MOSFET Output Resistance

Recall that due to channel-length modulation, the MOSFET drain current is slightly dependent on $v_{DS}$, and thus is more accurately described as:

$$i_d = K(v_{GS} - V_t)^2 (1 + \lambda v_{DS})$$

In order to determine the relationship between the small-signal voltage $v_{gs}$ and small-signal current $i_d$ we can apply a small-signal analysis of this equation:

$$i_d = \frac{d i_d}{d v_{GS}} \bigg|_{v_{gs}=v_{GS}} v_{gs}$$

$$= 2K(v_{GS} - V_t) \bigg|_{v_{gs}=v_{GS}} v_{gs}$$

$$= 2K(v_{GS} - V_t)v_{gs}$$

$$= g_m v_{gs}$$

Note that we evaluated the derivative at the DC bias point $V_{GS}$. The result, as we expected, was the transconductance $g_m$.

We can likewise determine the relationship between small-signal voltage $v_{ds}$ and the small-signal current $i_d$: 
\[ i_d = \frac{d i_D}{d v_{DS}} \bigg|_{v_{GS}=V_{GS}} v_{ds} \]

\[ = \lambda K (v_{GS} - V_t)^2 \bigg|_{v_{GS}=V_{GS}} v_{ds} \]

\[ = \frac{v_{ds}}{r_o} \]

where \( r_o \) is defined as the MOSFET output resistance:

\[ r_o = \frac{1}{\lambda K (v_{GS} - V_t)^2} \]

\[ = \frac{1}{\lambda I_D} \]

The small signal drain current \( i_d \) of a MOSFET (biased at a DC operating point \( I_D, V_{GS} \)) is therefore:

\[ i_d = g_m v_{gs} + \frac{v_{ds}}{r_o} \]

where:

\[ g_m = 2K (v_{GS} - V_t) \]

\[ r_o = \frac{1}{\lambda K (v_{GS} - V_t)^2} \]