The CMOS Transfer Function

Now, instead of determining the output $v_O$ of a CMOS inverter for just two specific input voltages ($v_I = 0$ and $v_I = V_{DD}$), we can determine the value of $v_O$ for any and all input voltages $v_I$—in other words, we can determine the CMOS inverter transfer function $v_O = f(v_I)$!

Determining this transfer function is a bit laborious, so we will simply present the result (the details are in your book):
Look at how close this transfer function is to the ideal transfer function!

The transition region for this transfer function is very small; note that:

1. $V_{IL}$ is just a bit less than $V_{DD}/2$

2. $V_{IH}$ is just a bit more than $V_{DD}/2$

In fact, by taking the derivative of the transfer function, we can determine the two points on the transfer function (i.e., $V_{IL}$ and $V_{IH}$) where the slope is equal to -1.0. I.E.:

$$v_i \text{ where } \frac{dv_o}{dv_i} = -1.0$$

Taking this derivative and solving for $v_i$, we can determine explicit values for $V_{IL}$ and $V_{IH}$ (again, the details are in your book):

$$V_{IL} = \frac{1}{8}(3V_{DD} + 2V_f)$$

$$V_{IH} = \frac{1}{8}(5V_{DD} - 2V_f)$$
Now, recall earlier we determined that the CMOS inverter provides **ideal** values for $V_{OL}$ and $V_{OH}$:

\[
\begin{align*}
V_{OL} &= 0.0 \\
V_{OH} &= V_{DD}
\end{align*}
\]

Thus, we can determine the **noise margins** of a CMOS inverter:

\[
\begin{align*}
NM_L &= V_{IL} - V_{OL} \\
&= \frac{1}{8} (3V_{DD} + 2V_t) - 0.0 \\
&= \frac{1}{8} (3V_{DD} + 2V_t)
\end{align*}
\]

and:

\[
\begin{align*}
NM_H &= V_{OH} - V_{IH} \\
&= V_{DD} - \frac{1}{8} (5V_{DD} - 2V_t) \\
&= \frac{1}{8} (3V_{DD} + 2V_t)
\end{align*}
\]

Therefore, the two noise margins are equal, and thus we can say that the noise margin for a **CMOS inverter** is:

\[
NM_L = NM_H = \frac{1}{8} (3V_{DD} + 2V_t)
\]