3.5 Rectifier Circuits

A. Junction Diode 2-Port Networks

\[ + \quad V_I(t) \quad + \quad V_O(t) \quad - \]

\[ \text{Junction Diode Circuit} \]

**HO:** The Transfer Function of Diode Circuits

**Q:**

**A:** *HO: Steps for finding a Junction Diode Circuit Transfer Function*

**Example:** Diode Circuit Transfer Function
B. Diode Rectifiers

**HO: Signal Rectification**

**Q:**

**A:**

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**HO: The Full-Wave Rectifier**  

**HO: The Bridge Rectifier**

**HO: Peak Inverse Voltage**
The Transfer Function of Diode Circuits

For many junction diode circuits, we find that one of the voltage sources is in fact unknown! This unknown voltage is typically some input signal of the form $v_I(t)$, which results in an output voltage $v_O(t)$.

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**Q:** How the heck do you expect us to determine $v_O(t)$ if we have no idea what $v_I(t)$ is??

**A:** We of course cannot determine an explicit value or expression for $v_O(t)$, since it depends on the input $v_I(t)$. Instead, we will attempt to explicitly determine this dependence of $v_O(t)$ on $v_I(t)$!
In other words, we seek to find an expression for $v_O$ in terms of $v_I$. Mathematically speaking, our goal is to determine the function:

$$v_O = f(v_I)$$

We refer to this as the circuit **transfer function**.

Note that we can plot a circuit transfer function on a 2-dimensional plane, just as if the function related values $x$ and $y$ (e.g. $y = f(x)$). For example, say our circuit transfer function is:

$$v_O = f(v_I) = 3v_I + 2$$

Note this is simply the **equation of a line** (e.g., $y = 3x + 2$), with slope $m=3$ and intercept $b=2$. 

![Diagram of a circuit transfer function](image)
Q: A “function” eh? Isn’t a “function” just your annoyingly pretentious way of saying we need to find some mathematic equation relating $v_O$ and $v_I$?

A: Actually no! Although a function is a mathematical equation, there are in fact scads of equations relating $v_O$ and $v_I$ that are not functions!

$\Rightarrow$ The set of all possible functions $y = f(x)$ are a subset of the set of all possible equations relating $y$ and $x$.

A function $v_O = f(v_I)$ is a mathematical expression such that for any value of $v_I$ (i.e., $-\infty < v_I < \infty$), there is one, but only one, value $v_O$.

Note this definition of a function is consistent with our physical understanding of circuits—we can place any voltage on the input that we want (i.e., $-\infty < v_I < \infty$), and the result will be one specific voltage value $v_O$ on the output.

Therefore, examples of valid circuit transfer functions include:
Conversely, the transfer “functions” below are invalid—they cannot represent the behavior of circuits, since they are not functions!
Moreover, we find that circuit transfer functions must be continuous. That is, $v_O$ cannot “instantaneously change” from one value to another as we increase (or decrease) the value $v_I$.

![A Discontinuous Function](https://via.placeholder.com/150)

![A Continuous Function](https://via.placeholder.com/150)

Remember, the transfer function of every junction diode circuit must be a continuous function. If it is not, you’ve done something wrong!
Steps for Finding a Junction Diode Circuit Transfer Function

Determining the transfer function of a junction diode circuit is in many ways very similar to the analysis steps we followed when analyzing previous junction diode circuits (i.e., circuits where all sources were explicitly known).

However, there are also some important differences that we must understand completely if we wish to successfully determine the correct transfer function!

**Step 1:** Replace all junction diodes with an appropriate junction diode model.

*Just* like before! We will now have an IDEAL diode circuit.

**Step 2:** Assume some mode for all ideal diodes.

*Just* like before! An IDEAL diode can be either forward or reverse biased.
**Step 3:** ENFORCE the bias assumption.

Just like before! ENFORCE the bias assumption by replacing the ideal diode with short circuit or open circuit.

**Step 4:** ANALYZE the remaining circuit.

Sort of, kind of, like before!

1. If we assumed an IDEAL diode was forward biased, we must determine $i_D$ -- just like before! However, instead of finding the numeric value of $i_D$, we determine $i_D$ as a function of the unknown source (e.g., $i_D = f(V_I)$).

2. Or, if we assumed an IDEAL diode was reversed biased, we must determine $v_D'$ -- just like before! However, instead of finding the numeric value of $v_D'$, we determine $v_D'$ as a function of the unknown source (e.g., $v_D' = f(V_I)$).

3. Finally, we must determine all the other voltages and/or currents we are interested in (e.g., $v_O$) -- just like before! However, instead of finding its numeric value, we determine it as a function of the unknown source (e.g., $v_O = f(V_I)$).
**Step 5:** Determine WHEN the assumption is valid.

**Q:** OK, we get the picture. Now we have to CHECK to see if our IDEAL diode assumption was correct, right?

**A:** Actually, no! This step is very different from what we did before!

We cannot determine IF \( i_D^f > 0 \) (forward bias assumption), or IF \( \nu_D^r < 0 \) (reverse bias assumption), since we cannot say for certain what the value of \( i_D^f \) or \( \nu_D^r \) is!

Recall that \( i_D^f \) and \( \nu_D^r \) are functions of the unknown voltage source (e.g., \( i_D^f = f(\nu_I) \) and \( \nu_D^r = f(\nu_I) \)). Thus, the values of \( i_D^f \) or \( \nu_D^r \) are dependent on the unknown source (\( \nu_I \), say). For some values of \( \nu_I \), we will find that \( i_D^f > 0 \) or \( \nu_D^r < 0 \), and so our assumption (and thus our solution for \( \nu_O = f(\nu_I) \)) will be correct.

However, for other values of \( \nu_I \), we will find that \( i_D^f < 0 \) or \( \nu_D^r > 0 \), and so our assumption (and thus our solution for \( \nu_O = f(\nu_I) \)) will be incorrect!

**Q:** Yikes! What do we do? How can we determine the circuit transfer function if we can't determine IF our ideal diode assumption is correct??
A: Instead of determining IF our assumption is correct, we must determine WHEN our assumption is correct!

In other words, we must determine for what values of $v_I$ is $i_D > 0$ (forward bias), or for what values of $v_I$ is $i_D < 0$ (reverse bias).

We can do this since we earlier (in step 4) determined the function $i_D = f(v_I)$ or the function $v_D = f(v_I)$.

Perhaps this step is best explained by an example. Let's say we assumed that our ideal diode was forward biased and, say we determined (in step 4) that $v_O$ is related to $v_I$ as:

$$v_O = f(v_I) = 2v_I - 3$$

Likewise, say that we determined (in step 4) that our ideal diode current is related to $v_I$ as:

$$i_D = f(v_I) > \frac{v_I - 5}{4}$$

Thus, in order for our forward bias assumption to be correct, the function $i_D = f(v_I)$ must be greater than zero:
We can now “solve” this inequality for $v_I$:

$$\frac{v_I - 5}{4} > 0$$

$$v_I > 5$$

Q: What does this mean? Does it mean that $v_I$ is some value greater than 5.0V??

A: NO! Recall that $v_I$ can be any value. What the inequality above means is that $i_D^i > 0$ (i.e., the ideal diode is forward biased) WHEN $v_D^i > 5.0$.

Thus, we know $v_o = 2v_I - 3$ is valid WHEN the ideal diode is forward biased, and the ideal diode is forward biased WHEN (for this example) $v_D^i > 5.0$. As a result, we can mathematically state that:

$$v_o = 2v_I - 3 \quad \text{when} \quad v_I > 5.0 \text{ V}$$
Conversely, this means that if $v_I < 5.0 \text{ V}$, the ideal diode will be reverse biased—our forward bias assumption would not be valid, and thus our expression $v_O = 2v_I - 3$ is not correct ($v_O \neq 2v_I - 3$ for $v_I < 5.0 \text{V}$!)

Q: So how do we determine $v_O$ for values of $v_I < 5.0 \text{ V}$?

A: Time to move to the last step!

Step 6: Change assumption and repeat steps 2 through 5!

For our example, we would change our bias assumption and now assume reverse bias. We then enforce $i_D = 0$, and then analyze the circuit to find both $v'_D = f(v_I)$ and a new expression $v_O = f(v_I)$ (it will no longer be $v_O = 2v_I - 3$!).

We then determine when our reverse bias assumption is valid, by solving the inequality $v'_D = f(v_I) > 0$ for $v_I$. For the example used here, we would find that the ideal diode is reverse biased when $v_I < 5.0 \text{V}$.

For junction diode circuits with multiple diodes, we may have to repeat this entire process multiple times, until all possible bias conditions are analyzed.
If we have done our analysis properly, the result will be a valid continuous function! That is, we will have an expression (but only one expression) relating $v_O$ to all possible values of $v_I$.

This transfer function will typically be piecewise linear. An example of a piece-wise linear transfer function is:

$$v_O = \begin{cases} 
2v_I - 3 & \text{for } v_I > 5.0 \\
12 - v_I & \text{for } v_I < 5.0 
\end{cases}$$

Just to make sure that we understand what a function is, note that the following expression is not a function:

$$v_O = \begin{cases} 
2v_I - 3 & \text{for } v_I > 7.0 \\
12 - v_I & \text{for } v_I < 3.0 
\end{cases}$$
Nor is this expression a function:

\[
v_o = \begin{cases} 
  2v_i - 3 & \text{for } v_i > 3.0 \\
  12 - v_i & \text{for } v_i < 7.0 
\end{cases}
\]
Finally, note that the following expression is a function, but it is not continuous:

\[
V_o = \begin{cases} 
2V_I - 3 & \text{for } V_I > 5.0 \\
5 - V_I & \text{for } V_I < 5.0
\end{cases}
\]

Make sure that the piece-wise transfer function that you determine is in fact a function, and is continuous!
Example: Diode Circuit
Transfer Function

Consider the following circuit, called a half-wave rectifier:

Let’s use the CVD model to determine the output voltage $v_O$ in terms of the input voltage $v_S$. In other words, let’s determine the diode circuit transfer function $v_O = f(v_S)$!

**ASSUME** the ideal diode is forward biased, ENFORCE $i_D^f = 0$.

From KVL, we find that:

$$v_O(t) = v_S(t) - 0.7$$
This result is of course true if our original assumption is correct—it is valid if the ideal diode is forward biased (i.e., \( i_d' > 0 \))!

From Ohm’s Law, we find that:

\[
i_d' = \frac{V_D}{R} = \frac{v_s - 0.7}{R}
\]

**Q:** I’m so confused! Is this current greater than zero or less than zero? Is our assumption correct? How can we tell?

**A:** The ideal diode current is dependent on the value of source voltage \( v_s(t) \). As such, we cannot determine if our assumption is correct, we instead must find out when our assumption is correct!

In other words, we know that the forward bias assumption is correct when \( i_d' > 0 \). We can rearrange our diode current expression to determine for what values of source voltage \( v_s(t) \) this is true:

\[
\begin{align*}
i_d' &> 0 \\
\frac{v_s(t) - 0.7}{R} &> 0 \\
v_s(t) - 0.7 &> 0 \\
v_s(t) &> 0.7
\end{align*}
\]
So, we have found that when the source voltage $v_s(t)$ is greater than 0.7 V, the output voltage $v_o(t)$ is:

$$v_o(t) = v_s(t) - 0.7$$

Now we change our assumption and ASSSUME the ideal diode in the CVD model is reverse biased, an assumption ENFORCEd with the condition that $i_d' = 0$ (i.e., an open circuit).

From Ohm's Law, we find that the output voltage is:

$$v_o = R i_d'$$

$$= R (0)$$

$$= 0 \text{ V} !!!$$

Q: OK, I've got this result written down. However, I still don't know what the output voltage $v_o(t)$ is when the source voltage $v_s(t)$ is less than 0.7V!??

Q: Fascinating! The output voltage is zero when the ideal diode is reverse biased. But, precisely when is the ideal diode reverse biased? For what values of $v_s$ does this occur?
**A:** To answer these questions, we must determine the **ideal** diode voltage in **terms** of \( v_S \) (i.e., \( v_D^i = f(v_S) \)):

From KVL:

\[
v_S - v_D^i - 0.7 = v_O
\]

Therefore:

\[
v_D^i = v_S - 0.7 - v_O
\]

\[
= v_S - 0.7 - 0.0
\]

\[
= v_S - 0.7
\]

Thus, the ideal diode is in reverse bias **when**:

\[
v_D^i < 0
\]

\[
v_S - 0.7 < 0
\]

Solving for \( v_S \), we find:

\[
v_S - 0.7 < 0
\]

\[
v_S < 0.7 \text{ V}
\]

In other words, we have determined that the **ideal** diode will be reverse biased **when** \( v_S < 0.7 \text{ V} \), and that the output voltage will be \( v_O = 0 \).

**Q:** So, we have found that:

\[
v_O = v_S - 0.7 \quad \text{when} \quad v_S > 0.7 \text{ V}
\]

and,

\[
v_O = 0.0 \quad \text{when} \quad v_S < 0.7 \text{ V}
\]

It appears we have a valid, continuous, function!
A: That's right! The **transfer function** for this circuit is therefore:

\[ v_O = \begin{cases} 
  v_s - 0.7 & \text{for } v_s > 0.7 \\
  0 & \text{for } v_s < 0.7 
\end{cases} \]

Although the circuit in this example may *seem* trivial, it is actually *very important*!

*It is called a half-wave rectifier, and provides signal rectification.*

Rectifiers are an *essential* part of every AC to DC power supply!
Signal Rectification

An important application of junction diodes is signal rectification.

There are two types of signal rectifiers, half-wave and full-wave.

Let's first consider the ideal half-wave rectifier. It is a circuit with the transfer function $v_O = f(v_S)$:

$$v_O = \begin{cases} 
0 & \text{for } v_S < 0 \\
1 & \text{for } v_S > 0 
\end{cases}$$
Pretty simple! **When** the input is negative, the output is **zero**, whereas **when** the input is positive, the output is the **same** as the input.

**Q:** Pretty simple and pretty **stupid** I’d say! This might be your most **pointless** circuit yet. How in the world is **this** circuit useful??

**A:** To see **why** a half-wave rectifier is useful, consider the typical case where the input source voltage is a **sinusoidal** signal with **frequency** \( \omega \) and peak **magnitude** \( A \):

\[
v_s(t) = A \sin \omega t
\]

Think about what the **output** of the half-wave rectifier would be! Remember the rule: when \( v_s(t) \) is **negative**, the output is **zero**, when \( v_s(t) \) is **positive**, the output is **equal** to the input.
The output of the half-wave rectifier for this example is therefore:

\[ V_S(t) = \frac{1}{T} \int_0^T v_S(t) \, dt \]

\[ = \frac{1}{T} \int_0^T A \sin \omega t \, dt = 0 \]

**Q:** That's maybe the lamest result I've ever seen. What good is half a sine wave? Why do we even bother?

**A:** Although it may appear that our rectifier had little useful effect on the input signal \( v_S(t) \), in fact the difference between input \( v_S(t) \) and output \( v_{\Delta}(t) \) is both important and profound.

To see how, consider first the **DC component** (i.e. the time-averaged value) of the input sine wave:
Thus, (as you probably already knew) the DC component of a sine wave is zero—a sine wave is an AC signal!

Now, contrast this with the output $v_o(t)$ of our half-wave rectifier. The DC component of the output is:

$$V_o = \frac{1}{T} \int_0^T v_o(t) \, dt$$

$$= \frac{1}{T} \int_0^{T/2} A \sin \omega t \, dt + \frac{1}{T} \int_{T/2}^T 0 \, dt = \frac{A}{\pi}$$

Unlike the input, the output has a non-zero (positive) DC component ($V_o = A/\pi$)!

Q: I see. A non-zero DC component eh? So refresh my memory, why is that important?
A: Recall that the **power distribution** system we use is an **AC** system. The source voltage $v_s(t)$ that we get when we plug our “**power cord**” into the wall socket is a 60 Hz **sinewave**—a source with a **zero DC component**!

The **problem** with this is that most **electronic devices** and systems, such as TVs, stereos, computers, etc., require a **DC voltage(s)** to operate!

Q: But, how can we create a **DC supply voltage** if our power source $v_s(t)$ has no **DC component**??

A: That’s why the half-wave rectifier is so **important**! It takes an **AC source** with no **DC component** and creates a signal with both a **DC** and **AC component**.

We can then pass the output of a half-wave rectifier through a **low-pass filter**, which **suppresses** the **AC** component but lets the **DC** value ($V_O = A/\pi$) pass through. We then **regulate** this output and form a **useful DC voltage source**—one suitable for powering our electronic systems!
A: An *ideal* half-wave rectifier can be “built” if we use an *ideal* diode.

If we follow the transfer function analysis steps we studied earlier, then we will find that this circuit is indeed an *ideal* half-wave rectifier!

\[
v_0 = \begin{cases} 
0 & \text{for } v_s < 0 \\ 
v_s & \text{for } v_s > 0 
\end{cases}
\]
Of course, since ideal diodes do not exist, we must use a junction diode instead:

Q: This circuit looks so familiar! Haven't we studied it before?

A: Yes! It was an example where we determined the junction diode circuit transfer function. Recall that the result was:

\[
 v_o = \begin{cases} 
 v_s - 0.7 & \text{for } v_s > 0.7 \\
 0 & \text{for } v_s < 0.7 
\end{cases}
\]

Note that this result is slightly different from that of the ideal half-wave rectifier! The 0.7 V drop across the junction diode causes a horizontal “shift” of the transfer function from the ideal case.

Q: So, this junction diode circuit is worthless?
A: Hardly! Although the transfer function is not quite ideal, it works well enough to achieve the goal of signal rectification—it takes an input with no DC component and creates an output with a significant DC component!

Note what the transfer function “rule” is now:

1. When the input is greater than 0.7 V, the output voltage is equal to the input voltage minus 0.7 V.

2. When the input is less than 0.7 V, the output voltage is zero.

So, let’s consider again the case where the source voltage is sinusoidal (just like the source from a “wall socket”!):

The output of our junction diode half-wave rectifier would therefore be:
Although the output is shifted downward by 0.7 V (note in the plot above this is exaggerated, typically $A \gg 0.7V$), it should be apparent that the output signal $v_o(t)$, unlike the input signal $v_s(t)$, has a non-zero (positive) DC component.

Because of the 0.7 V shift, this DC component is slightly smaller than the ideal case. In fact, we find that if $A \gg 0.7$, this DC component is approximately:

$$V_o \approx \frac{A}{\pi} - 0.35 \text{ V}$$

In other words, just 350 mV less than ideal!

**Q:** Way back on the first page you said that there were two types of rectifiers. I now understand half-wave rectification, but what about these so-called full-wave rectifiers?
Almost forgot! Let's examine the transfer function of an ideal full-wave rectifier:

If the ideal half-wave rectifier makes negative inputs zero, the ideal full-wave rectifier makes negative inputs—positive! For example, if we again consider our sinusoidal input, we find that the output will be:

\[ v_o = \begin{cases} 
- v_s & \text{for } v_s < 0 \\
 v_s & \text{for } v_s > 0 
\end{cases} \]
The result is that the output signal will have a DC component twice that of the ideal half-wave rectifier!

\[ V_O = \frac{1}{T} \int_{0}^{T} v_o(t) \, dt \]

\[ = \frac{1}{T} \int_{0}^{T/2} A \sin \omega t \, dt - \frac{1}{T} \int_{T/2}^{T} A \sin \omega t \, dt = \frac{2A}{\pi} \]

\[ V_O = \frac{2A}{\pi} \]

Q: Wow! Full-wave rectification appears to be twice as good as half-wave. Can we build an ideal full-wave rectifier with junction diodes?

A: Although we cannot build an ideal full-wave rectifier with junction diodes, we can build full-wave rectifiers that are very close to ideal with junction diodes!
Consider the following junction diode circuit:

![Diagram of the Full-Wave Rectifier circuit](image)

Note that we are using a transformer in this circuit. The job of this transformer is to step-down the large voltage on our power line (120 V rms) to some smaller magnitude (typically 20-70 V rms).

Note the secondary winding has a center tap that is grounded. Thus, the secondary voltage is distributed symmetrically on either side of this center tap.

For example, if $v_S = 10$ V, the anode of $D_1$ will be 10V above ground potential, while the anode of $D_2$ will be 10V below ground potential (i.e., -10V):
Conversely, if $\nu_S = -10 \text{ V}$, the anode of $D_1$ will be 10V below ground potential (i.e., -10V), while the anode of $D_2$ will be 10V above ground potential:
The more important question is, what is the value of output \( v_O \)? More specifically, how is \( v_O \) related to the value of source \( v_S \)—what is the transfer function \( v_O = f(v_S) \)?

To help simplify our analysis, we are going to redraw this circuit in another way. First, we will split the secondary winding into two explicit pieces:

We will now ignore the primary winding of the transformer and redraw the remaining circuit as:
Note that the secondary voltages at either end of this circuit are the same, but have opposite polarity. As a result, if $v_S=10$, then the anode of diode $D_1$ will be 10 V above ground, and the anode at diode $D_2$ will be 10V below ground—just like before!

Now, let's attempt to determine the transfer function $v_O = f(v_S)$ of this circuit.

First, we will replace the junction diodes with CVD models.

Then let's ASSUME $D_1$ is forward biased and $D_2$ is reverse biased, thus ENFORCE $v'_{D_1} = 0$ and $i'_{D_2} = 0$. Thus ANALYZE:
Note that we need to determine 3 things: the ideal diode current \( i_{D1}' \), the ideal diode voltage \( v_{D2}' \), and the output voltage \( v_O \). However, instead of finding numerical values for these 3 quantities, we must express them in terms of source voltage \( v_S \)!

From KCL:  
\[ i = i_{D1}' + i_{D2}' = i_{D1}' + 0 = i_{D1}' \]

From KVL:  
\[ v_S - v_{D1}' - 0.7 - R i_D' = 0 \]

Thus the ideal diode current is:
\[ i_{D1}' = \frac{v_S - 0.7}{R} \]

Likewise, from KVL:  
\[ v_S - v_{D1}' - 0.7 + 0.7 + v_{D2}' + v_S = 0 \]

Thus, the ideal diode voltage is:
\[ v_{D2}' = -2v_S \]

And finally, from KVL:  
\[ v_S - v_{D1}' - 0.7 = v_O \]

Thus, the output voltage is:
\[ v_O = v_S - 0.7 \]
Now, we must determine when both \( i_{D1}^i > 0 \) and \( v_{D2}^i < 0 \). When both these conditions are true, the output voltage will be \( v_o = v_s - 0.7 \). When one or both conditions \( i_{D1}^i > 0 \) and \( v_{D2}^i < 0 \) are false, then our assumptions are invalid, and \( v_o \neq v_s - 0.7 \).

Using the results we just determined, we know that \( i_{D1}^i > 0 \) when:

\[
\frac{v_s - 0.7}{R} > 0
\]

Solving for \( v_s \):

\[
\frac{v_s - 0.7}{R} > 0
\]

\[
v_s - 0.7 > 0
\]

\[
v_s > 0.7 \text{ V}
\]

Likewise, we find that \( v_{D2}^i < 0 \) when:

\[-2v_s < 0\]

Solving for \( v_s \):

\[-2v_s < 0\]

\[2v_s > 0\]

\[v_s > 0\]

Thus, our assumptions are correct when \( v_s > 0.0 \) AND \( v_s > 0.7 \). This is the same thing as saying our assumptions are valid when \( v_s > 0.7 \)!
Thus, we have found that the following statement is true about this circuit:

\[ V_o = V_s - 0.7 \text{ V when } V_s > 0.7 \text{ V} \]

Note that this statement does not constitute a function (what about \( V_s < 0.7 \)?), so we must continue with our analysis!

Say we now assume that \( D_1 \) is reverse biased and \( D_2 \) is forward biased, so we enforce \( i_{D1} = 0 \) and \( V_{D2} = 0 \). Thus, we analyze this circuit:

Using the same procedure as before, we find that
\[ V_o = -V_s - 0.7 \text{, and both our assumptions are true when } V_s < -0.7 \text{ V. In other words:} \]

\[ V_o = -V_s - 0.7 \text{ V when } V_s < -0.7 \text{ V} \]

Note we are still not done! We still do not have a complete transfer function (what happens when \(-0.7 \text{ V} < V_s < 0.7 \text{ V}\)?)
Finally then, we ASSUME that both ideal diodes are reverse biased, so we ENFORCE $i_{d1}' = 0$ and $i_{d2}' = 0$. Thus ANALYZE:

Following the same procedures as before, we find that $v_s = 0$, and both assumptions are true when $-0.7 < v_s < 0.7$. In other words:

$$v_s = 0 \quad \text{when} \quad -0.7 < v_s < 0.7$$

Now we have a function! The transfer function of this circuit is:

$$v_o = \begin{cases} 
  v_s - 0.7 \text{V} & \text{for} \quad v_s > 0.7 \text{V} \\
  0 \text{V} & \text{for} \quad -0.7 > v_s > 0.7 \text{V} \\
  -v_s - 0.7 \text{V} & \text{for} \quad v_s < -0.7 \text{V}
\end{cases}$$

Plotting this function:
The output of this full-wave rectifier with a sine wave input is therefore:

Note how this **compares** to the transfer function of the **ideal** full-wave rectifier:

\[ v_O = \begin{cases} 
-v_s & \text{for } v_s < 0 \\
v_s & \text{for } v_s > 0 
\end{cases} \]

Very similar!
Likewise, compare the output of this junction diode full-wave rectifier to the output of an ideal full-wave rectifier:

Again we see that the junction diode full-wave rectifier output is very close to ideal. In fact, if $A \gg 0.7$ V, the DC component of this junction diode full wave rectifier is approximately:

$$V_o \approx \frac{2A}{\pi} - 0.7 \text{ V}$$

Just 700 mV less than the ideal full-wave rectifier DC component!
The Bridge Rectifier

Now consider this junction diode rectifier circuit:

We call this circuit the bridge rectifier. Let's analyze it and see what it does!

First, we replace the junction diodes with the CVD model:
A: True! However, there are only three of these sets of assumptions are actually possible!

Consider the current $i$ flowing through the rectifier. This current of course can be positive, negative, or zero. It turns out that there is only one set of diode assumptions that would result in positive current $i$, one set of diode assumptions that would lead to negative current $i$, and one set that would lead to zero current $i$.

Q: But what about the remaining 13 sets of dog gone diode assumptions?

A: Regardless of the value of source $v_s$, the remaining 13 sets of diode assumptions simply cannot occur for this particular circuit design!
Let's look at the three possible sets of assumptions:

\( i > 0 \)

The rectifier current \( i \) can be **positive** only if these assumptions are true:

\( D_1 \) and \( D_3 \) are reverse biased.

\( D_2 \) and \( D_4 \) are forward biased.

Analyzing this circuit, we find that the **output voltage** is:

\[
\nu_O = \nu_s - 1.4 \text{ V}
\]

and the f.b. **ideal diode currents** are:

\[
i = i_D^i = \frac{\nu_s - 1.4}{R}
\]
and, finally the r.b. ideal diode voltages are:

\[ v_D^i = -v_S \]

Thus, \( i_D^i > 0 \) when:

\[ \frac{v_S - 1.4}{R} > 0 \]
\[ v_S - 1.4 > 0 \]
\[ v_S > 1.4 \text{ V} \]

and \( v_D^i < 0 \) when:

\[ -v_S < 0 \]
\[ v_S > 0 \]

Therefore, we find that for this circuit:

\[ v_O = v_S - 1.4 \text{ V} \text{ when } v_S > 1.4 \text{ V} \]

\( i < 0 \)

The rectifier current \( i \) can be negative only if these assumptions are true:

\( D_1 \) and \( D_3 \) are forward biased.

\( D_2 \) and \( D_4 \) are reverse biased.
Analyzing this circuit, we find that the output voltage is:

\[ v_O = -v_s - 1.4 \text{ V} \]

while the f.b. ideal diode currents are both:

\[ -i = i_D^i = \frac{-v_s - 1.4}{R} \]

and the r.b. ideal diode voltages are both:

\[ v_D^i = v_s \]
Thus, \( i_d^i > 0 \) when:

\[
-\frac{v_s - 1.4}{R} > 0
\]

\[-v_s - 1.4 > 0\]

\[-v_s > 1.4 \text{ V}\]

\[v_s < -1.4 \text{ V}\]

and, \( i_d^i < 0 \) when:

\[v_s < 0\]

Therefore, we likewise find for this circuit:

\[v_o = -v_s - 1.4 \text{ V} \text{ when } v_s < -1.4 \text{ V}\]

\[i = 0\]

The rectifier current \( i \) can be zero only if these assumptions are true:

All ideal diodes are reverse biased!
Analyzing this circuit, we find that the output voltage is:

\[ v_o = R \cdot i = 0 \]

while the ideal diode voltages of \( D_2 \) and \( D_4 \) are each:

\[ v_{D2}^i = \frac{v_s - 1.4}{2} = v_{D4}^i \]

and the ideal diode voltages of \( D_1 \) and \( D_3 \) are each:

\[ v_{D1}^i = \frac{-v_s - 1.4}{2} = v_{D3}^i \]

Thus, \( v_{D2}^i < 0 \) when:

\[ \frac{v_s - 1.4}{2} < 0 \]
\[ v_s - 1.4 < 0 \]
\[ v_s < 1.4 \]

and, \( v_{D1}^i < 0 \) when:

\[ \frac{-v_s - 1.4}{2} < 0 \]
\[ -v_s - 1.4 < 0 \]
\[ -v_s < 1.4 \]
\[ v_s > -1.4 \]

Therefore, we also find for this circuit that:

\[ v_o = 0 \quad \text{when both} \quad v_s < 1.4 \text{V and} \quad v_s > -1.4 \text{V} \quad (-1.4 < v_s < 1.4 \text{V}) \]
Q: You know, that dang Mizzou grad said we only needed to consider these **three** sets of diode assumptions, yet I am still concerned about the other 13. How can we be sure that we have analyzed every **possible** set of valid diode assumptions?

A: We know that we have considered **every** possible case, because when we combine the three results we find that we have a piece-wise linear function! I.E.,

\[
v_o = \begin{cases} 
-v_s - 1.4 \text{ V} & \text{if } v_s < -1.4 \text{ V} \\
0 & \text{if } -1.4 < v_s < 1.4 \text{ V} \\
v_s - 1.4 \text{ V} & \text{if } v_s > 1.4 \text{ V}
\end{cases}
\]
Note that the bridge rectifier is a full-wave rectifier!

If the input to this rectifier is a sine wave, we find that the output is approximately that of an ideal full-wave rectifier:

We see that the junction diode bridge rectifier output is very close to ideal. In fact, if \( A \gg 1.4 \text{ V} \), the DC component of this junction diode bridge rectifier is approximately:

\[
V_O \approx \frac{2A}{\pi} - 1.4 \text{ V}
\]

Just 1.4 V less than the ideal full-wave rectifier DC component!
Peak Inverse Voltage

Q: I'm so confused! The bridge rectifier and the full-wave rectifier both provide full-wave rectification. Yet, the bridge rectifier use 4 junction diodes, whereas the full-wave rectifier only uses 2. Why would we ever want to use the bridge rectifier?

A: First, a slight confession—the results we derived for the bridge and full-wave rectifiers are not precisely correct!

Recall that we used the junction diode CVD model to determine the transfer function of each rectifier circuit. The problem is that the CVD model does not predict junction diode breakdown!

If the source voltage $v_s$ becomes too large, the junction diodes can in fact breakdown—but the transfer functions we derived do not reflect this fact!

Q: Yikes! You mean that we need to rework our analysis and find new transfer functions!
A: Fortunately no. Breakdown is an undesirable mode for circuit rectification. Our job as engineers is to design a rectifier that avoids it—that why the bridge rectifier is helpful!

To see why, consider the voltage across a reversed biased junction diode in each of our rectifier circuit designs.

Recall that the voltage across a reverse biased ideal diode in the full-wave rectifier design was:

\[ v_{D2}^i = -2v_s \]

so that the voltage across the junction diode is approximately:

\[ v_D = v_{D2}^i + 0.7 \\
= -2v_s + 0.7 \]

Now, assuming that the source voltage is a sine wave \( v_s = A \sin \omega t \), we find that diode voltage is at its most negative (i.e., breakdown danger!) when the source voltage is at its maximum value \( A \). I.E.,:

\[ v_D^{\text{min}} = -2A + 0.7 \]

Of course, the largest junction diode voltage occurs when in forward bias:

\[ v_D^{\text{max}} = 0.7 \text{ V} \]
Note that this minimum diode voltage $v_D$ is very negative, with an absolute value ($|v_D^{\text{min}}| = 2A - 0.7$) nearly twice as large as the source magnitude $A$.

We call the absolute value of the minimum diode voltage the **Peak Inverse Voltage (PIV)**:

$$PIV = |v_D^{\text{min}}|$$

Note that this value is dependent on both the rectifier design and the magnitude of the source voltage $v_S$.

**Q:** So, why do we need to determine PIV? I’m not sure I see what difference this value makes.
A: The Peak Inverse Voltage answers one important question—will the junction diodes in our rectifier breakdown?

→ If the PIV is less than the Zener breakdown voltage of our rectifier diodes (i.e., if $PIV < V_{zk}$), then we know that our junction diodes will remain in either forward or reverse bias for all time $t$. The rectifier will operate “properly”!

→ However, if the PIV is greater than the Zener breakdown voltage of our rectifier diodes (i.e., if $PIV > V_{zk}$), then we know that our junction diodes will breakdown for at least some small amount of time $t$. The rectifier will NOT operate properly!

Q: So what do we do if PIV is greater than $V_{zk}$? How do we fix this problem?

A: We have two possible solutions:

1. Use junction diodes with larger values of $V_{zk}$ (if they exist!).
2. Use the bridge rectifier design.

Q: The bridge rectifier! How would that solve our breakdown problem?
A: To see how a bridge rectifier can be useful, let's determine its Peak Inverse Voltage PIV.

First, we recall that the voltage across the reverse biased ideal diodes was:

\[ V_D^i = -V_S \]

so that the voltage across the junction diode is approximately:

\[ V_D = V_D^i + 0.7 = -V_S + 0.7 \]

Now, assuming that the source voltage is a sine wave \( V_S = A \sin \omega t \), we find that diode voltage is at its most negative (i.e., breakdown danger!) when the source voltage is at its maximum value \( A \). I.E.,:

\[ V_D^{\text{min}} = -A + 0.7 \]

Of course, the largest junction diode voltage occurs when in forward bias:

\[ V_D^{\text{max}} = 0.7 \text{ V} \]
Note that this minimum diode voltage is very negative, with an absolute value \( |v_{D}^{\text{min}}| = A - 0.7 \), approximately equal to the value of the source magnitude \( A \).

Thus, the PIV for a bridge rectifier with a sinusoidal source voltage is:

\[
\text{PIV} = A - 0.7
\]

Note that this bridge rectifier value is approximately half the PIV we determined for the full-wave rectifier design!

Thus, the source voltage (and the output DC component) of a bridge rectifier can be twice that of the full-wave rectifier design—this is why the bridge rectifier is a very useful rectifier design!