ 3.76 = V_{\text{gs}} > V_T = 2.0 \, \checkmark \]
\[ 5.52 = V_{\text{ds}} > V_{\text{gs}} - V_T = 1.76 \, \checkmark \]

Both answers are consistent! Our solutions are correct!