

# Transcendental Solutions of Junction Diode Circuits

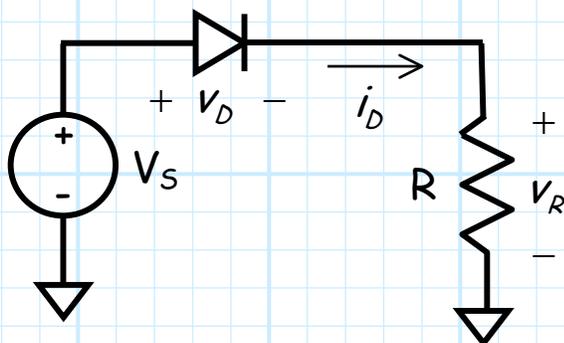
In a previous example, we were able to use the junction diode equation to **algebraically** analyze a circuit and find **numeric** solutions for all circuit currents and voltages.

However, we will find that this type of circuit analysis is, in general, often **impossible** to achieve using the junction diode equation!



**Q:** *Impossible !?! I must intercede, and point out that **you** are clearly wrong. If **I** have an explicit mathematical description of each device in a circuit (which **I** do for a junction diode), **I** can use KVL and KCL to analyze **any** circuit.*

**A:** Although we can always determine a numerical solution, it is often impossible to find this solution **algebraically**. Consider this **simple** junction diode circuit:



From KVL:

$$\begin{aligned}
 V_s - V_D - V_R &= 0 \\
 \therefore V_s - V_D - R i_D &= 0 \\
 \therefore i_D &= \frac{V_s - V_D}{R}
 \end{aligned}$$

Likewise, from the **junction** diode equation:

$$i_D = I_s \left( e^{v_D/nV_T} - 1 \right)$$

Equating these two, we have a **single** equation with a **single** unknown ( $v_D$ ):

$$\frac{V_s - v_D}{R} = I_s \left( e^{v_D/nV_T} - 1 \right)$$

**Q:** *Precisely! Just as I said! You have 1 equation with 1 unknown. Go solve this equation for  $v_D$ , and then you can determine all other unknown voltages and currents (i.e.,  $i_D$  and  $v_R$ ).*



**A:** But that's the problem! What is the algebraic solution of  $v_D$  for the equation:

$$\frac{V_s - v_D}{R} = I_s \left( e^{v_D/nV_T} - 1 \right)$$



????

The above equation is known as a **transcendental equation**. It is an algebraic expression for which there is **no algebraic solution!**

**Examples** of transcendental equations include:

$$x = \cos[x], \quad y^2 = \ln[y], \quad \text{or} \quad 4 - x = 2^x$$

**Q:** *But, we could build that simple junction diode circuit in the lab. Therefore  $v_D$ ,  $i_D$  and  $v_R$  must have **some** numeric value, right !?!*

**A:** Absolutely! For every value of source voltage  $V_s$ , resistance  $R$ , and junction diode parameters  $n$  and  $I_s$ , there is a specific numerical solution for  $v_D$ ,  $i_D$  and  $v_R$ . However, we **cannot** find this **numerical** solution with **algebraic** methods!

**Q:** *Well then how the heck do we find solution??*

**A:** We use what is know as **numerical methods**, often implementing some **iterative** approach, typically with the help of a **computer** (see example 3.4 on pp. 154-155).

This generally involves **more work** than we wish to do when analyzing junction diode circuits!

**Q:** *So just how **do** we analyze junction diode circuits??*

**A:** We replace the junction diodes with **circuit models** that **approximate** junction diode behavior!



**Q:** *Oh you're tricky, but you are still clearly wrong. Recall in an **earlier example** we analyzed a junction diode circuit, but we did **not** use "approximate models" nor "numerical methods" to find the answer!*

**A:** This is absolutely correct; we did **not** use approximate models or numerical methods to solve that problem. However, if you look back at that example, you will find that the problem was a bit **contrived**.

\* Recall that effectively, we were **given** the voltage across one diode as part of the problem statement. We were then asked to find the **source voltage**  $V_S$ .

\* This was a bit of an **academic** problem, as in the "real world" it is **unlikely** that we would somehow know the voltage across the diode without knowing the value of the voltage source that produced it!

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### Example: A Junction Diode Circuit

Consider the following circuit with two junction diodes:

The diodes are identical, with  $n = 1$  and  $I_S = 10^{-14}$  A.

**Q:** *If the current through the resistor is 6.5 mA, what is the voltage of source  $V_S$ ??*

**A:** This is a **difficult** problem to solve! Certainly, we cannot just write:

$$V_S =$$

and then the answer. Instead, let's just determine **what we can**, and see what happens!

- \* Thus, problems like this previous example are sometimes used by professors to create junction diode circuit problems that are solvable, **without** encountering a dreaded **transcendental equation!**
- \* In the real world, we typically know **neither** the diode voltage **nor** the diode current directly—transcendental equations are most often the **sad** result!
- \* **Instead** of applying numerical techniques, we will find it much faster (albeit slightly less accurate) to apply **approximate circuit models.**



*I wish I had a **nickel**  
for every time my  
software has **crashed**—  
Oh wait, **I do!***