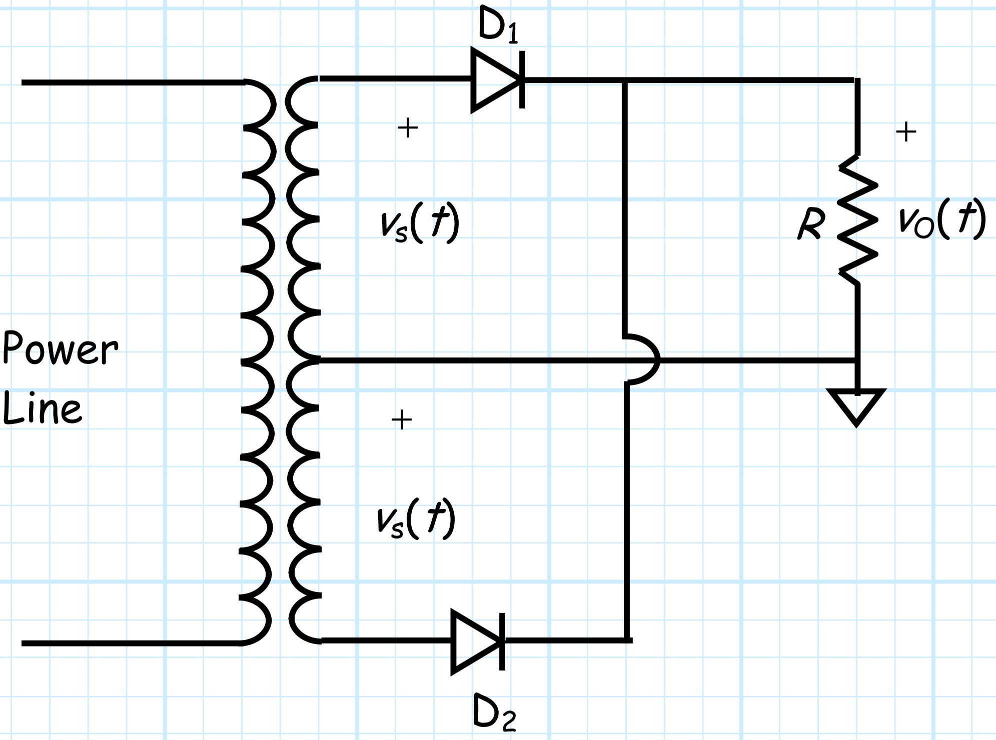
The Full-Wave Rectifier

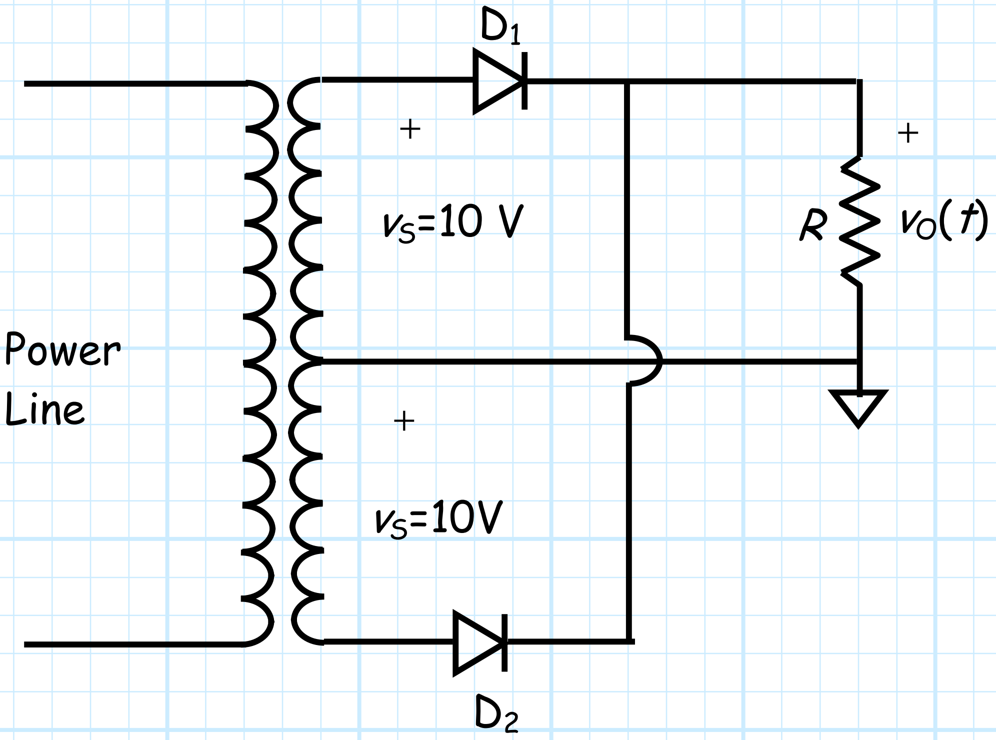
Consider the following **junction diode** circuit:



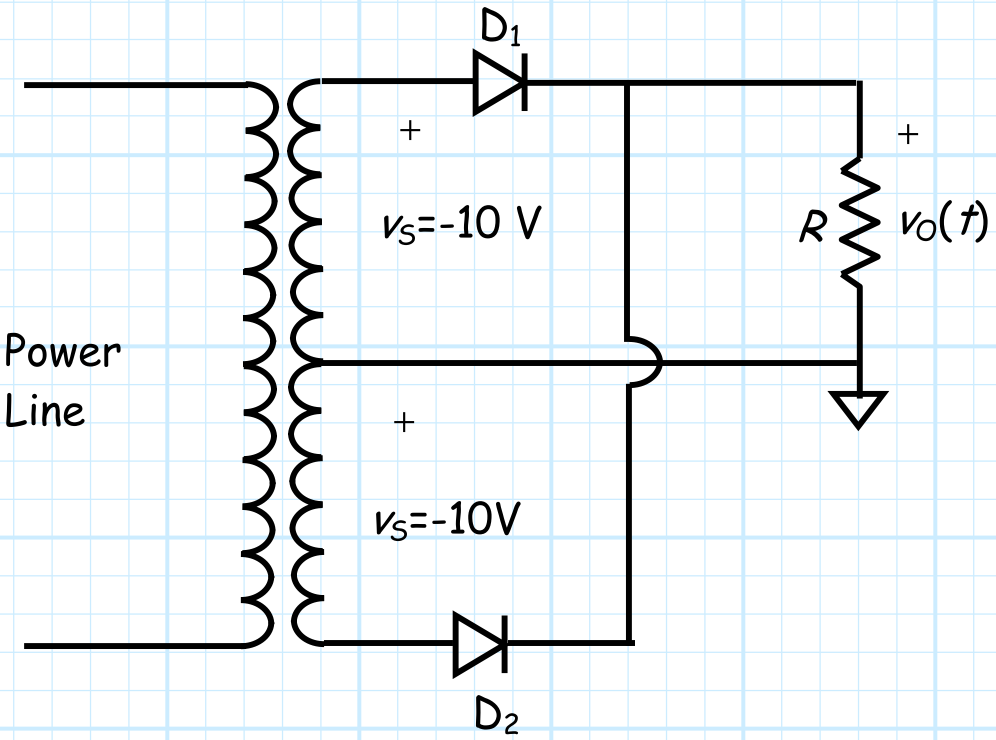
Note that we are using a **transformer** in this circuit. The job of this transformer is to **step-down** the large voltage on our power line (120 V rms) to some **smaller** magnitude (typically 20-70 V rms).

Note the secondary winding has a **center tap** that is **grounded**. Thus, the secondary voltage is distributed **symmetrically** on either side of this center tap.

For **example**, if $v_S = 10$ V, the anode of D_1 will be 10V **above** ground potential, while the anode of D_2 will be 10V **below** ground potential (i.e., -10V):

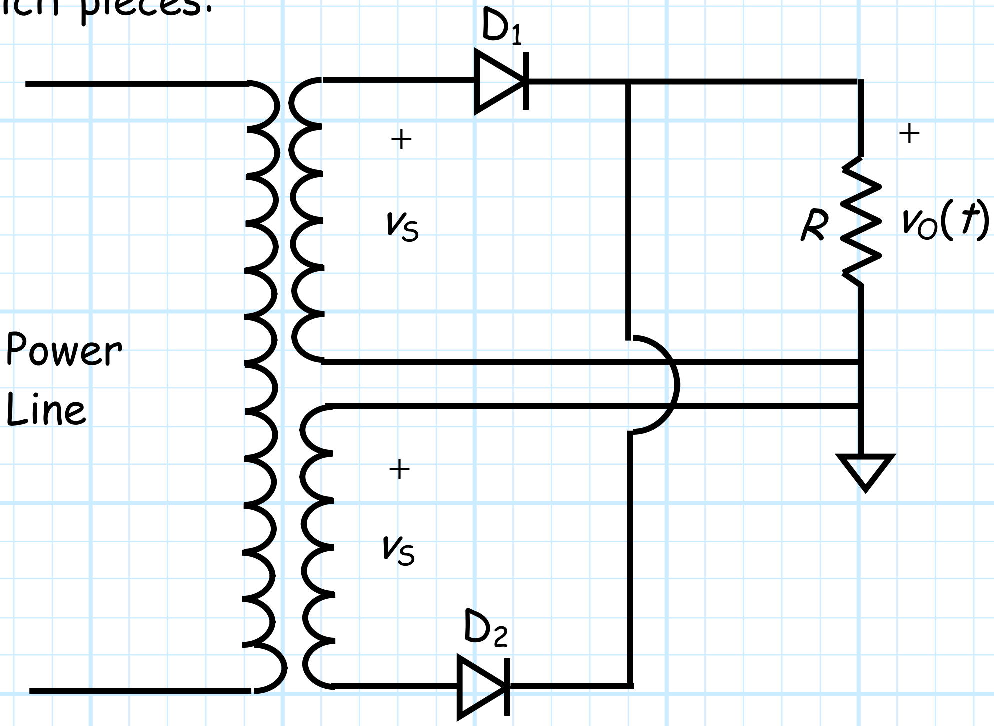


Conversely, if v_S=-10 V, the anode of D_1 will be 10V **below** ground potential (i.e., -10V), while the anode of D_2 will be 10V **above** ground potential:

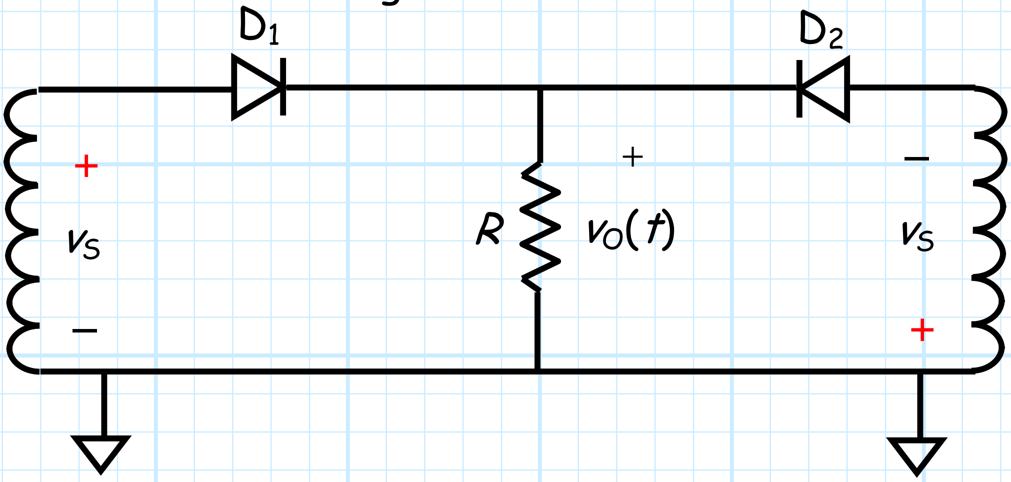


The more important question is, what is the value of **output** v_O? More specifically, how is v_O related to the value of source v_S—what is the **transfer fuction** $v_O = f(v_S)$?

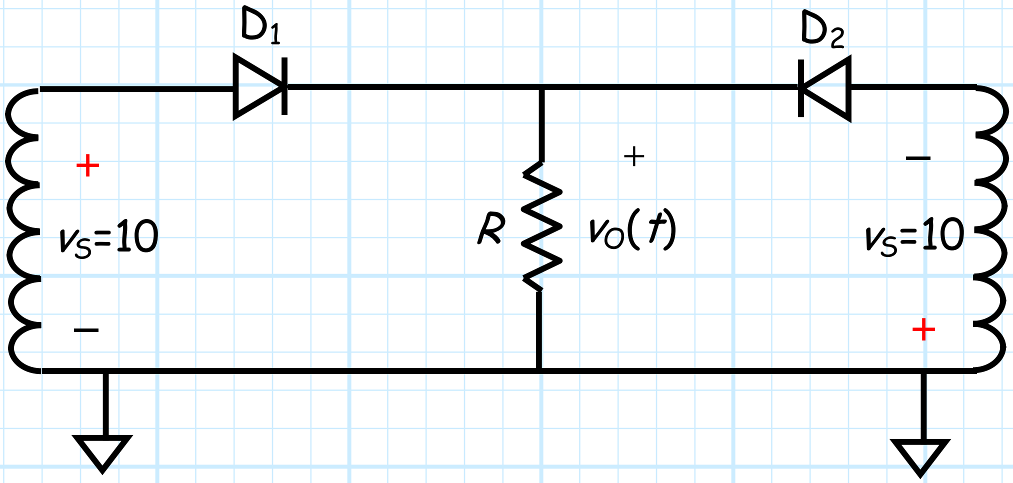
To help simplify our analysis, we are going **redraw** this cirucuit in another way. First, we will **split** the secondary winding into two explicit pieces:



We will now **ignore the primary** winding of the transformer and redraw the remaining circuit as:



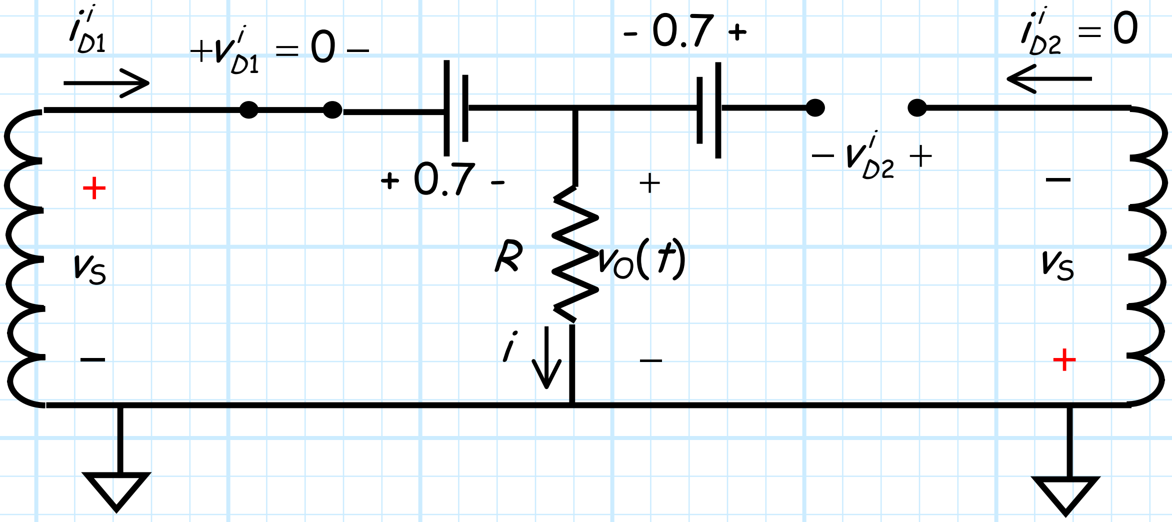
Note that the secondary voltages at either end of this circuit are the **same**, but have **opposite** polarity. As a result, if v_S=10, then the anode of diode D_1 will be 10 V **above** ground, and the anode at diode D_2 will be 10V **below** ground—just like before!



Now, let's attempt to determine the **transfer function** $v_O = f(v_S)$ of this circuit.

First, we will replace the junction diodes with **CVD models.**

Then let's ASSUME D_1 is **forward** biased and D_2 is **reverse** biased, thus ENFORCE $v_{D1}^i = 0$ and $i_{D2}^i = 0$. Thus ANALYZE:



Note that we need to determine **3** things: the **ideal diode current** i_{D1}^{i}, the **ideal diode voltage** v_{D2}^{i}, and the **output voltage** v_O. However, **instead** of finding numerical values for these 3 quantities, we must express them in terms of **source voltage** v_S!

From KCL: $i = i_{D1}^{i} + i_{D2}^{i} = i_{D1}^{i} + 0 = i_{D1}^{i}$

From KVL: $v_S - v_{D1}^{i} - 0.7 - R\, i_D^{i} = 0$

Thus the **ideal diode current** is:

$$i_{D1}^{i} = \frac{v_S - 0.7}{R}$$

Likewise, from KVL: $v_S - v_{D1}^{i} - 0.7 + 0.7 + v_{D2}^{i} + v_S = 0$

Thus, the **ideal diode voltage** is:

$$v_{D2}^{i} = -2v_S$$

And finally, from KVL: $v_S - v_{D1}^{i} - 0.7 = v_O$

Thus, the **output voltage** is:

$$v_O = v_S - 0.7$$

Now, we must determine **when** both $i_{D1}^{i} > 0$ and $v_{D2}^{i} < 0$. When **both** these conditions are true, the output voltage will be $v_O = v_S - 0.7$. When one **or** both conditions $i_{D1}^{i} > 0$ and $v_{D2}^{i} < 0$ are **false**, then our assuptions are **invalid**, and $v_O \neq v_S - 0.7$.

Using the results we just determined, we know that $i_{D1}^{i} > 0$ **when**:

$$\frac{v_S - 0.7}{R} > 0$$

Solving for v_S:

$$\begin{aligned} \frac{v_S - 0.7}{R} &> 0 \\ v_S - 0.7 &> 0 \\ v_S &> 0.7 \text{ V} \end{aligned}$$

Likewise, we find that $v_{D2}^{i} < 0$ **when**:

$$-2v_S < 0$$

Solving for v_S:

$$\begin{aligned} -2v_S &< 0 \\ 2v_S &> 0 \\ v_S &> 0 \end{aligned}$$

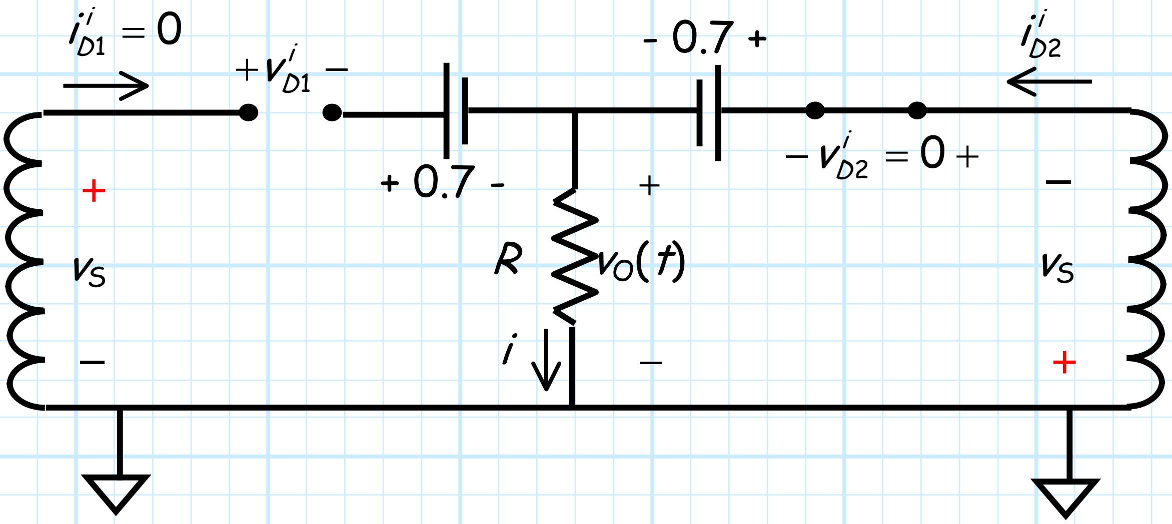
Thus, our assumptions are correct **when** $v_S > 0.0$ **AND** $v_S > 0.7$. This is the **same** thing as saying our assumptions are valid when $v_S > 0.7$!

Thus, we have found that the following statement is true about this circuit:

$$v_O = v_S - 0.7 \text{ V} \quad \text{when} \quad v_S > 0.7 \text{ V}$$

Note that this statement does **not** constitute a **function** (what about $v_S < 0.7$?), so we must **continue** with our analysis!

Say we now ASSUME that D_1 is **reverse** biased and D_2 is **forward** biased, so we ENFORCE $i_{D1}^i = 0$ and $v_{D2}^i = 0$. Thus, we ANALYZE **this** circuit:

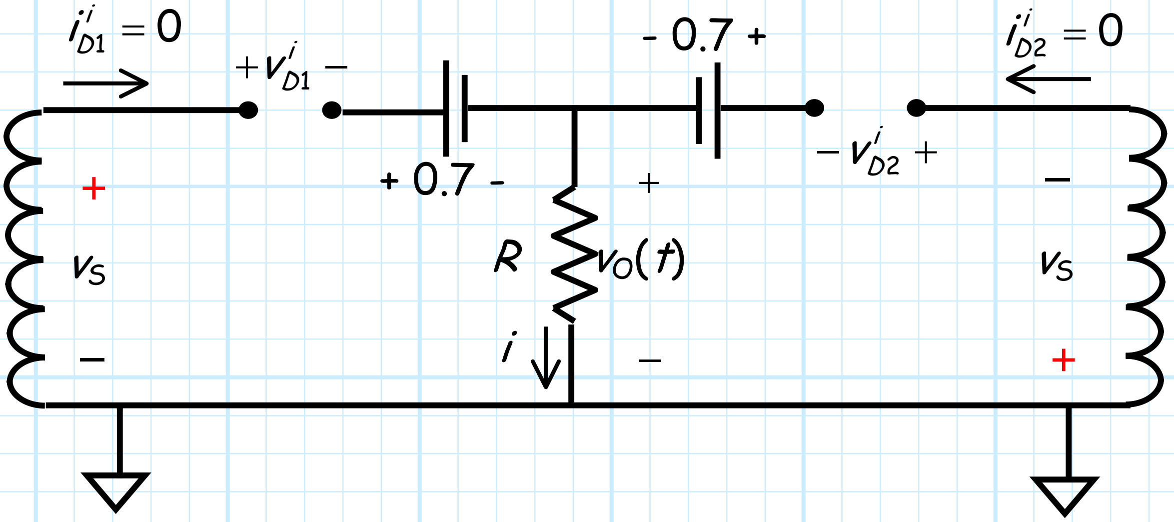


Using the **same proceedure** as before, we find that $v_O = -v_S - 0.7$, and both our assumptions are true **when** $v_S < -0.7$ V. In other words:

$$v_O = -v_S - 0.7 \text{ V} \quad \text{when} \quad v_S < -0.7 \text{ V}$$

Note we are still **not** done! We **still** do not have a complete transfer **function** (what happens when $-0.7 \text{ V} < v_S < 0.7 \text{ V}$?).

Finally then, we ASSUME that **both** ideal diodes are **reverse** biased, so we ENFORCE $i_{D1}^{i} = 0$ and $i_{D2}^{i} = 0$. Thus ANALYZE:



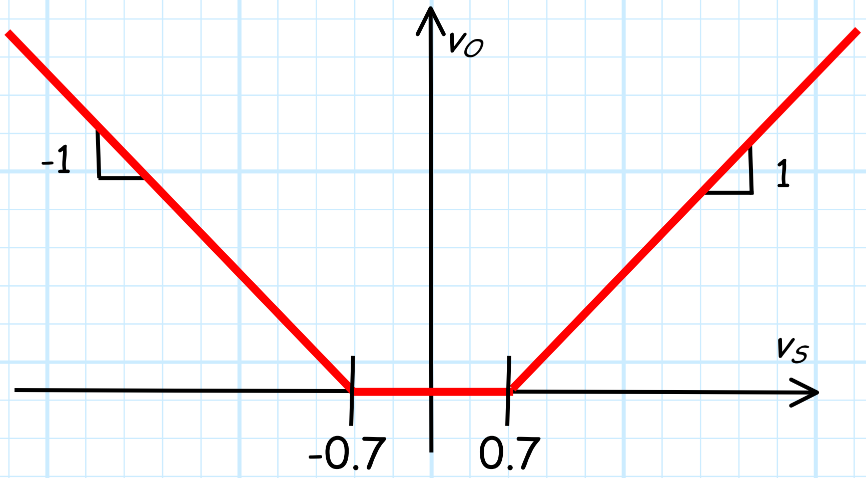
Following the **same proceedures** as before, we find that $v_s = 0$, and both assumptions are true **when** $-0.7 < v_s < 0.7$. In other words:

$$v_s = 0 \quad \text{when} \quad -0.7 < v_s < 0.7$$

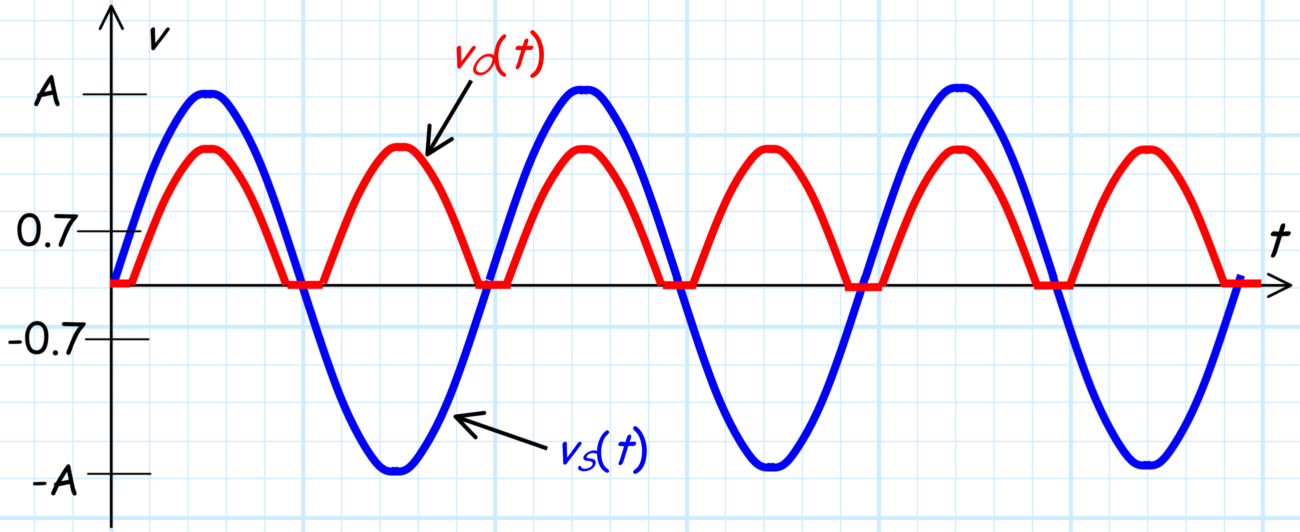
Now we have a **function**! The transfer function of this circuit is:

$$v_O = \begin{cases} v_s - 0.7\,V & \textit{for} \quad v_s > 0.7\,V \\ 0V & \textit{for} \quad -0.7 > v_s > 0.7\,V \\ -v_s - 0.7\,V & \textit{for} \quad v_s < -0.7\,V \end{cases}$$

Plotting this function:

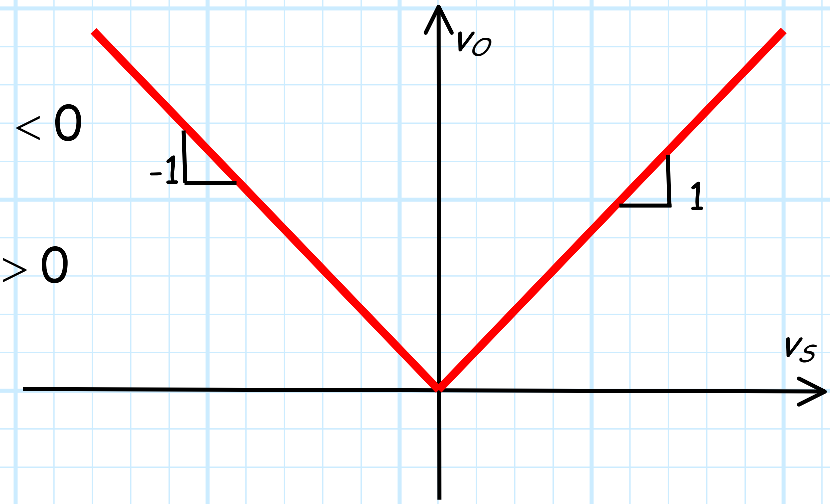


The output of this full-wave rectifier with a **sine wave input** is therefore:



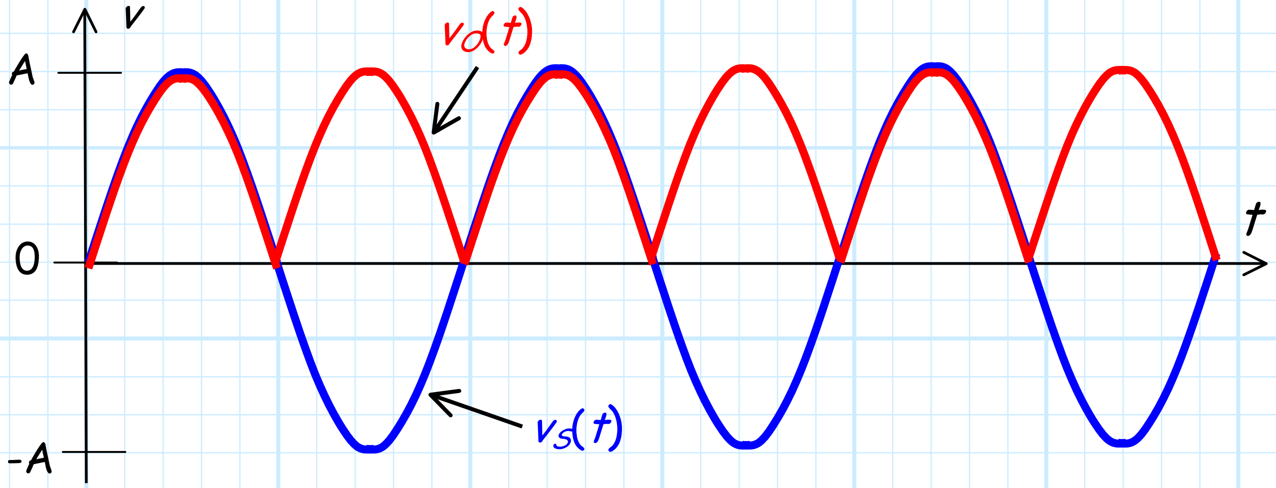
Note how this **compares** to the transfer function of the **ideal** full-wave rectifier:

$$v_O = \begin{cases} -v_S & \text{for} \quad v_S < 0 \\ v_S & \text{for} \quad v_S > 0 \end{cases}$$



Very similar!

Likewise, compare the output of this junction diode full-wave rectifier to the output of an **ideal** full-wave rectifier:



Again we see that the junction diode full-wave rectifier output is **very close** to ideal. In fact, if A>>0.7 V, the **DC component** of this junction diode full wave rectifier is approximately:

$$V_O \approx \frac{2A}{\pi} - 0.7 \text{ V}$$

Just 700 mV less than the **ideal** full-wave rectifier DC component!