3.2 Terminal Characteristics of Junction Diodes (pp.147-153)

HO: The Junction Diode Curve

HO: The Junction Diode Equation

A. The Forward Bias Region
\[ i_D = I_s \left( e^{v_D / nV_T} - 1 \right) \]

\[ \approx I_s e^{v_D / nV_T} \quad \text{for} \quad v_D \gg nV_T \]

Example: \( I_s = 10^{-12}, \, n=1 \)

\begin{array}{c|c|c|c|c|c|c|c}
 v_D (V) & i_D \\
 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \\
\end{array}
$V_D \approx 0.7 \ \text{V}$

b)

HO: The Junction Diode Forward Bias Equation

HO: Example: A Junction Diode Circuit

B. The Reverse Bias Region
\[ i_D = I_s \left( e^{v_D / nV_T} - 1 \right) \]
\[ \approx -I_s \quad \text{for} \quad v_D \ll -nV_T \]

**HO: Forward and Reverse Bias Approximations**

**C. The Breakdown Region**
D. Power Dissipation in Junction Diodes

\[ \text{f.b.} \Rightarrow \]
\[ \text{r.b} \Rightarrow \]
\[ \text{b.d} \Rightarrow \]
The Junction Diode Curve

In many ways, junction diode (i.e., real diode) behavior is similar to that of ideal diodes. However, there are some important and profound differences!

First, recall the ideal diode current voltage curve:

This curve is piece-wise linear, with two unambiguous regions—reverse bias (where \( v < 0 \) and \( i = 0 \)), and forward bias (where \( i > 0 \) and \( v = 0 \)).

Now consider the behavior of a junction diode:
Please note that unlike the ideal diode, the junction diode curve:

a) is continuous (not piece-wise linear).
b) Has three apparent regions of operation (not two).
c) Has, therefore, ambiguous boundaries between regions (i.e., continuous transitions occur between regions—the curve has two "knees"!).

By comparison to the ideal diode, we likewise define one region of the junction diode curve as the forward bias region, and another as the reverse bias region.

The third region has no similarity with ideal diode behavior (i.e., this is a “new” region). We call this region breakdown.
Note that the breakdown region occurs when the junction diode voltage (from anode to cathode) is approximately less than or equal to a voltage value $-V_{ZK}$. The value $V_{ZK}$ is known as the zener breakdown voltage, and is a fundamental performance parameter of any junction diode.

As we shall see later, the behavior of a junction diode in the forward and reverse bias region is a predictable result of semiconductor physics! As such we can write an explicit mathematical expression, simultaneously describing the behavior of a junction diode in both the forward and reverse bias regions (but not in breakdown!):

\[ i_d = I_s \left( e^{\frac{v_d}{nV_T}} - 1 \right) \text{ for } v_d > -V_{ZK} \]
The Junction Diode Equation

The relationship between the current through a junction diode ($i_D$) and the voltage across it ($v_D$) is:

$$i_D = I_s \left( e^{v_D/nV_T} - 1 \right) \quad \text{for} \quad v_D > -V_{ZK}$$

Note: this equation describes diode behavior in the forward and reverse biased region only (i.e., not valid for breakdown).

Q: Good golly! Just what do those dog-gone parameters $n$, $I_s$ and $V_T$ mean?

A: Similar to the resistance value $R$ of a resistor, or the capacitance $C$ of a capacitor, these three parameters specify the performance of a junction diode. Specifically, they are:

1. $I_s$ = Saturation (or scale) Current. Depends on diode material, size, and temperature.

   Typical values range from $10^{-8}$ to $10^{-15}$ A (i.e., tiny)!
2. \( V_T = \text{Thermal Voltage} = \frac{kT}{q} \)

Where:

- \( k = \text{Boltzman's Constant} \)
- \( T = \text{Diode Temperature (°K)} \)
- \( q = \text{Charge on an electron (coulombs)} \)

\[ \text{At 20 °C, } V_T \approx 25 \text{ mV} \]

**IMPORTANT NOTE!**: Unless otherwise stated, we will assume that each and every junction diode is at room temperature (i.e., \( T = 20 \text{ °C} \)). Thus, we will always assume that the thermal voltage \( V_T \) of all junction diodes is 25 mV (i.e., \( V_T = 25 \text{ mV} \))!

3. \( n = \) a constant called the **ideality factor** (i.e. “fudge factor”).

Typically, \( 1 \leq n \leq 2 \)
The Junction Diode
Forward Bias Equation

In forward bias, we have learned that the diode current $i_D$ can be related to the diode voltage $v_D$ using the following approximation:

$$i_D = I_S \left(e^{v_D/nV_T} - 1\right) \approx I_S e^{v_D/nV_T},$$

provided that $v_D \gg 25 \text{ mV}$.

We can invert this approximation to alternatively express $v_D$ in terms of diode current $i_D$:

$$I_S e^{v_D/nV_T} = i_D$$
$$e^{v_D/nV_T} = \frac{i_D}{I_S}$$
$$\frac{v_D}{nV_T} = \ln\left(\frac{i_D}{I_S}\right)$$
$$v_D = nV_T \ln\left(\frac{i_D}{I_S}\right)$$
Now, say a voltage $v_1$ across some junction diode results in a current $i_1$. Likewise, different voltage $v_2$ across this same diode a diode of course results in a different current $i_2$. We can define the difference between these two voltages as $\Delta v = v_2 - v_1$, and then using the above equation can express this voltage difference as:

$$\Delta v = nV_T \ln\left(\frac{i_2}{I_s}\right) - nV_T \ln\left(\frac{i_1}{I_s}\right) = nV_T \ln\left(\frac{i_2}{I_s} \cdot \frac{I_s}{i_1}\right) = nV_T \ln\left(\frac{i_2}{i_1}\right)$$

Yikes! Look at what this equation says:

* The difference in the two voltages is dependent on the ratio of the two currents.

* This voltage difference is independent of scale current $I_s$.

We can likewise invert the above equation and express the ratio of the two currents in terms of the difference of the two voltages:
\[
\begin{align*}
nV_T \ln \left[ \frac{i_2}{i_1} \right] &= v_2 - v_1 \\
\ln \left[ \frac{i_2}{i_1} \right] &= \frac{(v_2 - v_1)}{nV_T} \\
\frac{i_2}{i_1} &= \exp \left[ \frac{(v_2 - v_1)}{nV_T} \right]
\end{align*}
\]

Again, we find that this expression is independent of scale current \( I_s \).

**Q:** Stop wasting my time with these pointless derivations! Are these expressions even remotely useful?!

**A:** These expressions are often very useful! Frequently, instead of explicitly providing device parameters \( n \) and \( I_s \), a junction diode is specified by stating \( n \), and then a statement of the specific diode current resulting from a specific diode voltage.

For example, a junction diode might be specified as:

"A junction diode with \( n = 1 \) pulls 2mA of current at a voltage \( v_D = 0.6 \) V."
The above statement **completely specifies** the performance of this particular junction diode—we can now determine the current flowing through this diode for any other value of diode voltage $v_D$. Likewise, we can find the voltage across the diode for any other diode current value $i_D$.

For **example**, say we wish to find the current through the junction diode specified above when a potential difference of $v_D = 0.7$ V is placed across it. We have two options for finding this current:

**Option 1:**

We know that $n=1$ and that $i_D = 2$ mA when $v_D = 0.6$ V. Thus, we can use this information to solve for scale current $I_s$:

$$I_s e^{\frac{v_D}{nV_T}} = i_D$$

$$I_s e^{\frac{0.6}{0.025}} = 2$$

$$I_s = 2 e^{-\frac{0.6}{0.025}}$$

$$I_s = 7.55 \times 10^{-11} \text{ mA}$$

Now, we use the forward-biased junction diode equation to determine the current through this device at the new voltage of $v_D = 0.7$ V:

$$i_D = I_s e^{\frac{v_D}{nV_T}}$$

$$= (7.55 \times 10^{-11}) e^{\frac{0.7}{0.025}}$$

$$= 109.2 \text{ mA}$$
**Option 2**

Here, we directly determine the current at $v_D = 0.7$ using one of the expressions derived earlier in this handout! Using $i_T = 2 \text{ mA}$, $v_1 = 0.6$, and $v_2 = 0.7 \text{ V}$ we can find current $i_2$ as:

\[
\frac{i_2}{i_1} = \exp \left[ \frac{(v_2 - v_1)}{nV_T} \right]
\]
\[
i_2 = i_1 \exp \left[ \frac{(v_2 - v_1)}{nV_T} \right]
\]
\[
i_2 = 2 \exp \left[ \frac{(0.7 - 0.6)}{0.025} \right]
\]
\[
i_2 = 109.2 \text{ mA}
\]

Option 2 (using the equations we derived in this handout) is obviously quicker and easier (note in option 2 we did not have to deal with annoying numbers like $7.55 \times 10^{-11}$!).

Finally, we should also note that junction diodes are often specified simply as "a 2 mA diode" or "a 10 mA diode" or "a 100 mA diode". These statements implicitly provide the diode current at the standard diode test voltage of $v_D = 0.7 \text{ V}$.

**Q:** But what about the value of junction diode ideality factor $n$ ?
A: If no value of $n$ is provided (and there is not sufficient information given to determine it), we typically just assume that $n = 1$.

For example, consider the following problem:

"Determine the voltage across a 100 mA junction diode when there is 2 mA of current flowing through it."

A "100 mA junction diode" simply means a junction diode that will have a current of 100 mA flowing through it ($i_D = 100$ mA) if the voltage across it is $v_D = 0.7$ V. We will assume that $n=1$, since no other information about that parameter was given.

Thus, using $v_1 = 0.7$, $i_1 = 100$ mA, and $i_2 = 2$ mA, we can determine the value of $v_2$:

$$v_2 - v_1 = nV_t \ln \left( \frac{i_2}{i_1} \right)$$
$$v_2 - 0.7 = (0.025) \ln \left( \frac{2}{100} \right)$$
$$v_2 = 0.7 - 0.10$$
$$v_2 = 0.60 \text{ V}$$

**EXCELENT!**
Example: A Junction Diode Circuit

Consider the following circuit with two junction diodes:

![Diode Circuit Diagram]

The diodes are identical, with $n = 1$ and $I_S = 10^{-14} \text{ A}$.

Q: If the current through the resistor is 6.5 mA, what is the voltage of source $V_S$?

A: This is a difficult problem to solve! Certainly, we cannot just write:

$$V_S =$$

and then the answer. Instead, let's just determine what we can, and see what happens!
1) If 6.5 mA flows through a 0.1 K resistor, the voltage across that resistor is:

\[ V_R = \]

2) If the voltage across the resistor is 0.65 V, then the voltage across the diode \( D_2 \), which is parallel to the resistor, is the same value:

\[ V_{D2} = \]

3) If we know the voltage across a p-n junction diode, then we also know its current!

\[
     i_{D2} = I_S \exp \left( \frac{V_{D2}}{nV_T} \right) = 10^{-14} \exp \left[ \frac{0.650}{0.025} \right] = 1.96 \text{ mA}
\]

4) If we know \( i_{D2} \) and the current through the resistor, we know (using KCL) the current through \( D_1 \):

\[ i_{D1} = \]
5) If we know the current through a junction diode, then we can find the voltage across it:

\[ v_{D1} = nV_T \ln \left( \frac{i_{D1}}{I_S} \right) = 0.025 \ln \left( \frac{0.00846}{10^{-14}} \right) = 0.69V \]

6) Finally, if we know \( v_{D1} \) and \( v_{D2} \), we can find \( V_S \) using KVL:

\[ V_S = \]

\[ +v_{D1} = 0.69V \]

\[ +v_{D2} = 0.69V \]

\[ -v_{D1} = 0.69V \]

\[ -v_{D2} = 0.69V \]
Forward and Reverse Bias Approximations

Q: Man, am I ever befuddled! Is the behavior of a junction diode in the forward biased region described as this:

\[ i_D = I_s \left( e^{\frac{v_D}{nV_T}} - 1 \right) \]

or as this:

\[ i_D = I_s e^{\frac{v_D}{nV_T}} \]

or as this

\[ i_D > 0 \quad \text{and} \quad v_D = 0.7 \quad \text{V} \]

A: Actually, all three of the above statements are true (or, at least, approximately so)!

Let's review what we know about the junction diode in forward and reverse bias:

1. First, we know that if the diode is not in breakdown, the relationship between current and voltage can be precisely described as:

\[ i_D = I_s \left( e^{\frac{nV_T}{v_D}} - 1 \right) \]
\[ i_D = I_s \left( e^{\frac{v_D}{nV_T}} - 1 \right) \quad \text{for} \quad v_D > -V_{ZK} \]

**Q:** Here’s where I get confused. Is this equation valid for reverse bias, or is it valid for forward bias?

**A:** The above expression is valid for forward bias, and it is valid for reverse bias, and it is also valid for the transition region between forward and reverse bias!

In other words, the above equation is a very accurate description of the junction diode behavior—with the important exception of when the junction diode is in breakdown.

2. Now, let's simplify the previous expression further, separately examining the cases when the junction diode is in forward bias (i.e., \( v_D \gg nV_T \)), and reverse bias (i.e., \( -V_{ZK} < v_D \ll -nV_T \)).

For the forward bias case, we find that:

\[ e^{\frac{v_D}{nV_T}} \ll 1 \quad \text{if} \quad v_D \gg nV_T \]
Therefore, we can approximate the junction diode behavior in **forward bias** mode as:

\[ i_D \approx I_s e^{v_D/nVT} \quad \text{for} \quad v_D \gg nV_T \quad (\text{i.e., forward biased}) \]

Likewise, for the **reverse bias** case, we find that:

\[ e^{v_D/nVT} \ll 1 \quad \text{if} \quad v_D \ll -nV_T \]

Therefore, we can approximate the junction diode behavior in **reverse bias** mode as:

\[ i_D \approx -I_s \quad \text{for} \quad -V_{ZK} < v_D \ll -nV_T \quad (\text{i.e., reversed biased}) \]

Combining, we can approximate the expression at the top of the previous page as:

\[
\begin{align*}
\hat{i}_D &\approx \begin{cases} 
I_s e^{v_D/nVT} & \text{for} \quad v_D \gg nV_T \quad (\text{i.e., forward biased}) \\
-I_s & \text{for} \quad -V_{ZK} < v_D \ll -nV_T \quad (\text{i.e., reversed biased})
\end{cases}
\end{align*}
\]

**3.** We can now simplify these expressions even further! We rewrite the above approximation for forward bias so that the junction diode voltage is a function of junction diode current:
\[ I_s e^{v_D/nV_T} = i_D \]
\[ e^{v_D/nV_T} = \frac{i_D}{I_s} \]
\[ \frac{v_D}{nV_T} = \ln \left( \frac{i_D}{I_s} \right) \]
\[ v_D = nV_T \ln \left( \frac{i_D}{I_s} \right) \]

As a previous example demonstrated, as we vary the value of diode current \( i_D \) from microamps to kiloamps, the diode voltage will vary only a few hundred millivolts, from about 0.5 V to 0.9 V.

Thus, we can assume that if any appreciable current is flowing from junction diode anode to junction diode cathode (i.e., forward bias condition), the junction diode voltage will be approximately (i.e., within a few hundred millivolts) 0.7 V.

Q: It looks to me that you are saying a **forward biased** junction diode exhibits a diode voltage of \( v_D = 700 \text{mV} \), regardless of the diode current \( i_D \), right?
A: NO! This is not what I am saying! As is evident in the previous two equations, the junction diode current in forward bias is directly dependent on diode current—as the current increases, the voltage increases! For each possible diode current, there is a specific (and different) diode voltage.

* However, we find that this increase is logarithmically related to diode current, such that the voltage increases very slowly with increasing current—it takes a bunch of additional junction diode current to increase the junction diode voltage even a small amount.

* Thus, we are simply saying that for all appreciable (and plausible) diode currents, the junction diode voltage will be within of few hundred millivolts of, say, 700 mV.

* As a result, $v_D = 0.7 \text{ V}$ is not a bad approximation for forward biased junction diodes!

Now, we can likewise simplify further our approximation for a reverse biased junction diode. Recall that we now approximate the reverse bias diode current as $i_D = -I_s$.

However, recall that the diode saturation current $I_s$ is a very small value, typically $10^{-8}$ to $10^{-15}$ Amps!

Q: A billionth of an amp!? That’s so tiny it might as well be zero!
A: Precisely! The reverse bias current value $i_D = -I_s$ is so small that we can approximate it as zero:

$$i_D \approx 0 \quad \text{if} \quad -V_{zk} < v_D \ll -nV_T \quad \text{(reverse bias)}$$

Thus, we arrive at an even simpler (albeit less accurate) approximation of junction diode behavior in forward and reverse bias:

Each of the three expressions examined in this handout can be used to describe the behavior of junction diodes in forward and/or reverse bias. The first expression we examined is the most accurate, but it is likewise the most mathematically complex. Conversely, the third expression above is the simplest, but is likewise the least accurate.

We will find that all three of the expressions are useful to us, depending on what specifically we are attempting to determine, and how accurately we need to determine it!