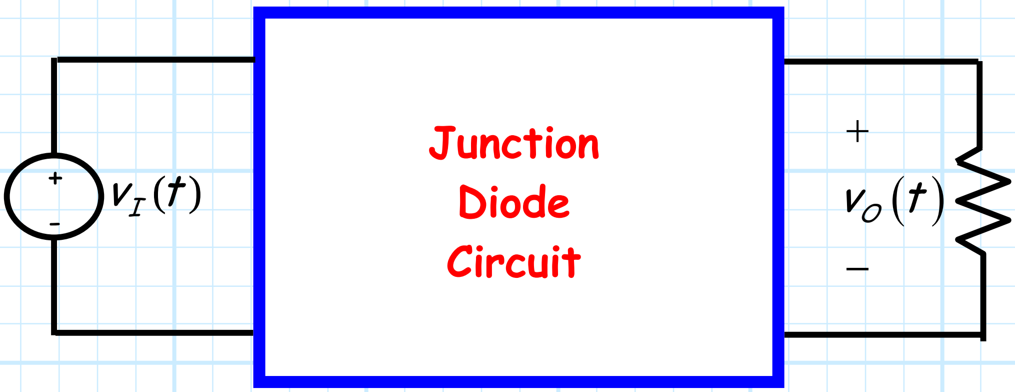
3.5 Rectifier Circuits

A. Junction Diode 2-Port Networks



HO: The Transfer Function of Diode Circuits

Q:

A: HO: Steps for finding a Junction Diode Circuit Transfer Function

Example: Diode Circuit Transfer Function

B. Diode Rectifiers

HO: Signal Rectification

Q:

A:

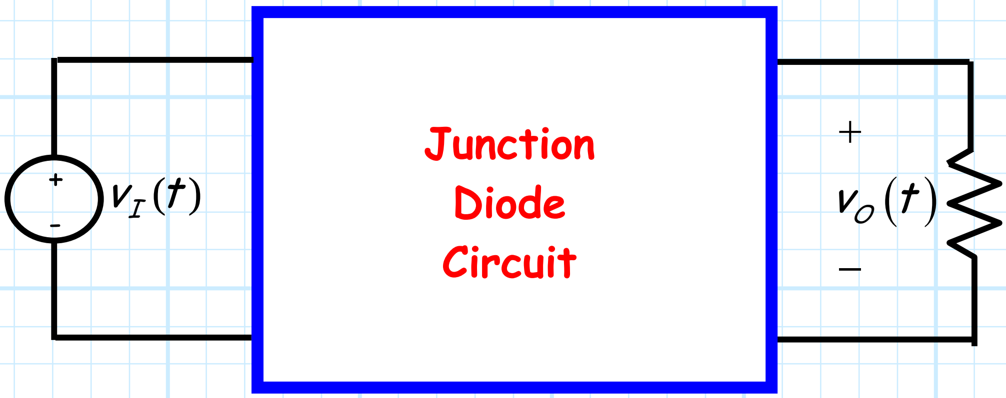
HO: The Full-Wave Rectifier

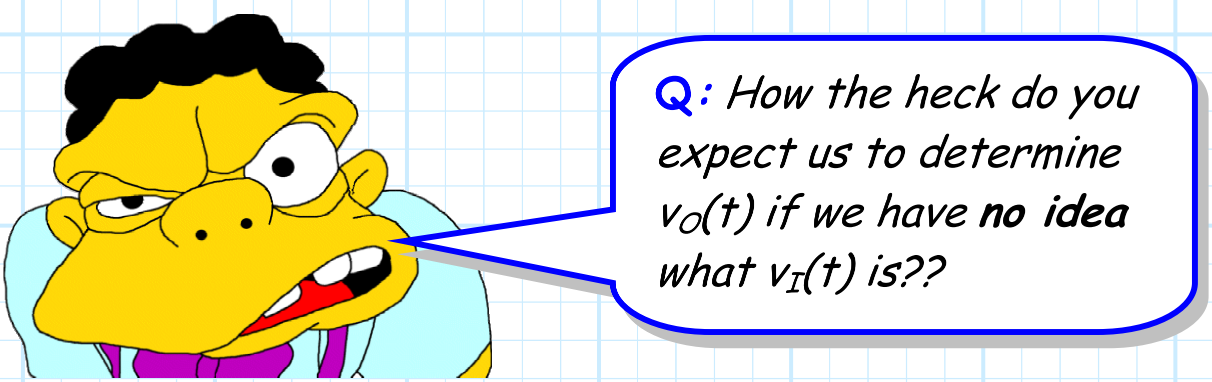
HO: The Bridge Rectifier

HO: Peak Inverse Voltage

The Transfer Function of Diode Circuits

For many junction diode circuits, we find that one of the voltage sources is in fact **unknown**! This unknown voltage is typically some **input** signal of the form $v_I(t)$, which results in an output voltage $v_O(t)$.





A: We of course cannot determine an **explicit** value or expression for $v_O(t)$, since it **depends** on the input $v_I(t)$. Instead, we will attempt to explicitly determine this **dependence** of $v_O(t)$ on $v_I(t)$!

In other words, we seek to find an expression for v_O in **terms** of v_I. Mathematically speaking, our goal is to determine the **function**:

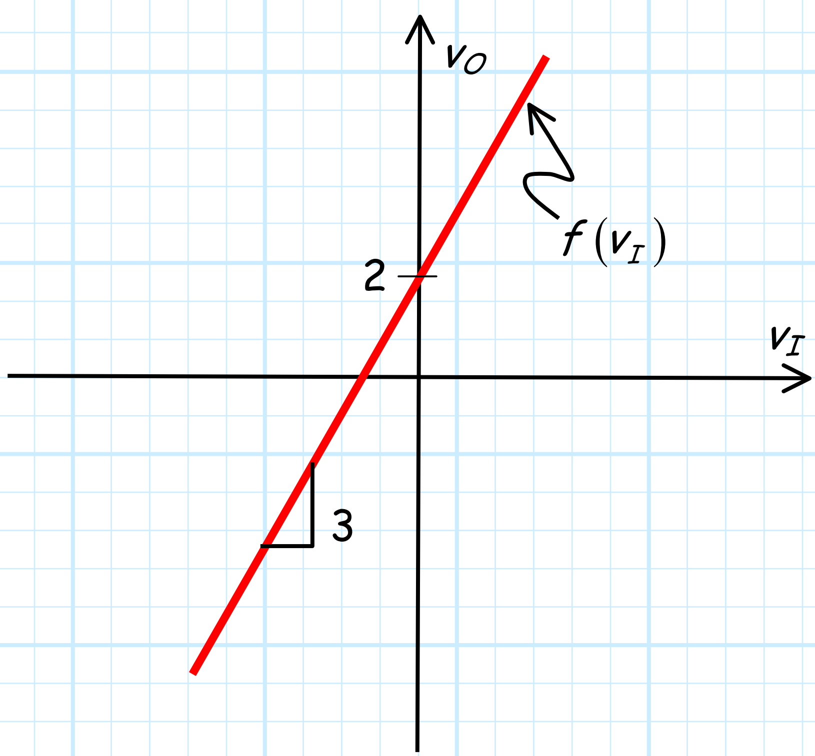
$$v_O = f(v_I)$$

We refer to this as the circuit **transfer function**.

Note that we can **plot** a circuit transfer function on a 2-dimensional plane, just as if the function related values x and y (e.g. $y = f(x)$). For **example**, say our circuit transfer function is:

$$\begin{aligned} v_O &= f(v_I) \\ &= 3v_I + 2 \end{aligned}$$

Note this is simply the **equation of a line** (e.g., $y = 3x + 2$), with slope m=3 and intercept b=2.



Q: *A **"function"** eh? Isn't a **"function"** just your annoyingly pretentious way of saying we need to find some mathematic **equation** relating v_O and v_I?*

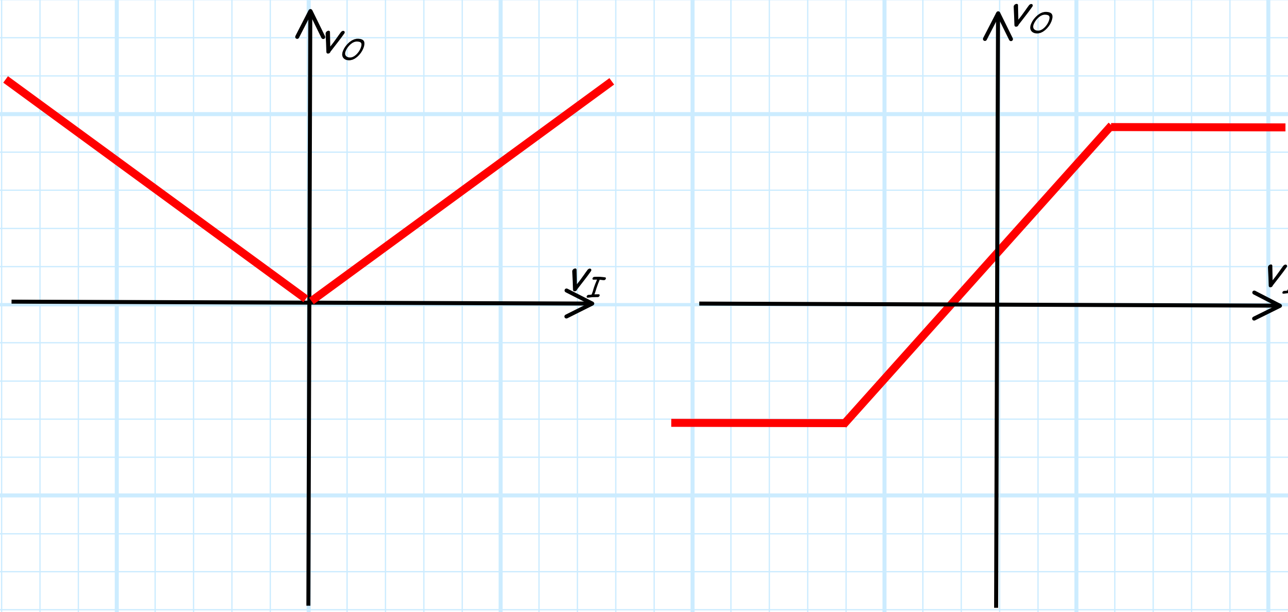
A: Actually **no**! Although a function **is** a mathematical equation, there are in fact **scads** of equations relating v_O and v_I that are **not** functions!

→ The set of all possible functions $y = f(x)$ are a **subset** of the set of all possible equations relating y and x.

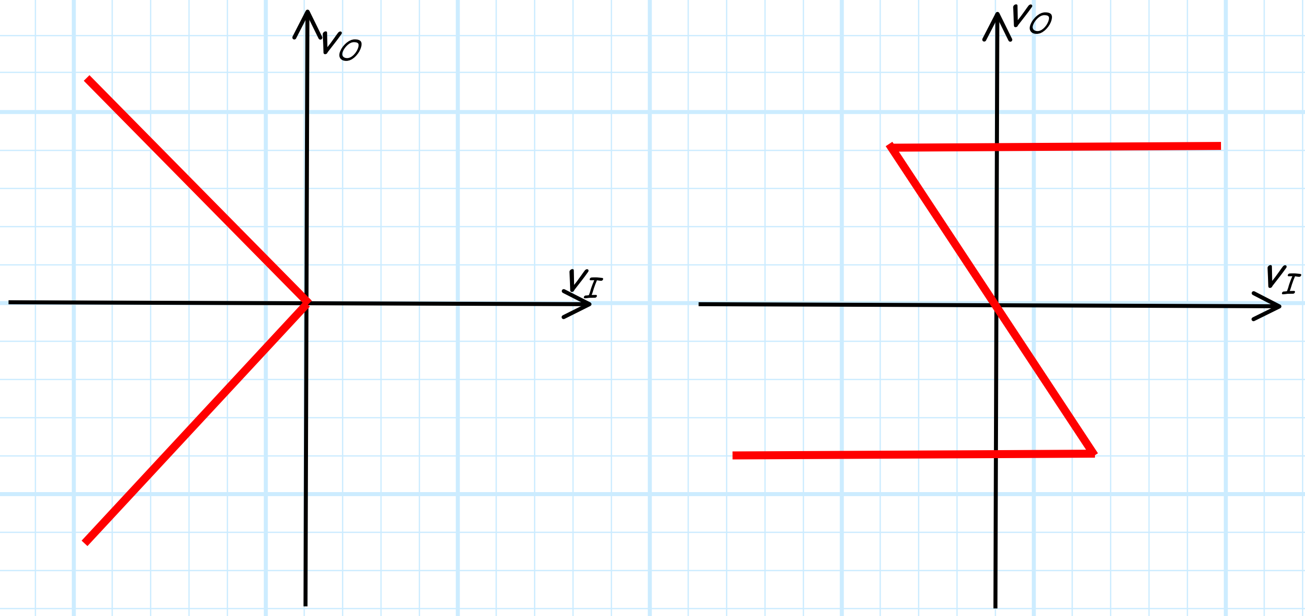
> A **function** $v_O = f(v_I)$ is a mathematical expression such that for **any** value of v_I (i.e., $-\infty < v_I < \infty$), there is **one**, but **only** one, value v_O.

Note this definition of a function is consistent with our **physical** understanding of circuits—we can place **any** voltage on the input that we want (i.e., $-\infty < v_I < \infty$), and the result will be **one** specific voltage value v_O on the output.

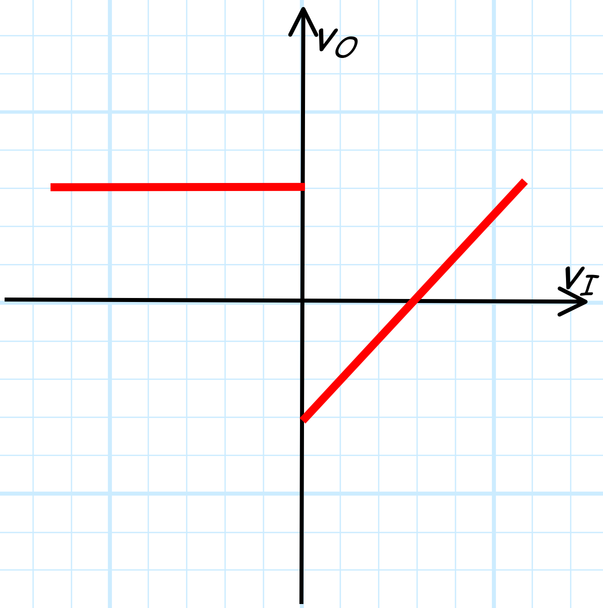
Therefore, examples of **valid** circuit transfer **functions** include:



Conversely, the transfer "functions" **below** are **invalid**—they **cannot** represent the behavior of circuits, since they **are not** functions!

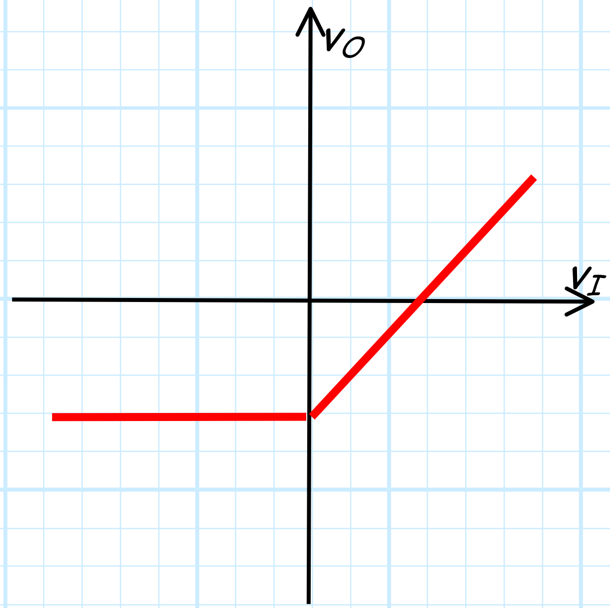


Moreover, we find that **circuit** transfer functions must be **continuous**. That is, v_O **cannot** "instantaneously change" from one value to another as we increase (or decrease) the value v_I.



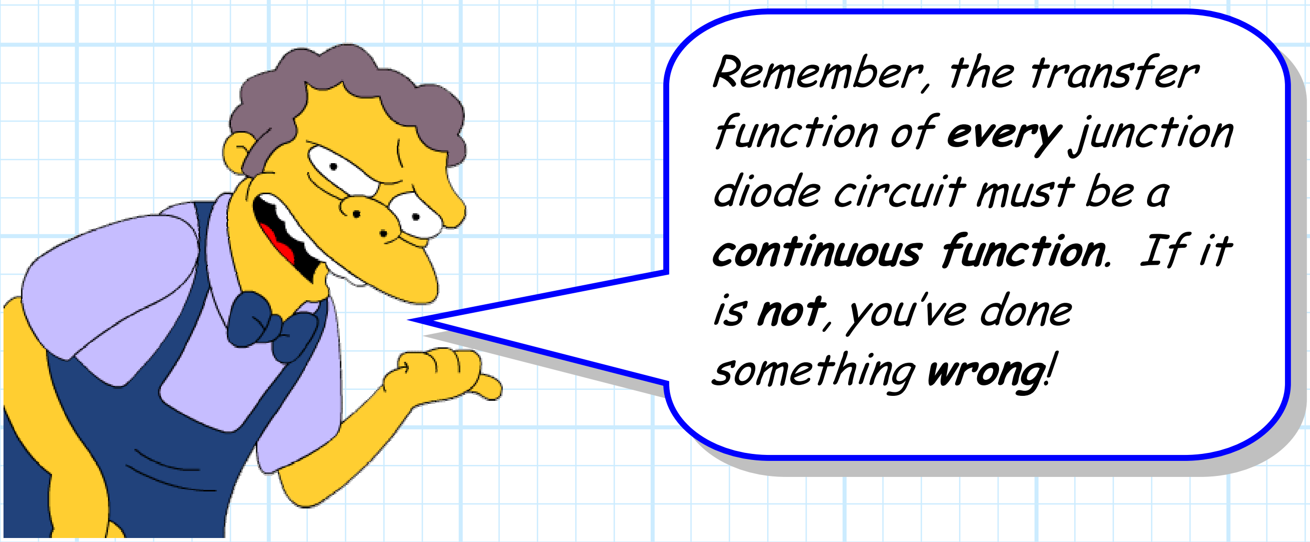
A Discontinuous Function

(*Invalid circuit transfer function*)



A Continuous Function

(*Valid circuit transfer function*)



Steps for Finding a Junction Diode Circuit Transfer Function

Determining the **transfer function** of a junction diode circuit is in many ways **very similar** to the analysis steps we followed when analyzing previous junction diode circuits (i.e., circuits where all sources were **explicitly known**).

However, there are also some **important differences** that we must understand completely if we wish to successfully determine the **correct transfer function**!

Step1: *Replace all junction diodes with an appropriate junction diode **model**.*

Just like before! We will now have an **IDEAL** diode circuit.

Step 2: *ASSUME some mode for all ideal diodes.*

Just like before! An IDEAL diode can be either forward or reverse biased.

Step 3: *ENFORCE the bias assumption.*

Just like before! ENFORCE the bias assumption by replacing the ideal diode with short circuit or open circuit.

Step 4: *ANALYZE the remaining circuit.*

Sort of, **kind** of, like before!

1. If we assumed an IDEAL diode was forward biased, we must determine i_D^i--**just** like before! However, **instead** of finding the numeric value of i_D^i, we determine i_D^i as a **function** of the unknown source (e.g., $i_D^i = f(v_I)$).

2. Or, if we assumed an IDEAL diode was reversed biased, we must determine v_D^i--**just** like before! However, **instead** of finding the numeric value of v_D^i, we determine v_D^i as a **function** of the unknown source (e.g., $v_D^i = f(v_I)$).

3. Finally, we must determine all the **other** voltages and/or currents we are interested in (e.g., v_O)--**just** like before! However, **instead** of finding its numeric value, we determine it as a **function** of the unknown source (e.g., $v_O = f(v_I)$).

Step 5: *Determine WHEN the assumption is valid.*

Q: *OK, we get the picture. Now we have to **CHECK** to see if our IDEAL diode assumption was correct, right?*

A: Actually, **no**! This step is **very different** from what we did before!

We **cannot** determine **IF** $i_D^i > 0$ (forward bias assumption), or **IF** $v_D^i < 0$ (reverse bias assumption), since we **cannot** say for certain what the value of i_D^i or v_D^i is!

Recall that i_D^i and v_D^i are **functions** of the unknown voltage source (e.g.,$i_D^i = f(v_I)$ and $v_D^i = f(v_I)$). Thus, the values of i_D^i or v_D^i are **dependent** on the unknown source (v_I, say). For **some** values of v_I, we will find that $i_D^i > 0$ or $v_D^i < 0$, and so our assumption (and thus our solution for $v_O = f(v_I)$) will be! **correct**

However, for **other** values of v_I, we will find that $i_D^i < 0$ or $v_D^i > 0$, and so our assumption (and thus our solution for $v_O = f(v_I)$) will be **incorrect**!

Q: *Yikes! What do we do? How can we determine the circuit transfer function if we can't determine **IF** our ideal diode assumption is correct??*

A: Instead of determining **IF** our assumption is correct, we must determine **WHEN** our assumption is correct!

In other words, we must determine for **what values** of v_I is $i_D^i > 0$ (forward bias), or for **what values** of v_I is $v_D^i < 0$ (reverse bias).

We can do this since we earlier (in step 4) determined the function $i_D^i = f(v_I)$ or the function $v_D^i = f(v_I)$.

Perhaps this step is best explained by an **example**. Let's say we assumed that our ideal diode was **forward biased** and, say we determined (in step 4) that v_O is related to v_I as:

$$\begin{aligned} v_O &= f(v_I) \\ &= 2v_I - 3 \end{aligned}$$

Likewise, say that we determined (in step 4) that our ideal diode current is related to v_I as:

$$\begin{aligned} i_D^i &= f(v_I) \\ &> \frac{v_I - 5}{4} \end{aligned}$$

Thus, in order for our forward bias assumption to be **correct**, the function $i_D^i = f(v_I)$ must be **greater than zero**:

$$i_D^i > 0$$

$$f(v_I) > 0$$

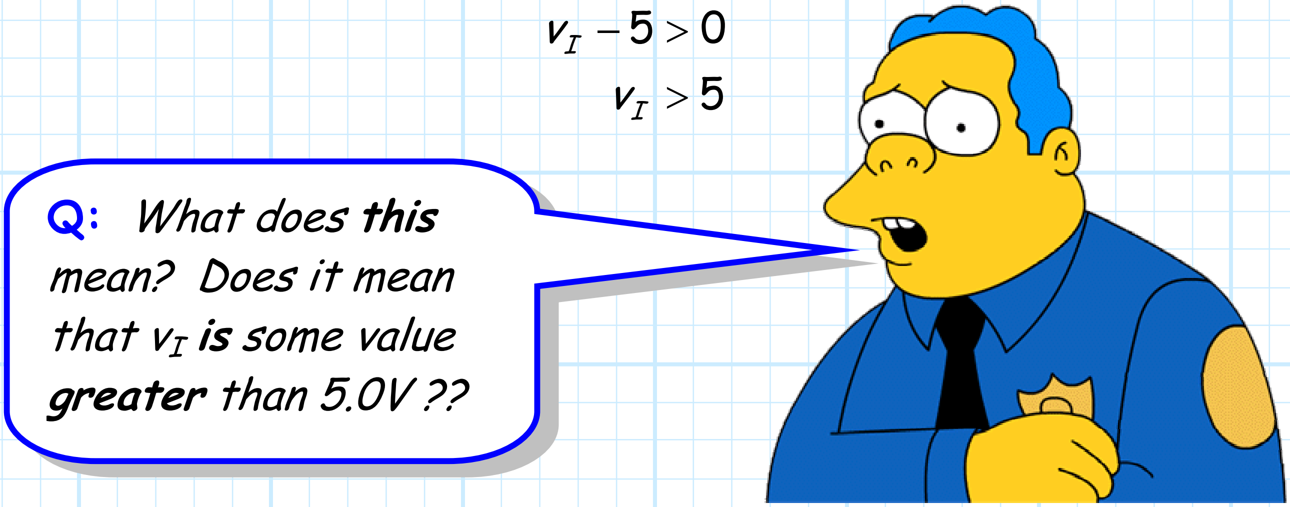
$$\frac{v_I - 5}{4} > 0$$

We can now "solve" this **inequality** for v_I:

$$\frac{v_I - 5}{4} > 0$$

$$v_I - 5 > 0$$

$$v_I > 5$$



A: **NO!** Recall that v_I can be **any** value. What the inequality above means is that $i_D^i > 0$ (i.e., the ideal diode is forward biased) **WHEN** $v_D^i > 5.0$.

Thus, we know $v_O = 2v_I - 3$ is valid **WHEN** the ideal diode is forward biased, and the ideal diode is forward biased **WHEN** (for this example) $v_D^i > 5.0$. As a result, we can mathematically state that:

$$v_O = 2v_I - 3 \quad \text{when} \quad v_I > 5.0 \text{ V}$$

Conversely, this means that **if** $v_I < 5.0$ V, the **ideal** diode will be **reverse biased**—our forward bias assumption would **not** be valid, and thus our expression $v_O = 2v_I - 3$ is **not** correct ($v_O \neq 2v_I - 3$ for $v_I < 5.0\text{V}$)!

Q: *So how **do** we determine v_O for values of $v_I < 5.0$ V ?*

A: Time to move to the **last** step !

Step 6: ***Change** assumption and **repeat** steps 2 through 5 !*

For our **example**, we would change our bias assumption and now ASSUME reverse bias. We then ENFORCE $i_D' = 0$, and then ANALYZE the circuit to find both $v_D' = f(v_I)$ and a **new** expression $v_O = f(v_I)$ (it will **no longer** be $v_O = 2v_I - 3$!).

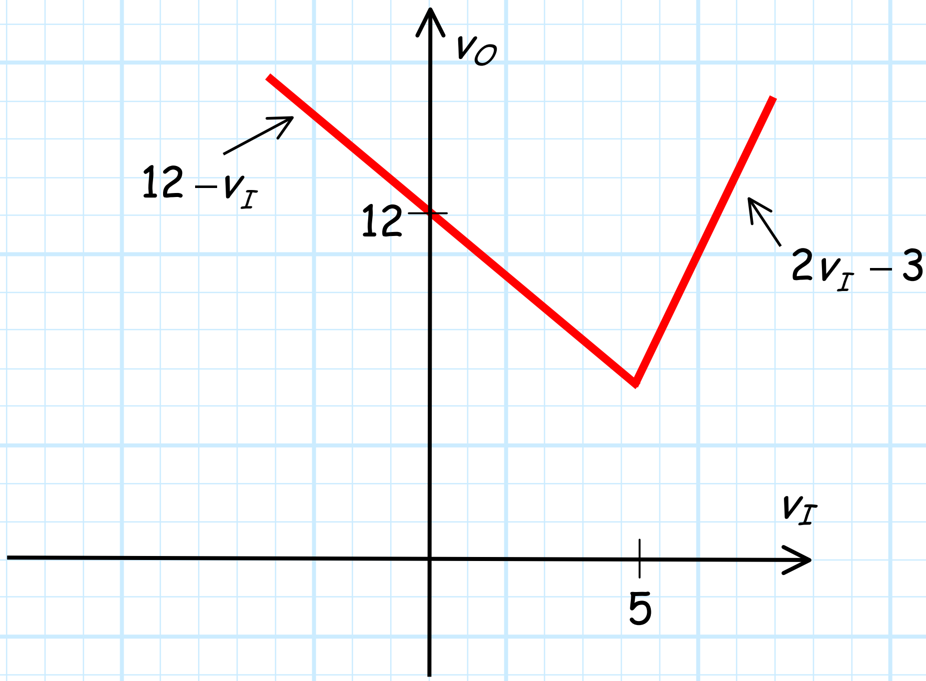
We then determine **WHEN** our reverse bias assumption is valid, by solving the **inequality** $v_D' = f(v_I) > 0$ for v_I. For the example used here, we would find that the **IDEAL** diode is reverse biased **WHEN** $v_I < 5.0$ V .

For junction diode circuits with **multiple** diodes, we may have to repeat this entire process **multiple** times, until **all possible** bias conditions are analyzed.

If we have done our analysis **properly**, the result will be a valid **continuous function**! That is, we will have an expression (but only **one** expression) relating v_O to **all** possible values of v_I.

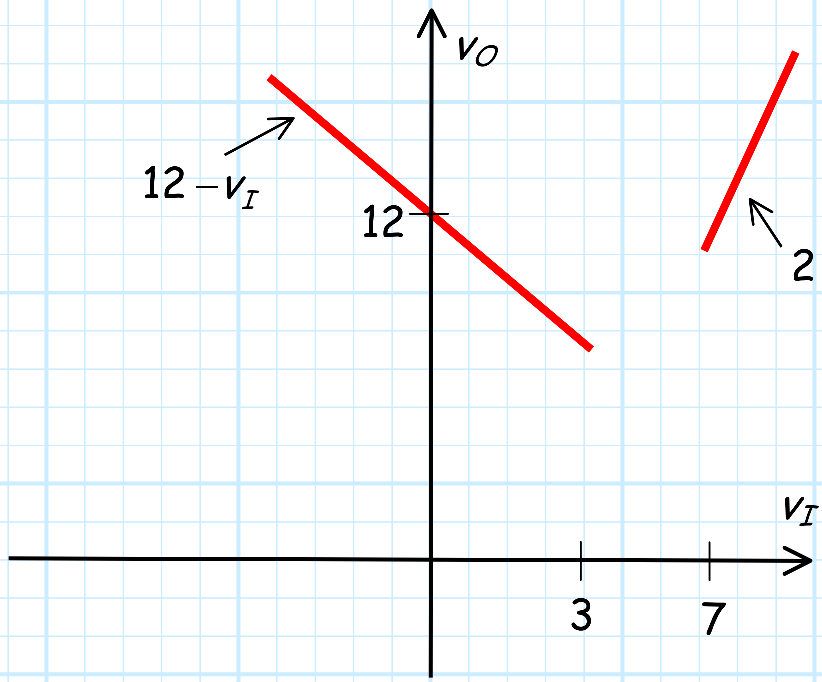
This transfer function will typically be **piecewise linear**. An **example** of a piece-wise linear transfer function is:

$$v_O = \begin{cases} 2v_I - 3 & for \quad v_I > 5.0 \\ 12 - v_I & for \quad v_I < 5.0 \end{cases}$$



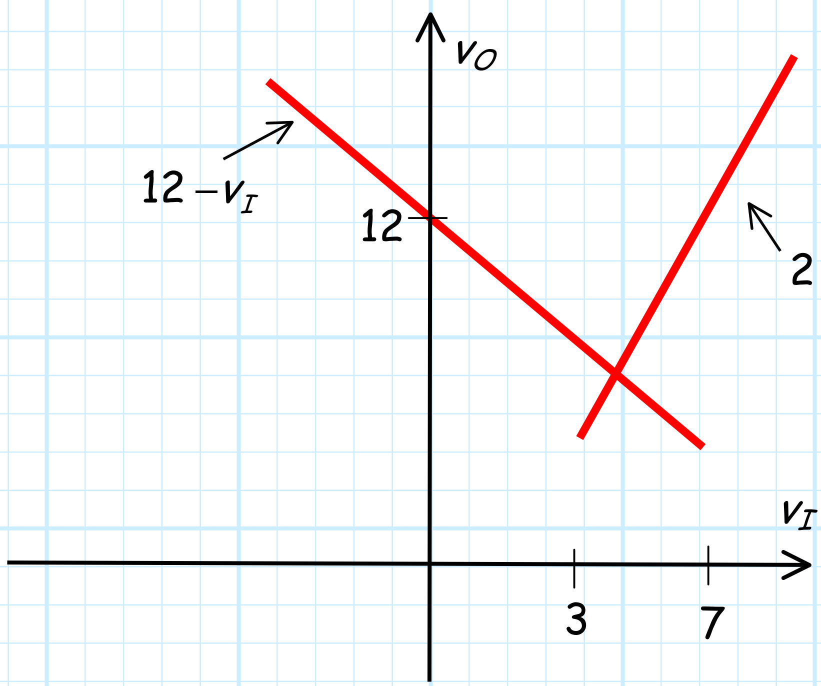
Just to make **sure** that we understand what a function is, note that the following expression is **not** a function:

$$v_O = \begin{cases} 2v_I - 3 & for \quad v_I > 7.0 \\ 12 - v_I & for \quad v_I < 3.0 \end{cases}$$



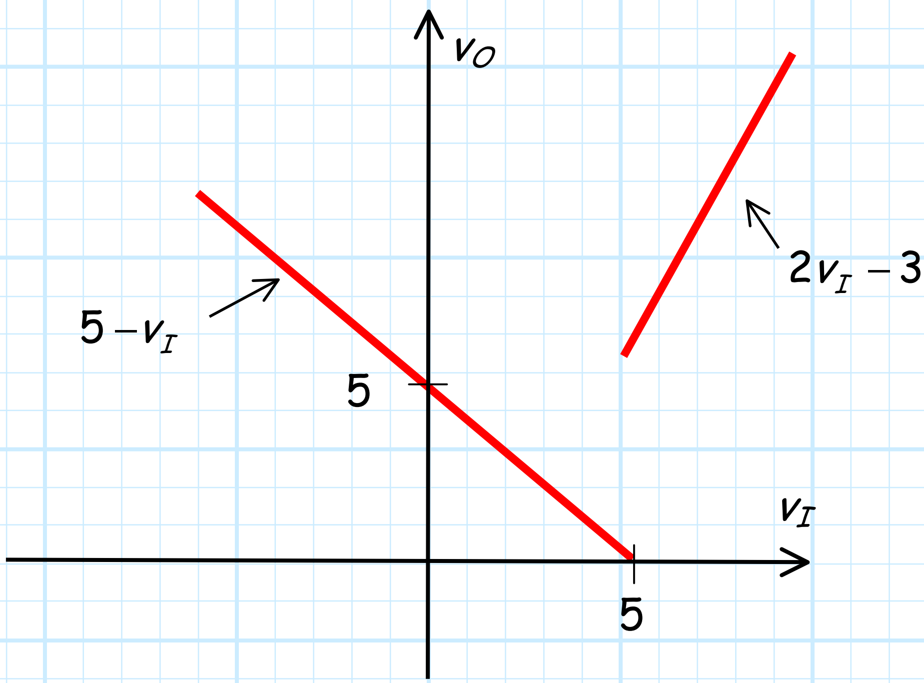
Nor is this expression a function:

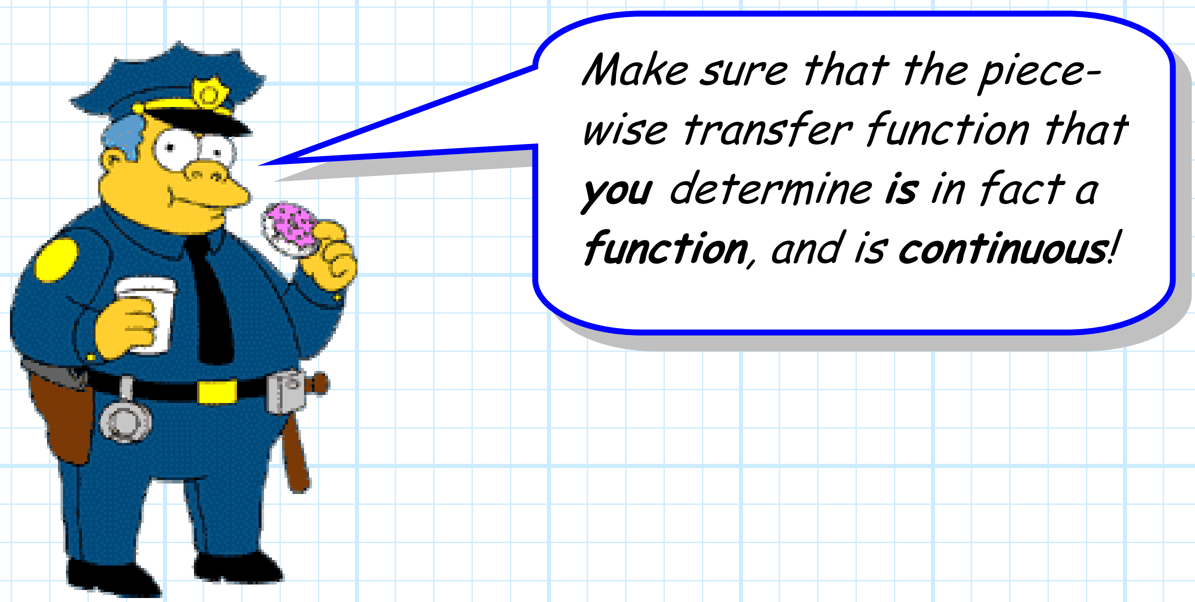
$$v_O = \begin{cases} 2v_I - 3 & for \quad v_I > 3.0 \\ 12 - v_I & for \quad v_I < 7.0 \end{cases}$$



Finally, note that the following expression **is** a function, but it is **not continuous**:

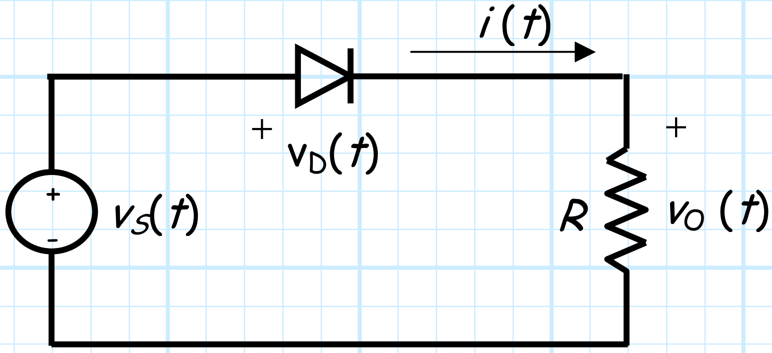
$$v_O = \begin{cases} 2v_I - 3 & \text{for} \quad v_I > 5.0 \\ \\ 5 - v_I & \text{for} \quad v_I < 5.0 \end{cases}$$





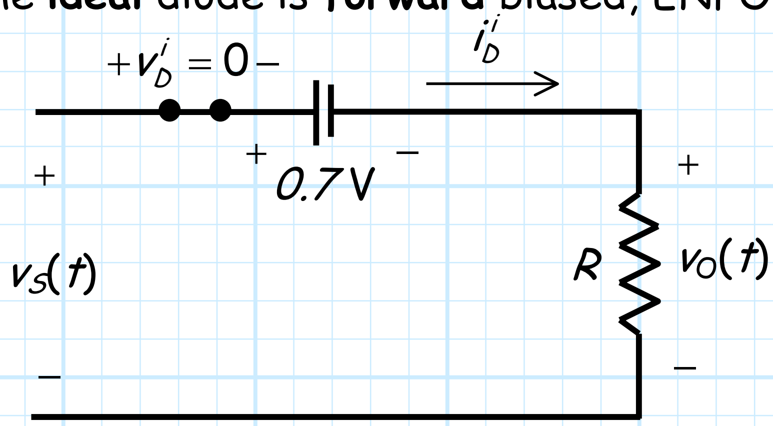
Example: Diode Circuit Transfer Function

Consider the following circuit, called a **half-wave rectifier**:



Let's use the **CVD model** to determine the output voltage v_O in terms of the input voltage v_S. In other words, let's determine the diode circuit **transfer function** $v_O = f(v_S)$!

ASSUME the **ideal** diode is **forward** biased, ENFORCE $v_D^i = 0$.



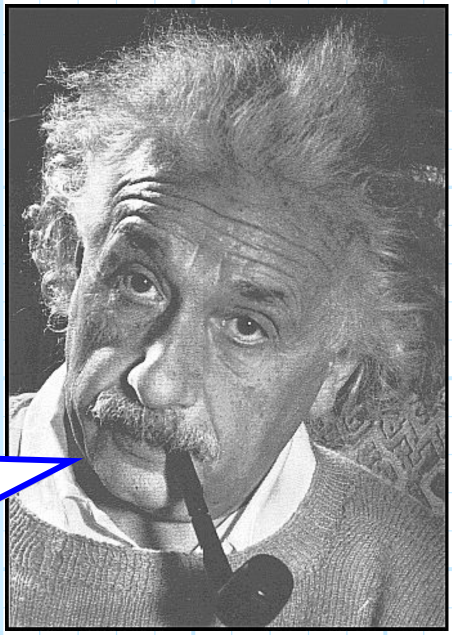
From KVL, we find that:

$$v_O(t) = v_S(t) - 0.7$$

This result is of course true **if** our original assumption is correct—it is valid **if** the ideal diode is forward biased (i.e., $i_D^i > 0$)!

From Ohm's Law, we find that:

$$i_D^i = \frac{v_O}{R} = \frac{v_S - 0.7}{R}$$



Q: *I'm so **confused!** Is this current **greater** than zero or **less** than zero? **Is** our assumption correct? **How** can we tell?*

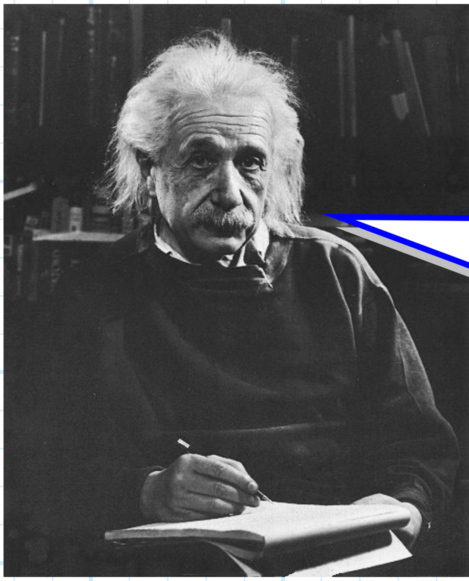
A: The ideal diode current is **dependent** on the value of source voltage $v_S(t)$. As such, we **cannot** determine **if** our assumption is correct, we **instead** must find out **when** our assumption is correct!

In other words, we know that the forward bias assumption is correct **when** $i_D^i > 0$. We can rearrage our diode current expression to determine for what values of source voltage $v_S(t)$ this is true:

$$\begin{aligned} i_D^i &> 0 \\ \frac{v_S(t) - 0.7}{R} &> 0 \\ v_S(t) - 0.7 &> 0 \\ v_S(t) &> 0.7 \end{aligned}$$

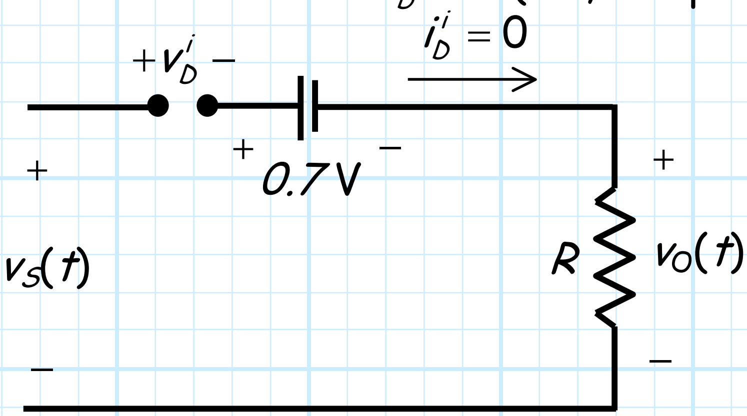
So, we have found that **when** the source voltage $v_S(t)$ is greater than 0.7 V, the output voltage $v_O(t)$ is:

$$v_O(t) = v_S(t) - 0.7$$



Q: *OK, I've got this result written down. However, I still don't know what the output voltage $v_O(t)$ is **when** the source voltage $v_S(t)$ is **less** than 0.7V!?!*

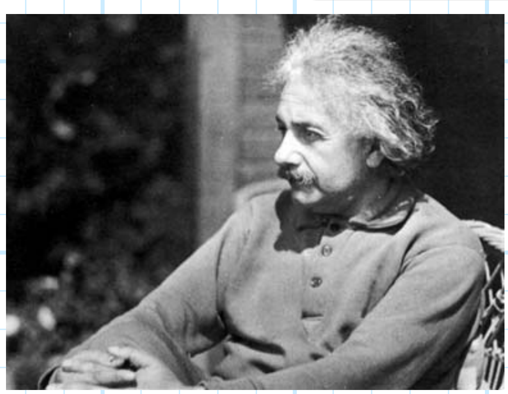
Now we **change** our assumption and ASSSUME the ideal diode in the CVD model is **reverse** biased, an assumption ENFORCEd with the condition that $i_D^i = 0$ (i.e., an open circuit).



Q: *Fascinating! The output voltage is **zero** when the ideal diode is reverse biased. But, precisely when **is** the ideal diode reverse biased? For **what** values of v_S does this occur ?*

From Ohm's Law, we find that the output voltage is:

$$\begin{aligned} v_O &= R\, i_D^i \\ &= R\,(0) \\ &= 0 \text{ V} \quad !!! \end{aligned}$$



A: To answer these questions, we must determine the **ideal** diode voltage in **terms** of v_S (i.e., $v_D^i = f(v_S)$):

From KVL: $$v_S - v_D^i - 0.7 = v_O$$

Therefore:

$$\begin{aligned} v_D^i &= v_S - 0.7 - v_O \\ &= v_S - 0.7 - 0.0 \\ &= v_S - 0.7 \end{aligned}$$

Thus, the ideal diode is in reverse bias **when:**

$$\begin{aligned} v_D^i &< 0 \\ v_S - 0.7 &< 0 \end{aligned}$$

Solving for v_S, we find:

$$\begin{aligned} v_S - 0.7 &< 0 \\ v_S &< 0.7 \text{ V} \end{aligned}$$

In other words, we have determined that the **ideal** diode will be reverse biased **when** $v_S < 0.7$ V, and that the output voltage will be $v_O = 0$.

Q: *So, we have found that:*

$$v_O = v_S - 0.7 \quad \textit{when} \quad v_S > 0.7 \text{ V}$$

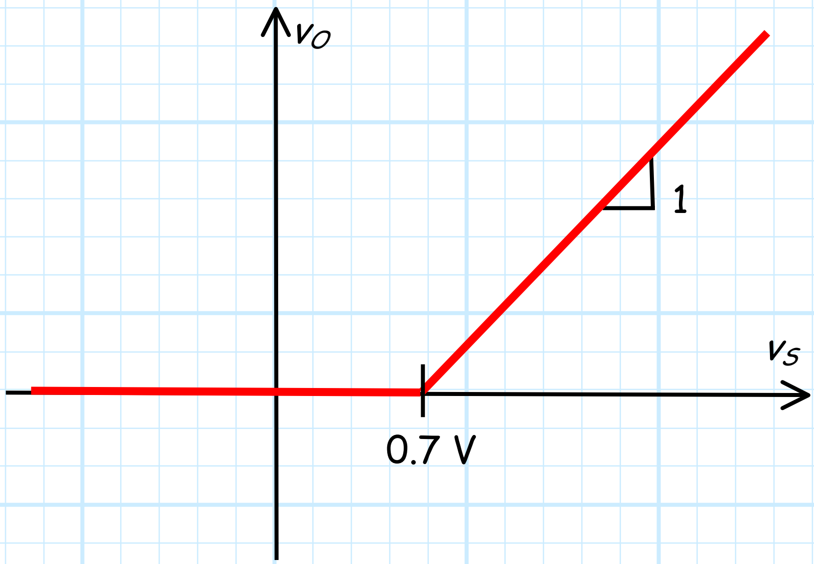
and,

$$v_O = 0.0 \quad \textit{when} \quad v_S < 0.7 \text{ V}$$

***It appears** we have a **valid, continuous, function**!*

A: That's right! The **transfer function** for this circuit is therefore:

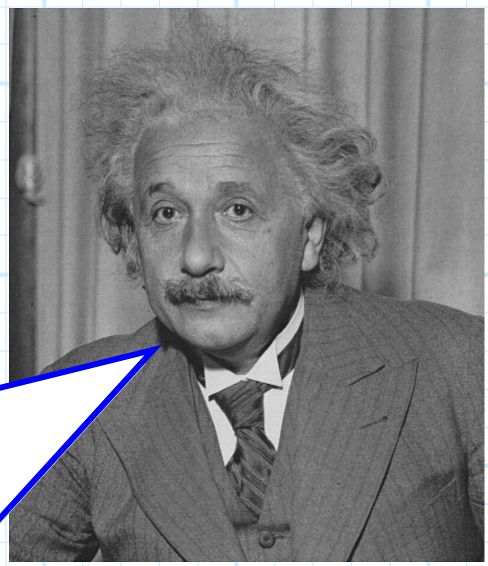
$$v_O = \begin{cases} v_S - 0.7 & \text{for} \quad v_S > 0.7 \\ 0 & \text{for} \quad v_S < 0.7 \end{cases}$$



*Although the circuit in this example may **seem** trivial, it is actually **very important**!*

*It is called a **half-wave rectifier**, and provides signal **rectification**.*

*Rectifiers are an **essential** part of every AC to DC **power supply**!*

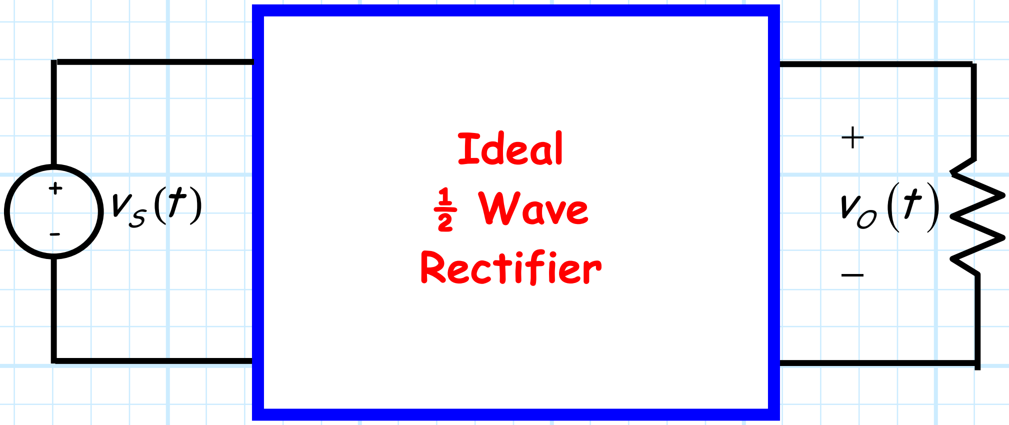


Signal Rectification

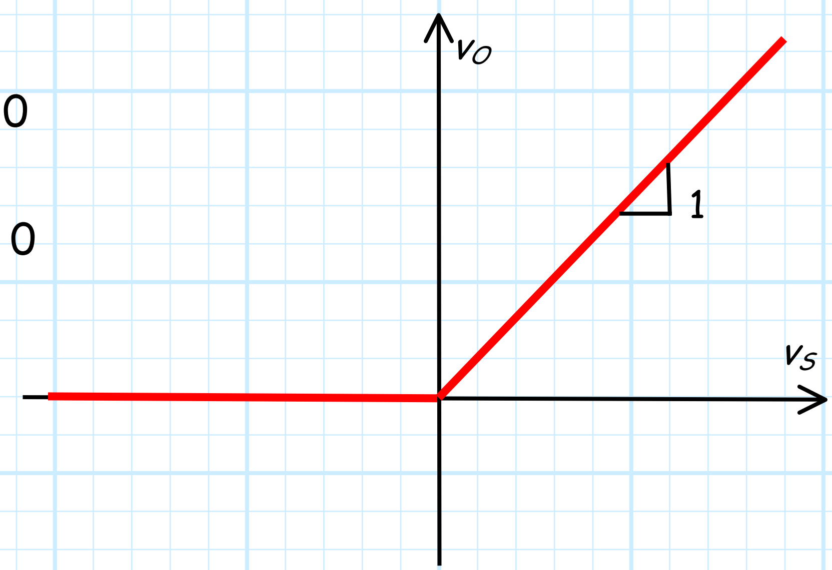
An important application of junction diodes is **signal rectification**.

There are **two types** of signal rectifiers, **half-wave** and **full-wave**.

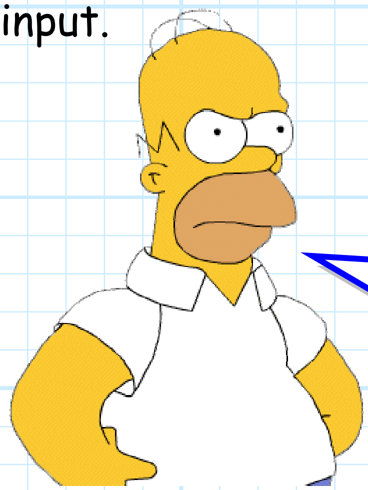
Let's first consider the **ideal half-wave rectifier**. It is a circuit with the transfer function $v_O = f(v_S)$:



$$v_O = \begin{cases} 0 & for \quad v_S < 0 \\ v_S & for \quad v_S > 0 \end{cases}$$

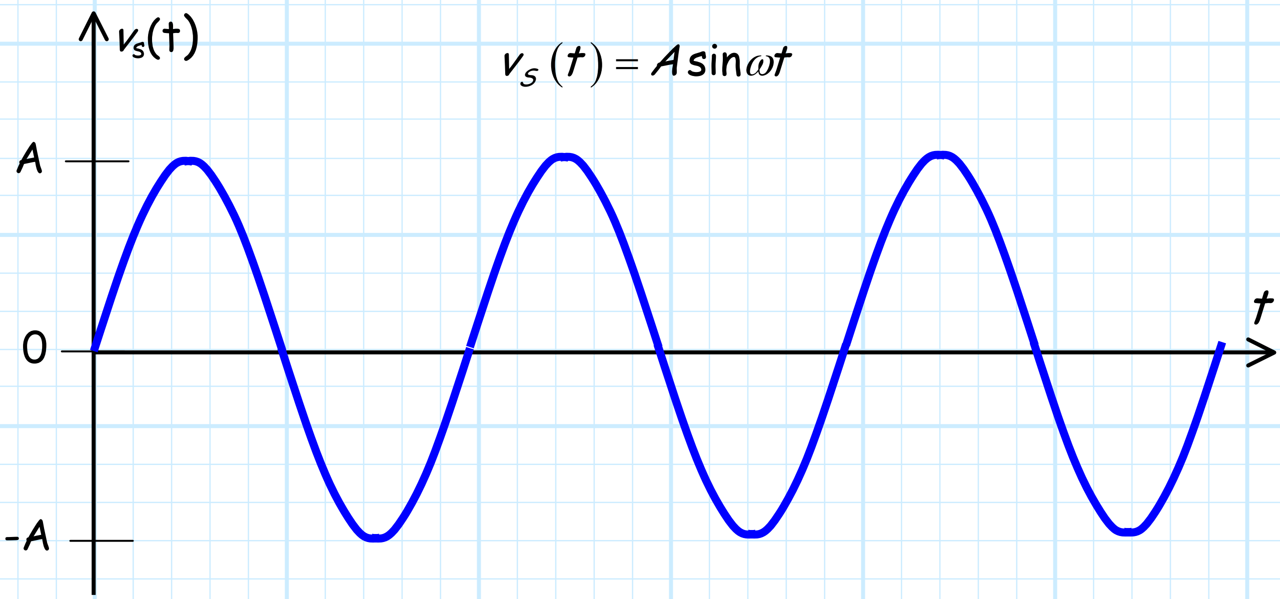


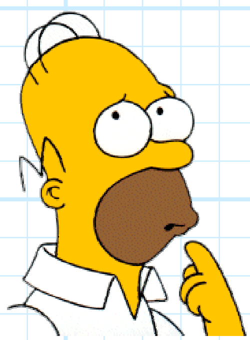
Pretty simple! **When** the input is negative, the output is **zero**, whereas **when** the input is positive, the output is the **same** as the input.



Q: *Pretty simple and pretty **stupid** I'd say! This might be your most **pointless** circuit yet. How in the world is **this** circuit useful??*

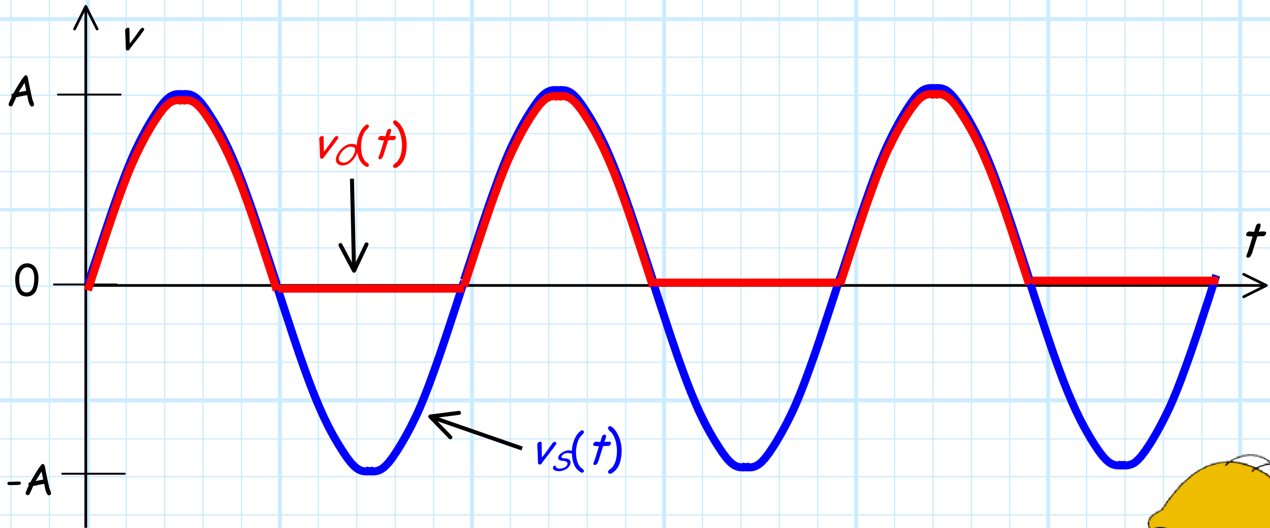
A: To see **why** a half-wave rectifier is useful, consider the **typical** case where the input source voltage is a **sinusoidal** signal with **frequency** ω and peak **magnitude** A:



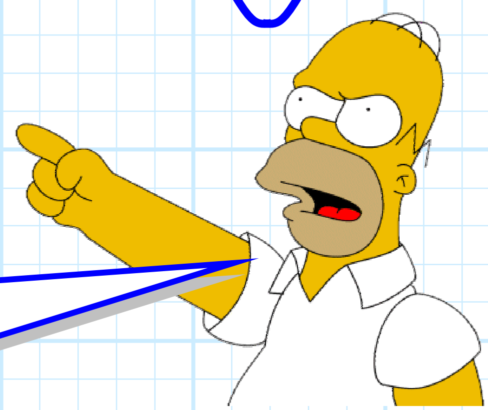


Think about what the **output** of the half-wave rectifier would be! Remember the rule: when $v_S(t)$ is **negative**, the output is **zero**, when $v_S(t)$ is **positive**, the output is **equal** to the input.

The **output** of the half-wave rectifier for **this** example is therefore:



Q: *That's maybe the **lamest** result I've ever seen. What good is **half** a sine wave? Why do we even bother?*



A: Although it may appear that our rectifier had **little** useful effect on the input signal $v_S(t)$, in fact the difference between input $v_S(t)$ and output $v_O(t)$ is both **important** and **profound**.

To see how, consider first the **DC component** (i.e. the time-averaged value) of the **input** sine wave:

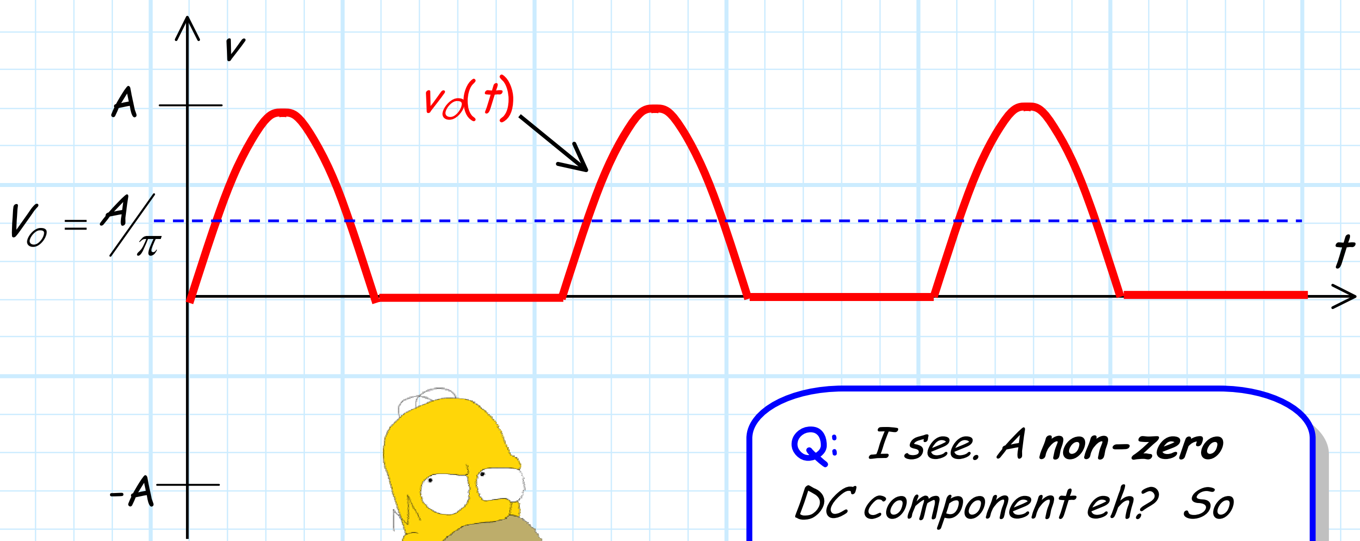
$$\begin{aligned} V_S &= \frac{1}{T}\int_0^T v_S(t)\,dt \\ &= \frac{1}{T}\int_0^T A \sin \omega t\, dt = 0 \end{aligned}$$

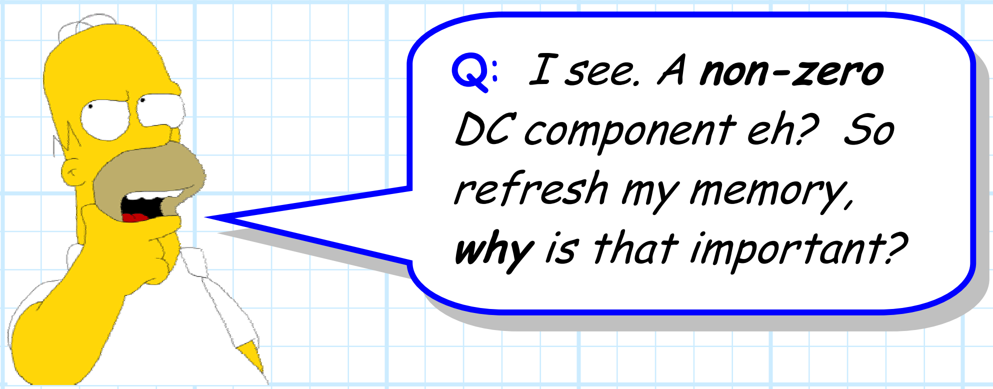
Thus, (as you probably already knew) the **DC component** of a sine wave is **zero**—a sine wave is an ***AC* signal**!

Now, contrast this with the **output** $v_O(t)$ of our half-wave rectifier. The **DC component** of the **output** is:

$$
\begin{aligned}
V_O &= \frac{1}{T}\int_0^T v_O(t)\,dt \\
&= \frac{1}{T}\int_0^{T/2} A\sin\omega t\,dt + \frac{1}{T}\int_{T/2}^{T} 0\,dt = \frac{A}{\pi}
\end{aligned}
$$

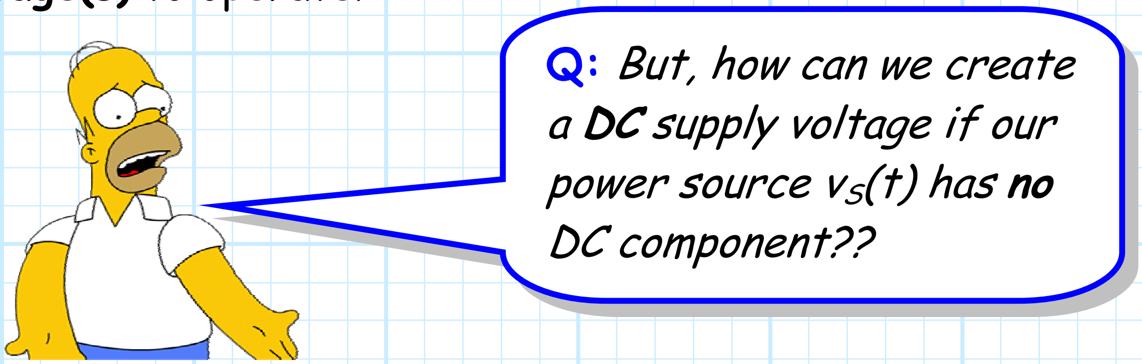
Unlike the input, the **output** has a **non-zero** (positive) **DC component** ($V_O = A/\pi$)!





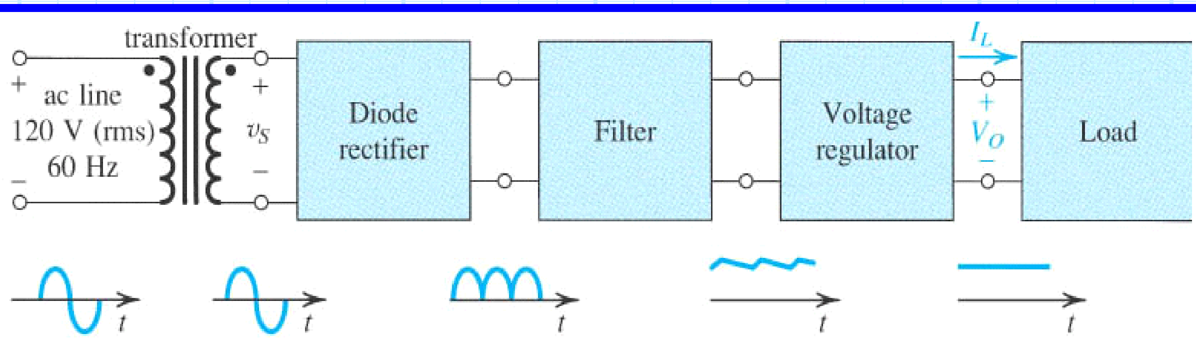
A: Recall that the **power distribution** system we use is an ***AC*** system. The source voltage $v_S(t)$ that we get when we plug our "**power cord**" into the wall socket is a 60 Hz **sinewave**—a source with a **zero DC component**!

The **problem** with this is that most **electronic devices** and systems, such as TVs, stereos, computers, etc., require a **DC voltage(s)** to operate!

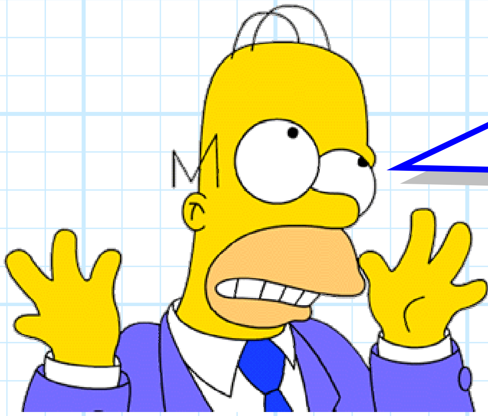


A: That's **why** the half-wave rectifier is so **important**! It takes an AC source with **no** DC component and creates a signal with **both** a DC and AC component.

We can then pass the output of a half-wave rectifier through a **low-pass filter**, which **suppresses** the AC component but lets the **DC** value ($V_O = A/\pi$) pass through. We then **regulate** this output and form a **useful DC voltage source**—one suitable for powering our electronic systems!

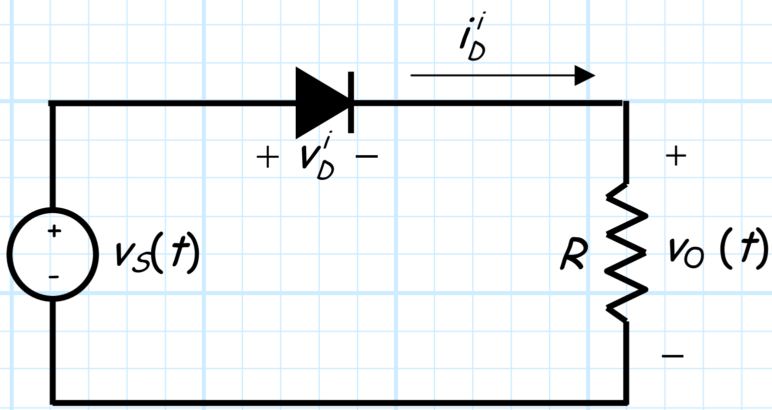


A Power Supply



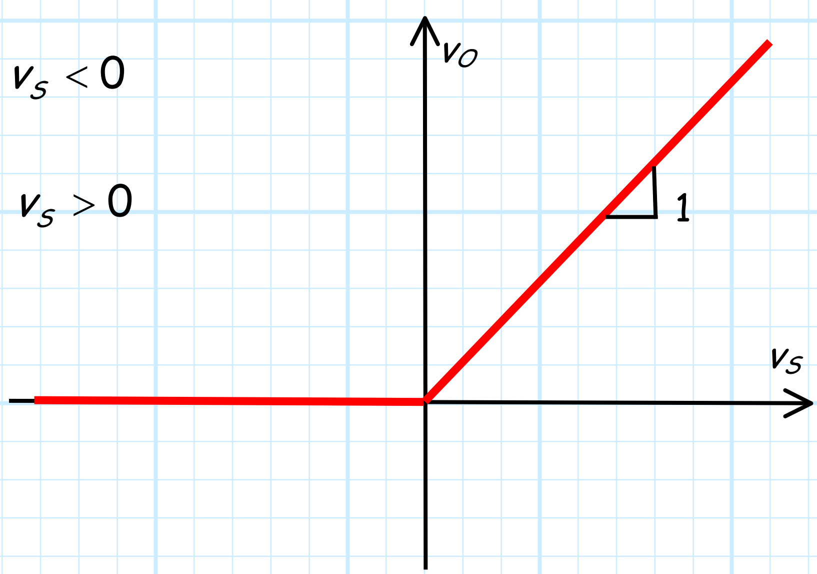
Q: *OK, now I see **why** the **ideal** half-wave rectifier might be useful. But, is there any way to actually **build** this magical device?*

A: An **ideal** half-wave rectifier can be "built" if we use an **ideal** diode.

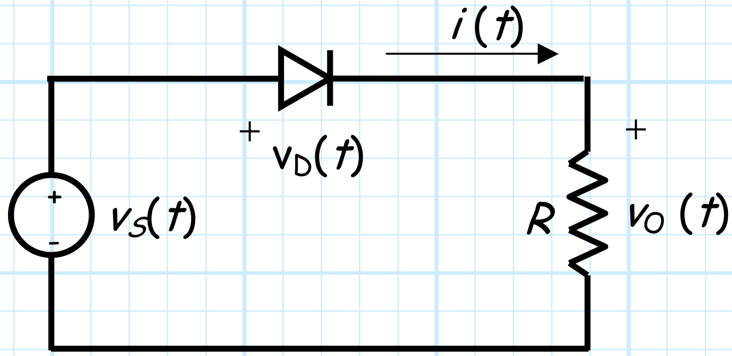


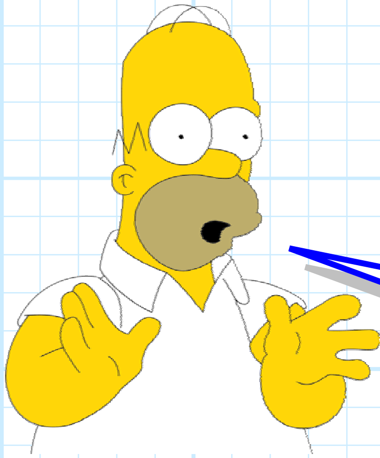
If we follow the transfer function **analysis steps** we studied earlier, then we will find that this circuit is indeed an **ideal half-wave rectifier**!

$$v_O = \begin{cases} 0 & \text{for} \quad v_S < 0 \\ v_S & \text{for} \quad v_S > 0 \end{cases}$$



Of course, since **ideal** diodes do **not** exist, we must use a **junction** diode instead:

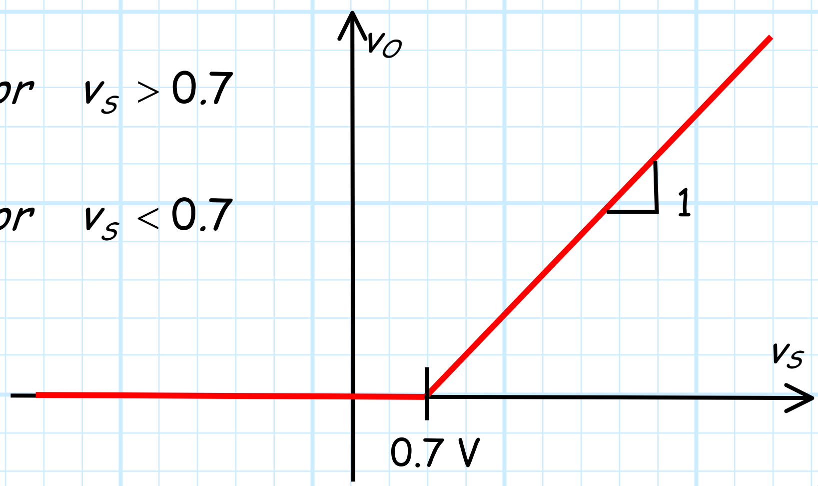




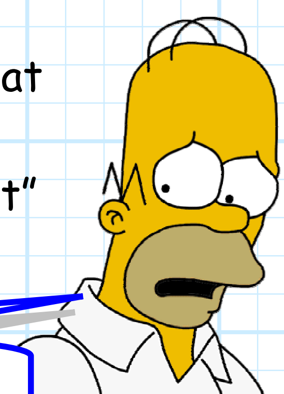
Q: *This circuit looks so **familiar**! Haven't we studied it before?*

A: Yes! It was an **example** where we determined the junction diode circuit transfer function. Recall that the **result** was:

$$v_O = \begin{cases} v_s - 0.7 & \text{for} \quad v_s > 0.7 \\ 0 & \text{for} \quad v_s < 0.7 \end{cases}$$



Note that this result is **slightly different** from that of the **ideal** half-wave rectifier! The **0.7 V drop** across the junction diode causes a horizontal "shift" of the transfer function from the ideal case.



Q: *So, this junction diode circuit is **worthless**?*

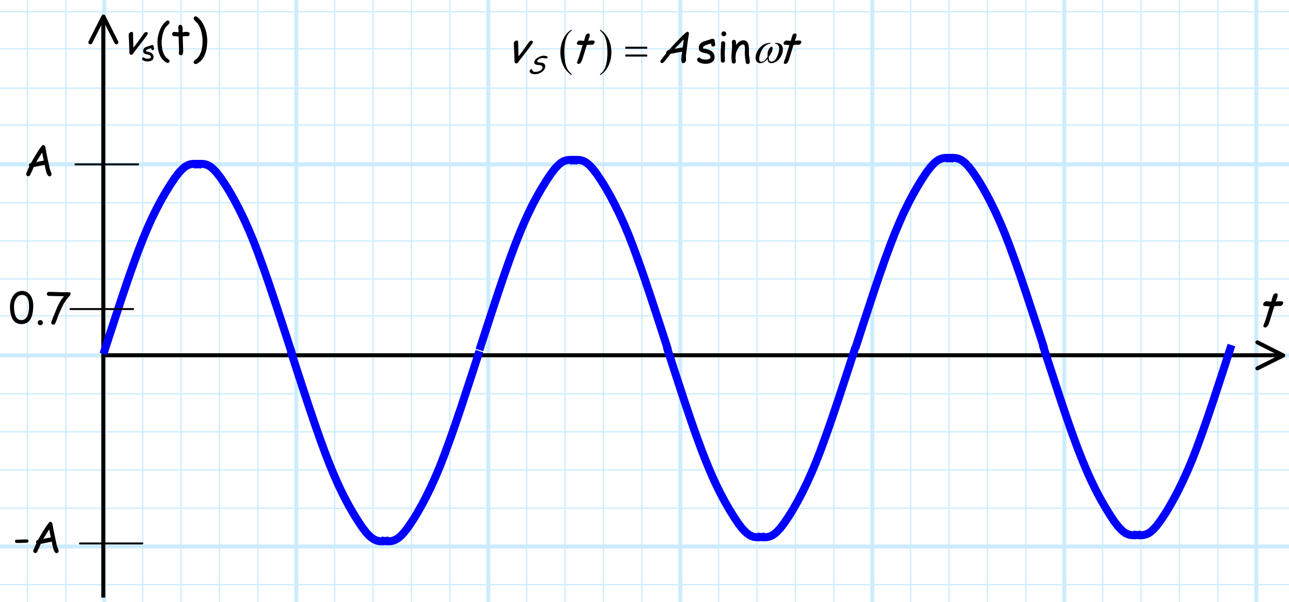
A: Hardly! Although the transfer function is **not quite** ideal, it works **well enough** to achieve the goal of signal rectification—it takes an input with **no** DC component and creates an output with a **significant** DC component!

Note what the transfer function "**rule**" is now:

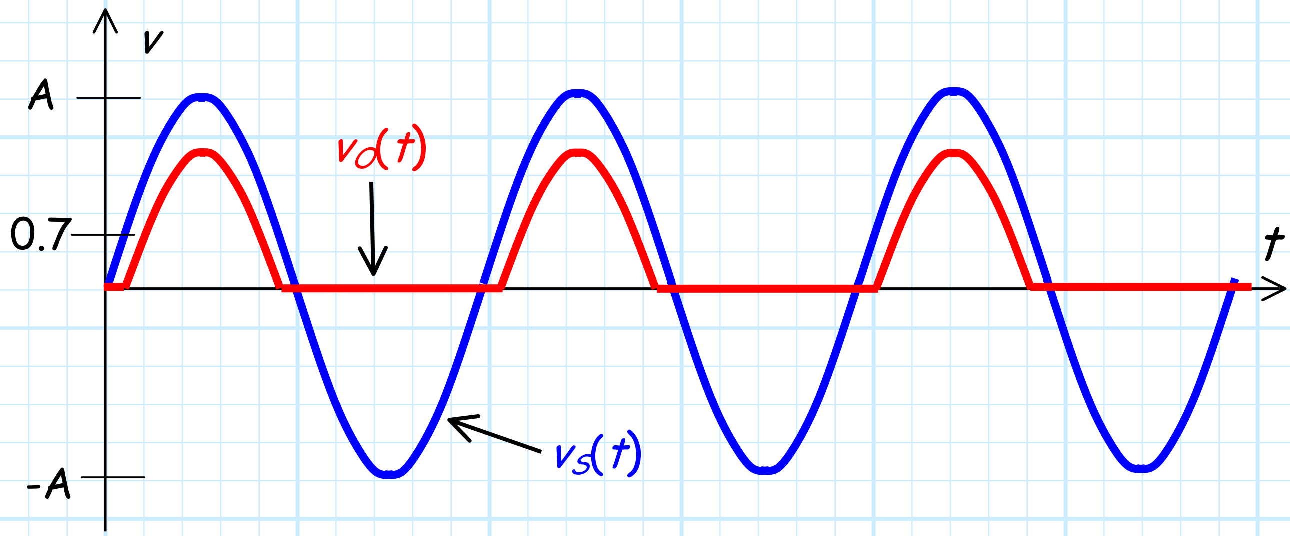
1. When the input is **greater** than 0.7 V, the output voltage is **equal** to the input voltage minus 0.7 V.

2. When the input is **less** than 0.7 V, the output voltage is **zero**.

So, let's consider **again** the case where the source voltage is **sinusoidal** (just like the source from a "wall socket"!):



The output of our **junction diode** half-wave rectifier would therefore be:



Although the output is **shifted downward** by 0.7 V (note in the plot above this is **exaggerated**, typically A>>0.7V), it should be apparent that the **output** signal $v_O(t)$, unlike the input signal $v_S(t)$, has a **non-zero** (positive) **DC component**.

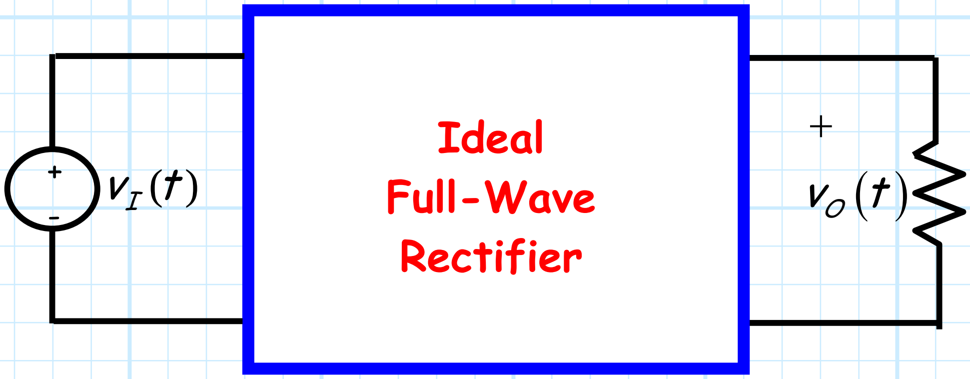
Because of the 0.7 V shift, this DC component is **slightly smaller** than the **ideal** case. In fact, we find that if A>>0.7, this **DC component** is approximately:

$$V_O \approx \frac{A}{\pi} - 0.35 \text{ V}$$

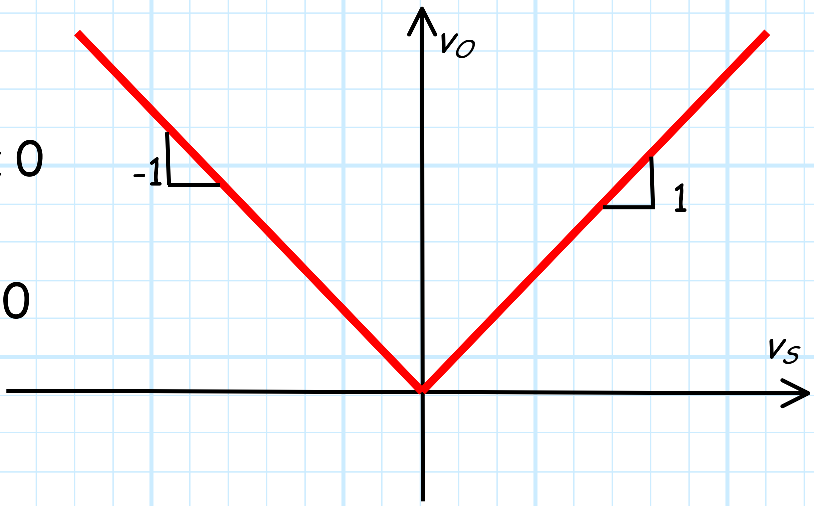
In other words, **just** 350 mV less than **ideal**!

Q: *Way back on the first page you said that there were **two** types of rectifiers. I now understand **half-wave** rectification, but what about these so-called **full-wave** rectifiers?*

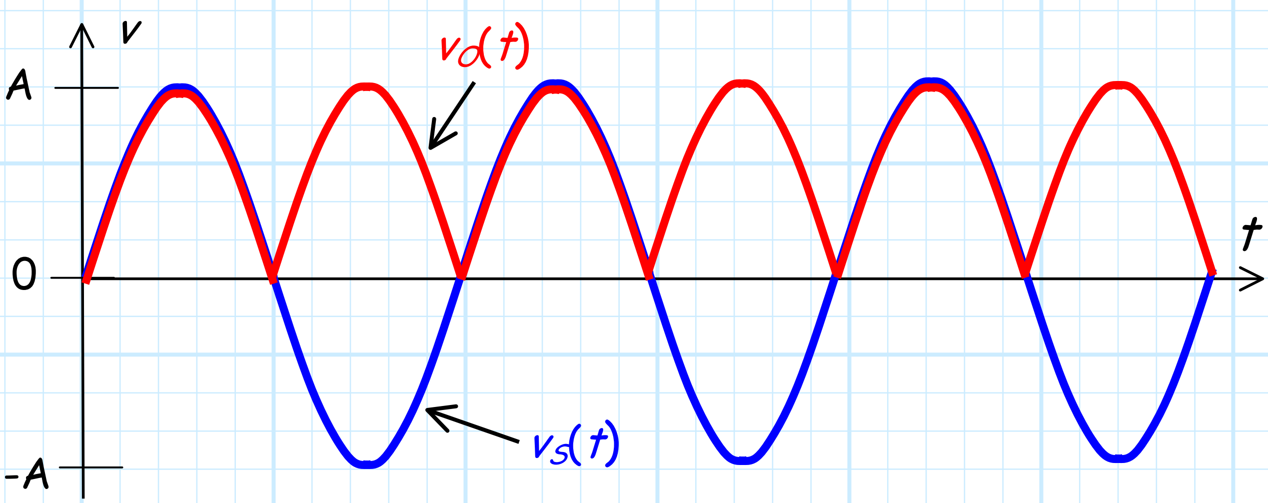
A: Almost forgot! Let's examine the transfer function of an **ideal full-wave rectifier**:



$$v_O = \begin{cases} -v_S & \text{for} \quad v_S < 0 \\ v_S & \text{for} \quad v_S > 0 \end{cases}$$



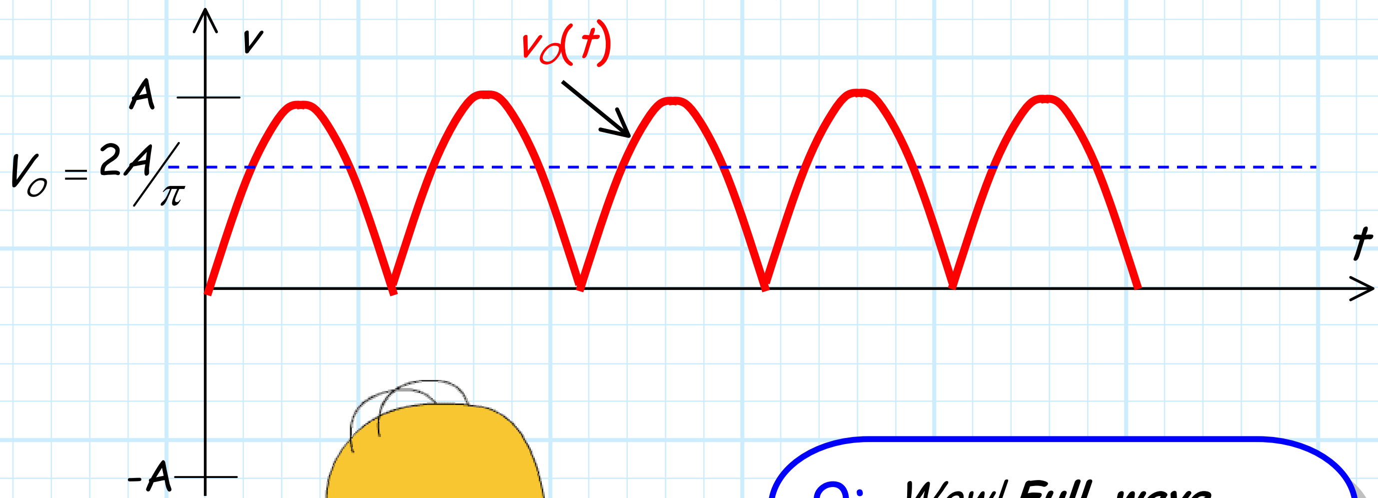
If the ideal half-wave rectifier makes **negative** inputs **zero**, the ideal full-wave rectifier makes **negative** inputs—**positive**! For **example**, if we again consider our **sinusoidal** input, we find that the output will be:

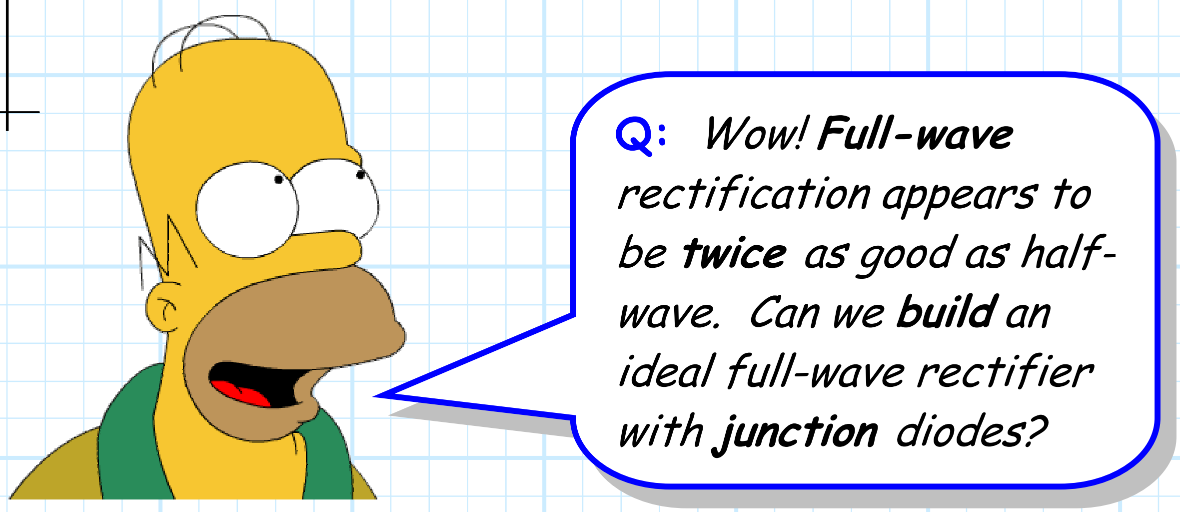


The result is that the output signal will have a DC component **twice** that of the ideal half-wave rectifier!

$$V_O = \frac{1}{T}\int_0^T v_O(t)\,dt$$

$$= \frac{1}{T}\int_0^{T/2} A\sin\omega t\,dt - \frac{1}{T}\int_{T/2}^{T} A\sin\omega t\,dt = \frac{2A}{\pi}$$

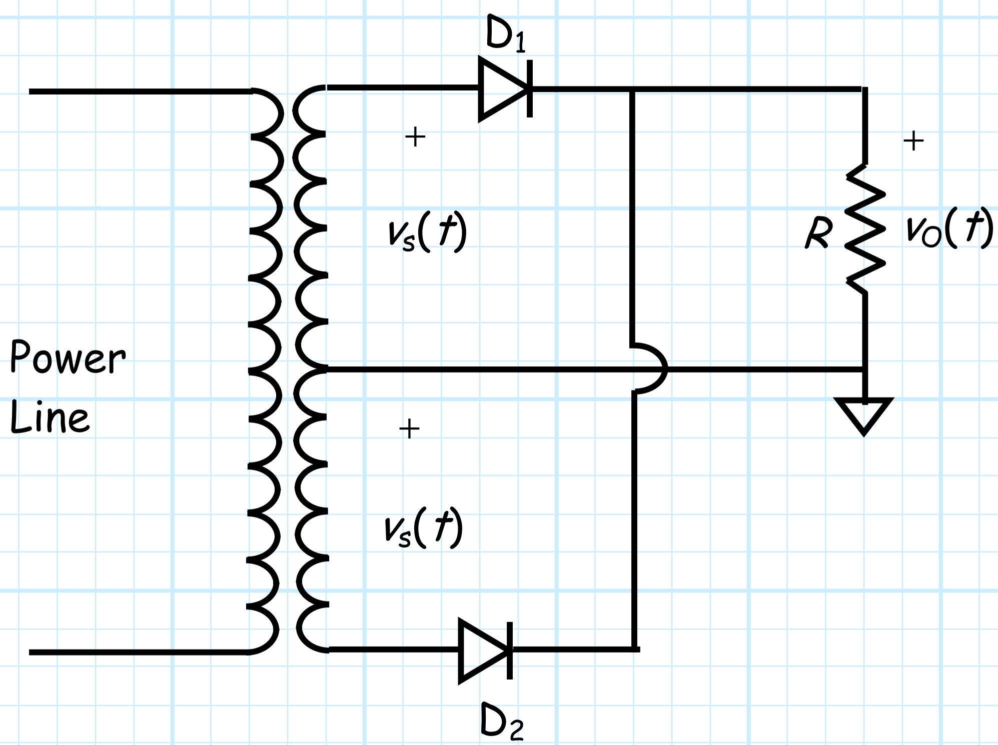




A: Although we cannot build an **ideal** full-wave rectifier with **junction** diodes, we can build full-wave rectifiers that are **very close** to ideal with junction diodes!

The Full-Wave Rectifier

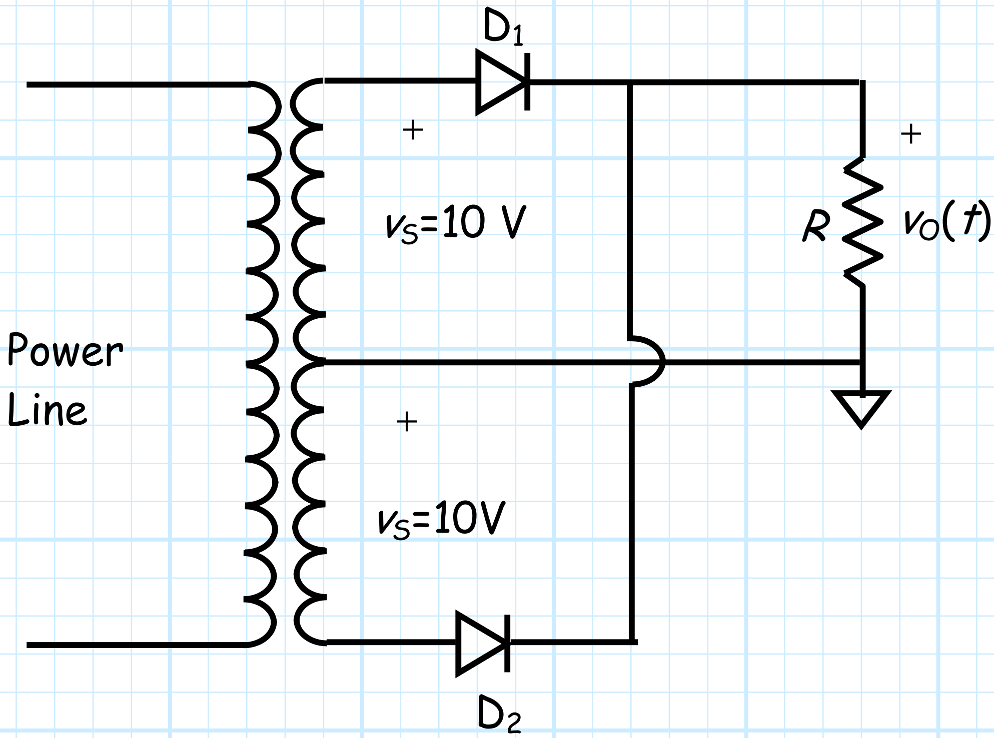
Consider the following **junction diode** circuit:



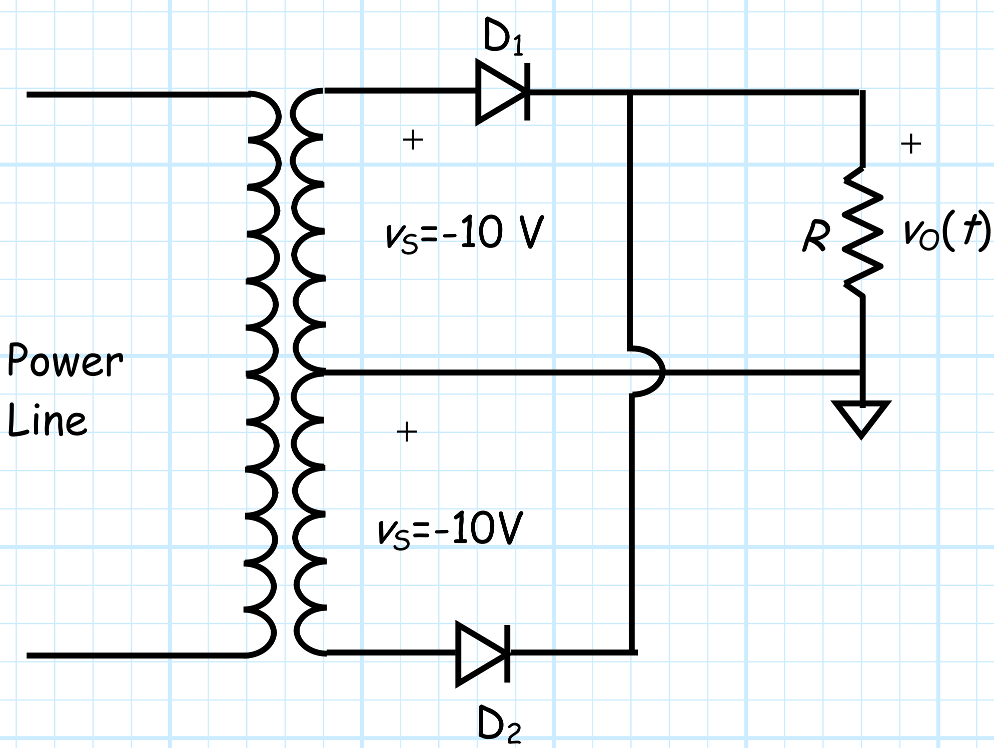
Note that we are using a **transformer** in this circuit. The job of this transformer is to **step-down** the large voltage on our power line (120 V rms) to some **smaller** magnitude (typically 20-70 V rms).

Note the secondary winding has a **center tap** that is **grounded**. Thus, the secondary voltage is distributed **symmetrically** on either side of this center tap.

For example, if v_S = 10 V, the anode of D_1 will be 10V **above** ground potential, while the anode of D_2 will be 10V **below** ground potential (i.e., -10V):

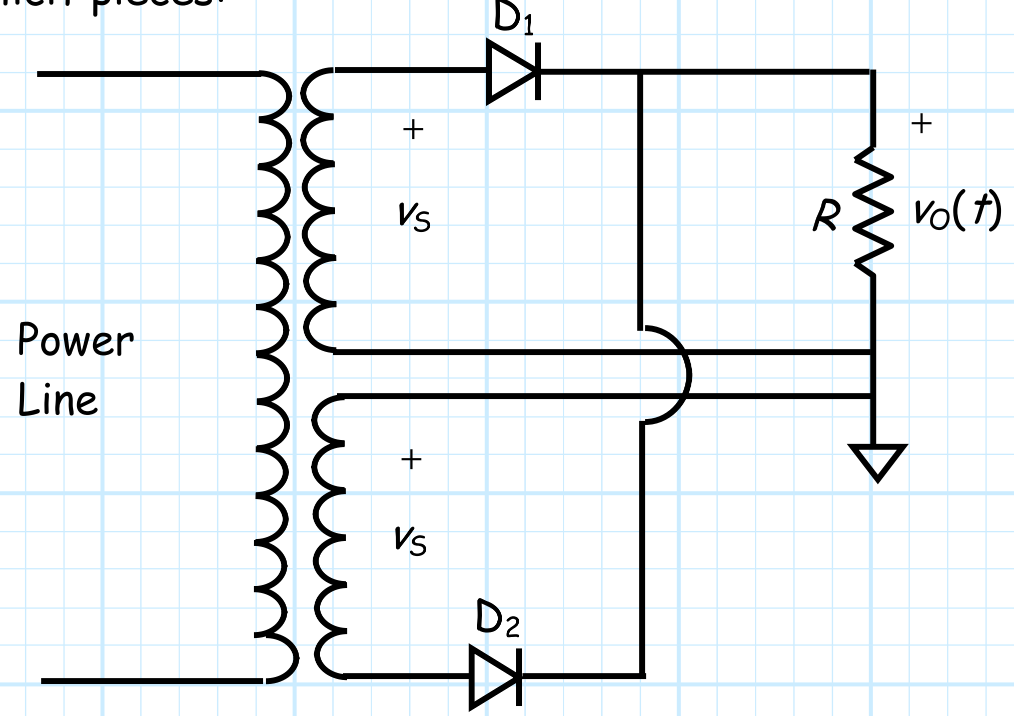


Conversely, if v_S=-10 V, the anode of D_1 will be 10V **below** ground potential (i.e., -10V), while the anode of D_2 will be 10V **above** ground potential:

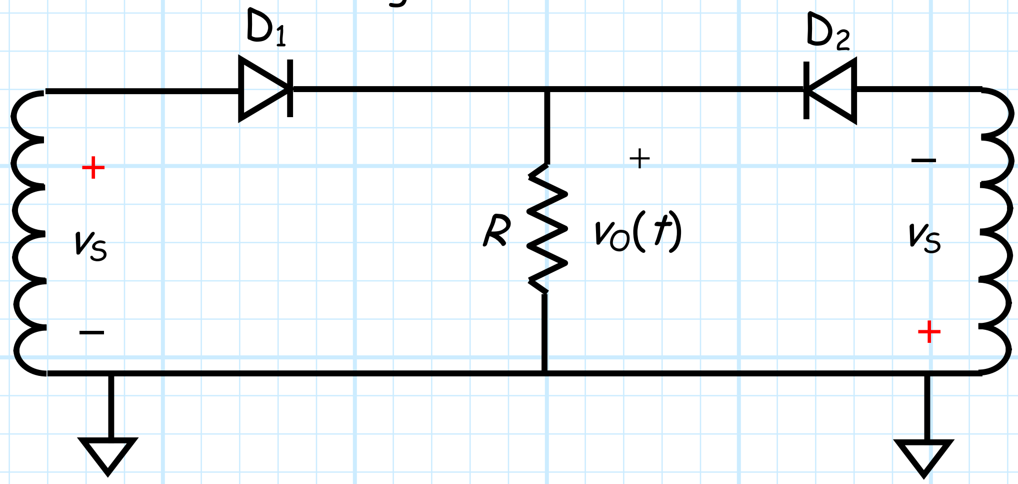


The more important question is, what is the value of **output** v_O? More specifically, how is v_O related to the value of source v_S—what is the **transfer fuction** $v_O = f(v_S)$?

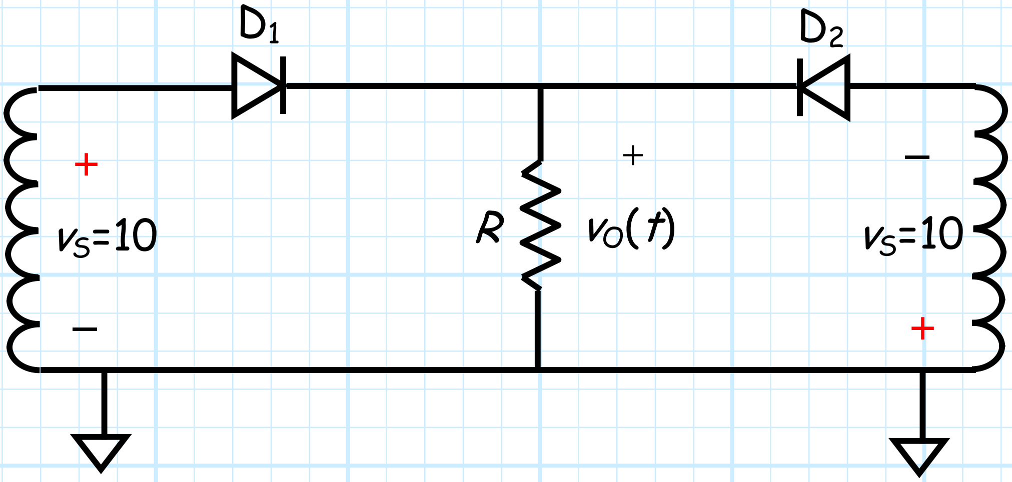
To help simplify our analysis, we are going **redraw** this cirucuit in another way. First, we will **split** the secondary winding into two explicit pieces:



We will now **ignore the primary** winding of the transformer and redraw the remaining circuit as:



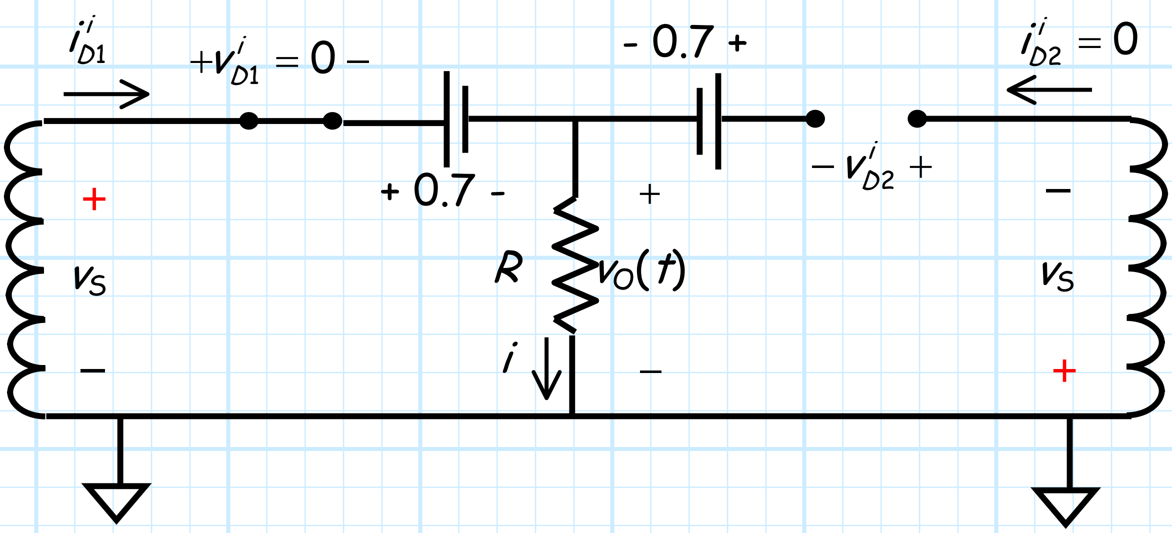
Note that the secondary voltages at either end of this circuit are the **same**, but have **opposite** polarity. As a result, if v_S=10, then the anode of diode D_1 will be 10 V **above** ground, and the anode at diode D_2 will be 10V **below** ground—just like before!



Now, let's attempt to determine the **transfer function** $v_O = f(v_S)$ of this circuit.

First, we will replace the junction diodes with **CVD models**.

Then let's ASSUME D_1 is **forward** biased and D_2 is **reverse** biased, thus ENFORCE $v_{D1}^i = 0$ and $i_{D2}^i = 0$. Thus ANALYZE:



Note that we need to determine **3** things: the **ideal diode current** i_{D1}^{i}, the **ideal diode voltage** v_{D2}^{i}, and the **output voltage** v_O. However, **instead** of finding numerical values for these 3 quantities, we must express them in terms of **source voltage v_S**!

From KCL: $i = i_{D1}^{i} + i_{D2}^{i} = i_{D1}^{i} + 0 = i_{D1}^{i}$

From KVL: $v_S - v_{D1}^{i} - 0.7 - R\, i_D^{i} = 0$

Thus the **ideal diode current** is:

$$i_{D1}^{i} = \frac{v_S - 0.7}{R}$$

Likewise, from KVL: $v_S - v_{D1}^{i} - 0.7 + 0.7 + v_{D2}^{i} + v_S = 0$

Thus, the **ideal diode voltage** is:

$$v_{D2}^{i} = -2 v_S$$

And finally, from KVL: $v_S - v_{D1}^{i} - 0.7 = v_O$

Thus, the **output voltage** is:

$$v_O = v_S - 0.7$$

Now, we must determine **when** both $i_{D1}^{i} > 0$ and $v_{D2}^{i} < 0$. When **both** these conditions are true, the output voltage will be $v_O = v_S - 0.7$. When one **or** both conditions $i_{D1}^{i} > 0$ and $v_{D2}^{i} < 0$ are **false**, then our assuptions are **invalid**, and $v_O \neq v_S - 0.7$.

Using the results we just determined, we know that $i_{D1}^{i} > 0$ **when**:

$$\frac{v_S - 0.7}{R} > 0$$

Solving for v_S:

$$\begin{aligned} \frac{v_S - 0.7}{R} &> 0 \\ v_S - 0.7 &> 0 \\ v_S &> 0.7 \text{ V} \end{aligned}$$

Likewise, we find that $v_{D2}^{i} < 0$ **when**:

$$-2v_S < 0$$

Solving for v_S:

$$\begin{aligned} -2v_S &< 0 \\ 2v_S &> 0 \\ v_S &> 0 \end{aligned}$$

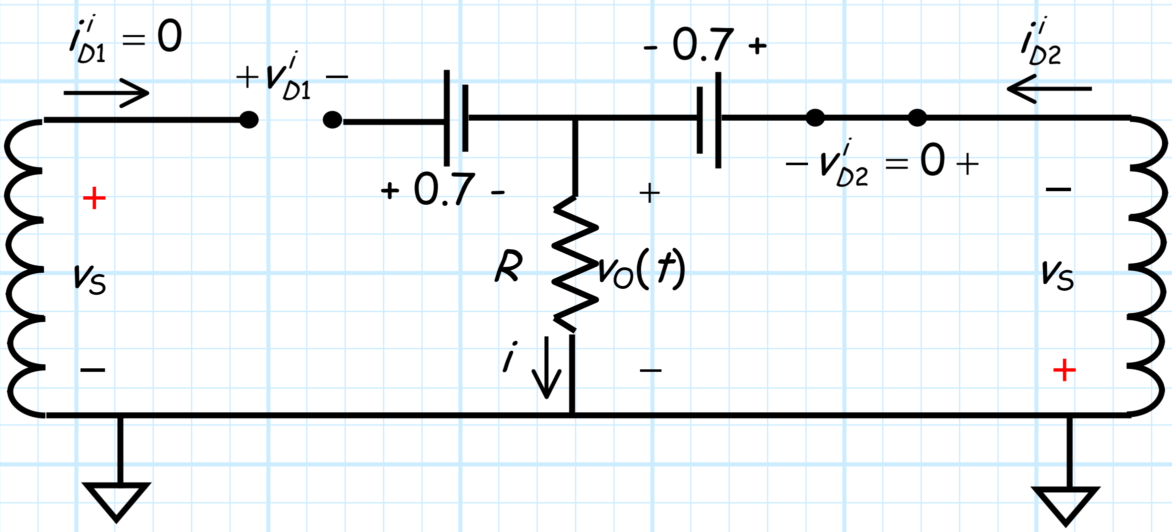
Thus, our assumptions are correct **when** $v_S > 0.0$ **AND** $v_S > 0.7$. This is the **same** thing as saying our assumptions are valid when $v_S > 0.7$!

Thus, we have found that the following statement is true about this circuit:

$$v_O = v_S - 0.7 \text{ V} \quad \text{when} \quad v_S > 0.7 \text{ V}$$

Note that this statement does **not** constitute a **function** (what about $v_S < 0.7$?), so we must **continue** with our analysis!

Say we now ASSUME that D_1 is **reverse** biased and D_2 is **forward** biased, so we ENFORCE $i_{D1}^i = 0$ and $v_{D2}^i = 0$. Thus, we ANALYZE **this** circuit:

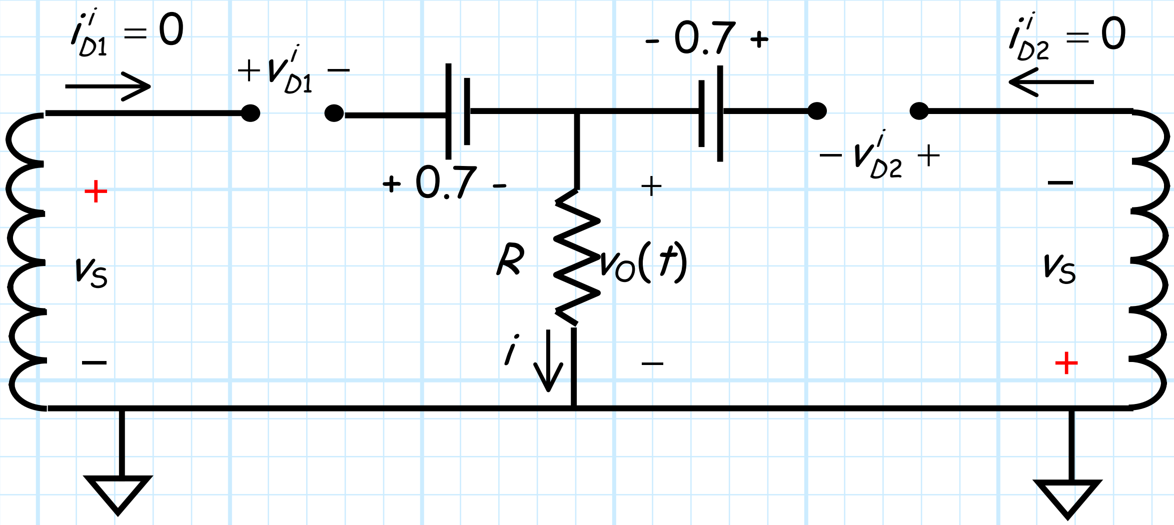


Using the **same proceedure** as before, we find that $v_O = -v_S - 0.7$, and both our assumptions are true **when** $v_S < -0.7$ V. In other words:

$$v_O = -v_S - 0.7 \text{ V} \quad \text{when} \quad v_S < -0.7 \text{ V}$$

Note we are still **not** done! We **still** do not have a complete transfer **function** (what happens when $-0.7 \text{ V} < v_S < 0.7 \text{ V}$?).

Finally then, we ASSUME that **both** ideal diodes are **reverse** biased, so we ENFORCE $i_{D1}^{i}=0$ and $i_{D2}^{i}=0$. Thus ANALYZE:



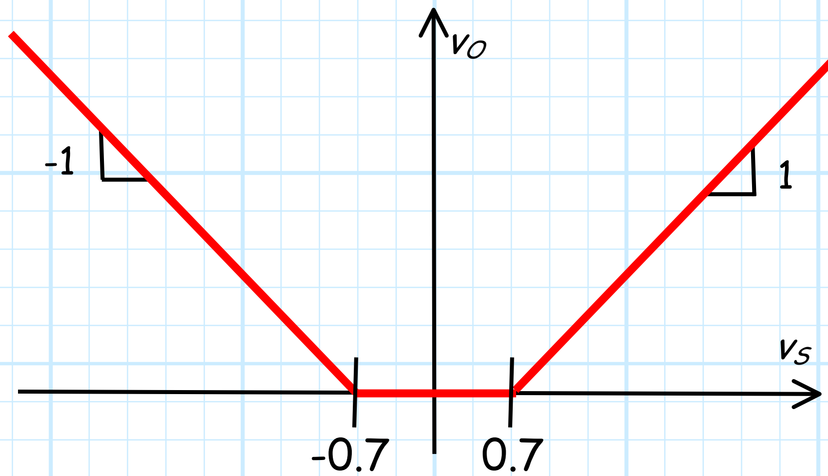
Following the **same proceedures** as before, we find that $v_S=0$, and both assumptions are true **when** $-0.7<v_S<0.7$. In other words:

$$v_S=0 \quad \text{when} \quad -0.7<v_S<0.7$$

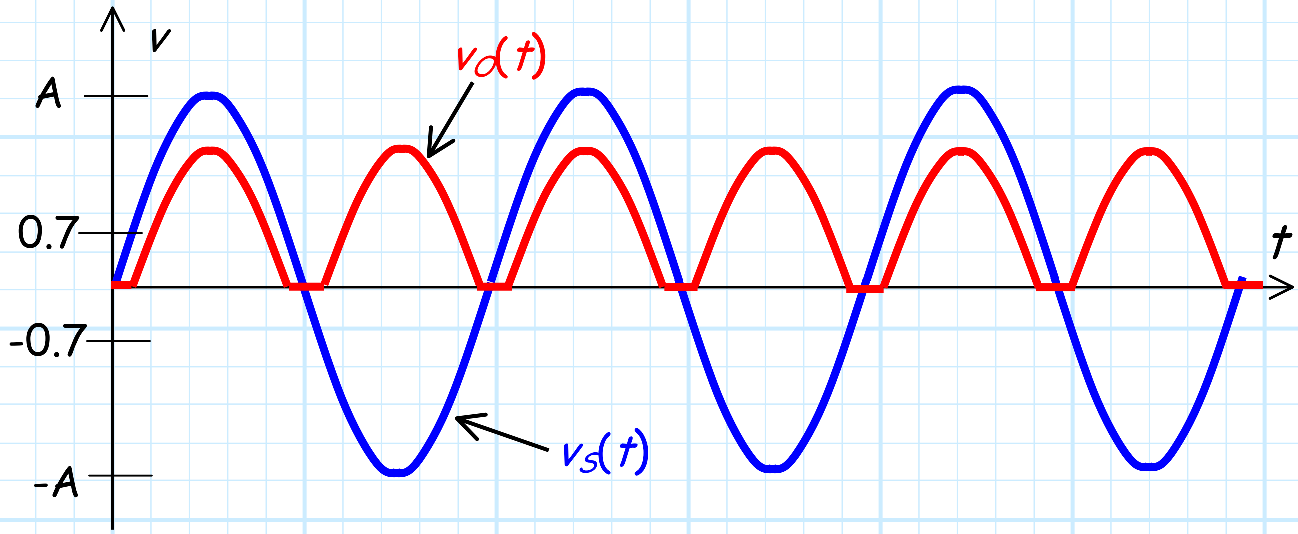
Now we have a **function**! The transfer function of this circuit is:

$$v_O=\begin{cases} v_S-0.7\,\text{V} & \textit{for} \quad v_S>0.7\,\text{V} \\ 0V & \textit{for} \quad -0.7>v_S>0.7\,\text{V} \\ -v_S-0.7\,\text{V} & \textit{for} \quad v_S<-0.7\,\text{V} \end{cases}$$

Plotting this function:

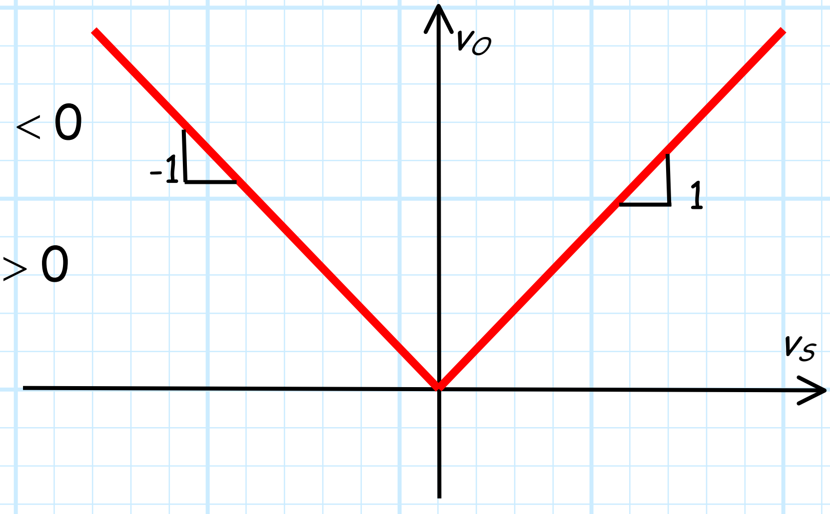


The output of this full-wave rectifier with a **sine wave input** is therefore:



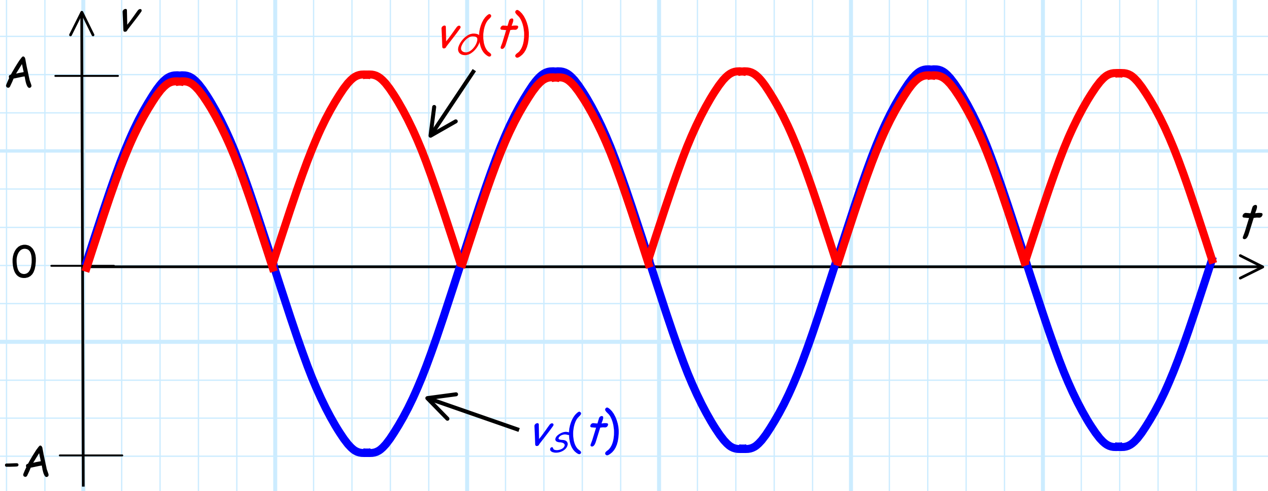
Note how this **compares** to the transfer function of the **ideal** full-wave rectifier:

$$v_O = \begin{cases} -v_S & for \quad v_S < 0 \\ v_S & for \quad v_S > 0 \end{cases}$$



Very similar!

Likewise, compare the output of this junction diode full-wave rectifier to the output of an **ideal** full-wave rectifier:



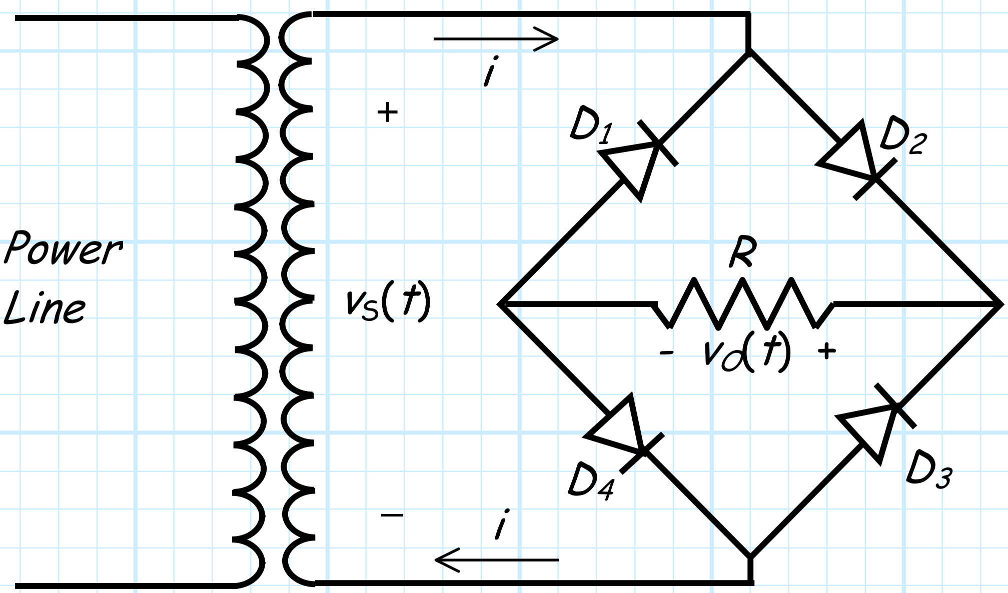
Again we see that the junction diode full-wave rectifier output is **very close** to ideal. In fact, if A>>0.7 V, the **DC component** of this junction diode full wave rectifier is approximately:

$$V_O \approx \frac{2A}{\pi} - 0.7 \text{ V}$$

Just 700 mV less than the **ideal** full-wave rectifier DC component!

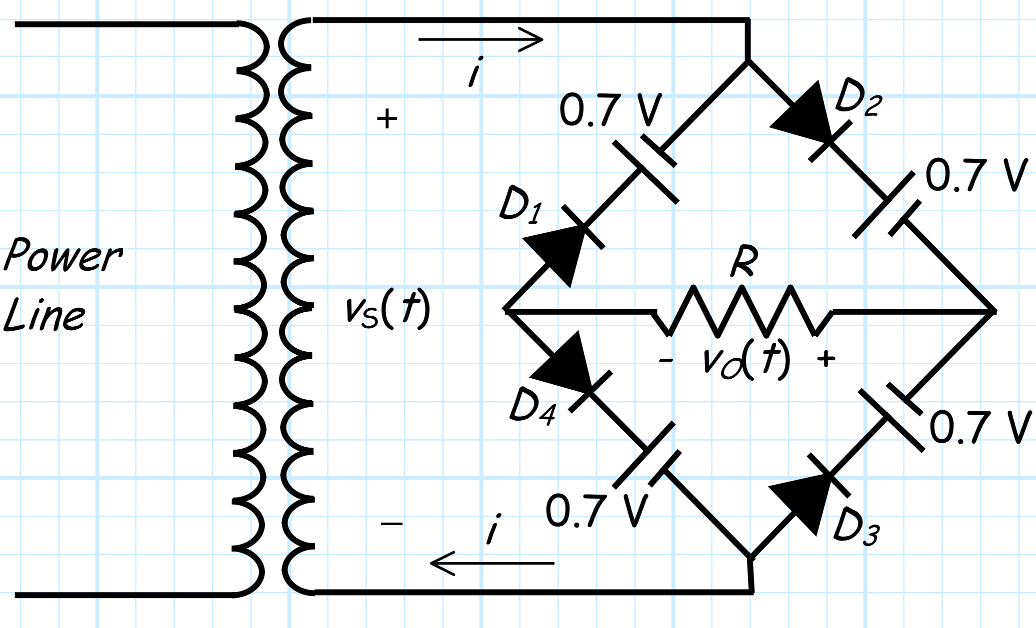
The Bridge Rectifier

Now consider this **junction diode** rectifier circuit:



We call this circuit the **bridge rectifier**. Let's **analyze** it and see what it does!

First, we **replace** the junction diodes with the **CVD model**:



Q: *__Four__ gul-durn ideal diodes! That means __16__ sets of dad-gum assumptions!*

A: True! However, there are only **three** of these sets of assumptions are actually **possible**!

Consider the **current** i flowing through the rectifier. This current of course can be positive, negative, or zero. It turns out that there is only **one** set of diode assumptions that would result in positive current i, **one** set of diode assumptions that would lead to negative current i, and **one** set that would lead to zero current i.

Q: *But what about the remaining __13__ sets of dog gone diode assumptions?*

A: **Regardless** of the value of source v_S, the remaining 13 sets of diode assumptions simply **cannot occur** for this particular circuit design!

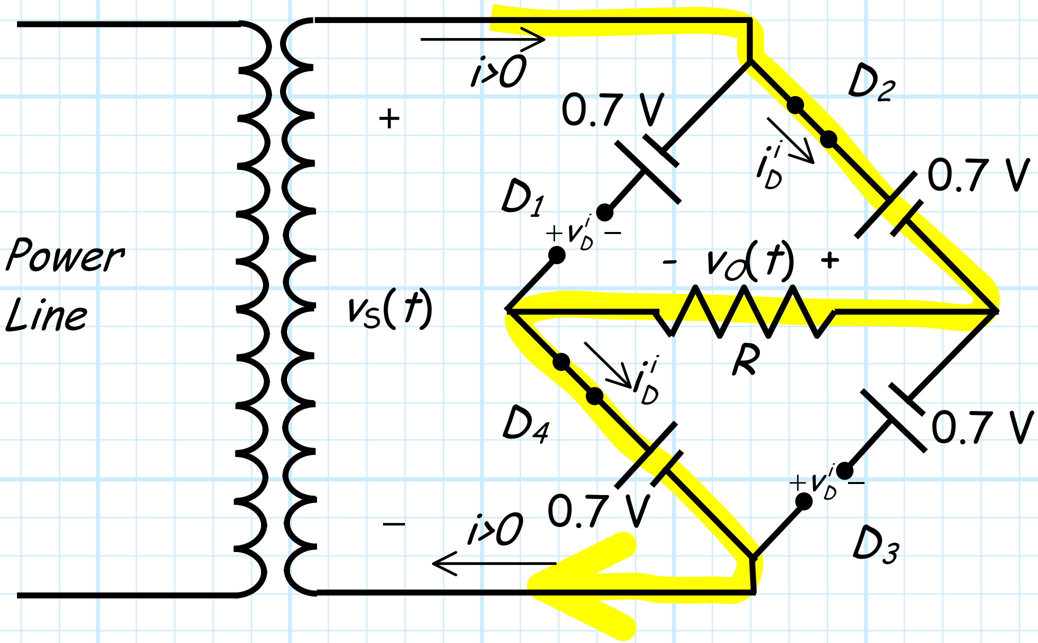
Let's look at the **three** possible sets of assumptions:

$i > 0$

The rectifier current i can be **positive** only **if** these assumptions are true:

D_1 and D_3 are reverse biased.

D_2 and D_4 are forward biased.



Analyzing this circuit, we find that the **output voltage** is:

$$v_O = v_S - 1.4 \text{ V}$$

and the f.b. **ideal diode currents** are:

$$i = i_D^i = \frac{v_S - 1.4}{R}$$

and, finally the r.b. **ideal diode voltages** are:

$$v_D^i = -v_S$$

Thus, $i_D^i > 0$ when:

$$\frac{v_S - 1.4}{R} > 0$$

$$v_S - 1.4 > 0$$

$$v_S > 1.4 \text{ V}$$

and $v_D^i < 0$ when:

$$-v_S < 0$$

$$v_S > 0$$

Therefore, we **find** that for this circuit:

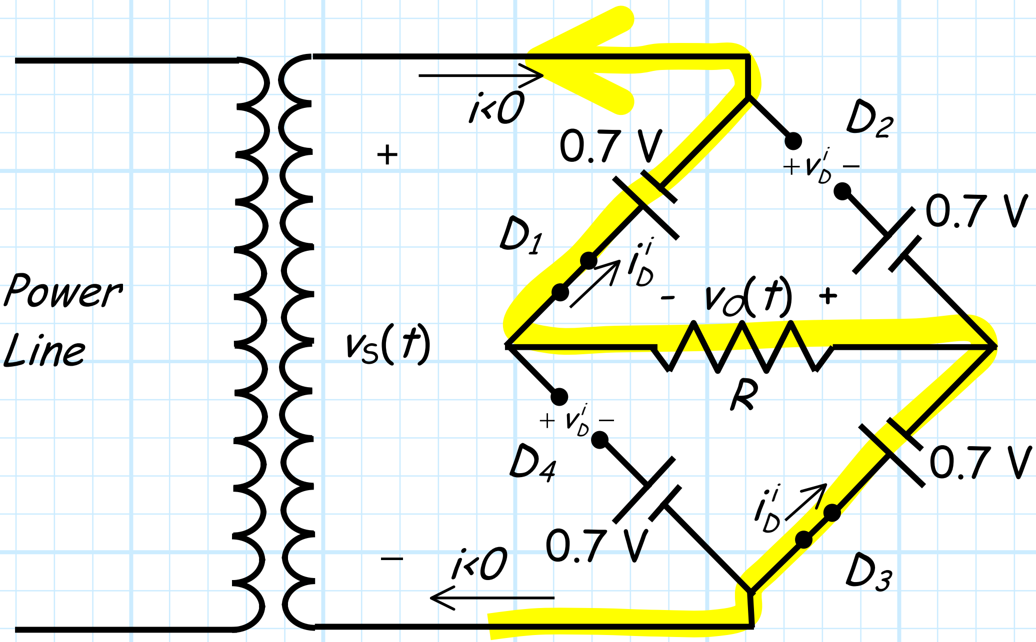
$$v_O = v_S - 1.4 \text{ V} \quad \text{when} \quad v_S > 1.4 \text{V}$$

i <0

The rectifier current i can be **negative** only **if** these assumptions are true:

D_1 and D_3 are forward biased.

D_2 and D_4 are reverse biased.



Analyzing this circuit, we find that the **output voltage** is:

$$v_O = -v_S - 1.4 \text{ V}$$

while the f.b. **ideal diode currents** are both
:

$$-i = i_D^i = \frac{-v_S - 1.4}{R}$$

and the r.b. **ideal diode voltages** are both:

$$v_D^i = v_S$$

Thus, $i_D^i > 0$ when:

$$\frac{-v_S - 1.4}{R} > 0$$

$$-v_S - 1.4 > 0$$

$$-v_S > 1.4 \text{ V}$$

$$v_S < -1.4 \text{ V}$$

and, $v_D^i < 0$ when:

$$v_S < 0$$

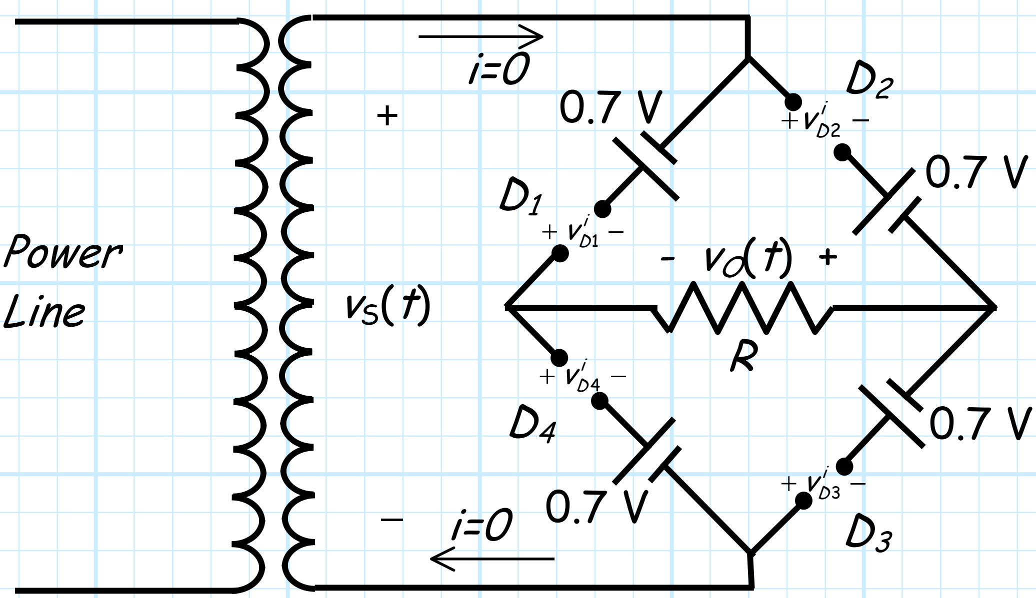
Therefore, we likewise **find** for this circuit:

$$v_O = -v_S - 1.4\text{ V} \quad \text{when} \quad v_S < -1.4\text{V}$$

i =0

The rectifier current *i* can be **zero** only **if** these assumptions are true:

All ideal diodes are **reverse** biased!



Analyzing this circuit, we find that the **output voltage** is:

$$v_O = R\,i = 0$$

while the **ideal diode voltages** of D_2 and D_4 are each:

$$v_{D2}^i = \frac{v_S - 1.4}{2} = v_{D4}^i$$

and the **ideal diode voltages** of D_1 and D_3 are each:

$$v_{D1}^i = \frac{-v_S - 1.4}{2} = v_{D3}^i$$

Thus, $v_{D2}^i < 0$ when:

$$\begin{aligned} \frac{v_S - 1.4}{2} &< 0 \\ v_S - 1.4 &< 0 \\ v_S &< 1.4 \end{aligned}$$

and, $v_{D1}^i < 0$ when:

$$\begin{aligned} \frac{-v_S - 1.4}{2} &< 0 \\ -v_S - 1.4 &< 0 \\ -v_S &< 1.4 \\ v_S &> -1.4 \end{aligned}$$

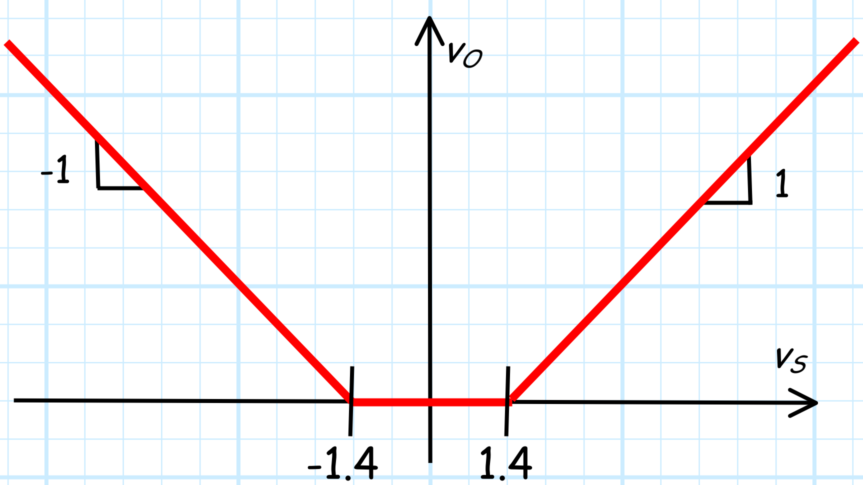
Therefore, we also **find** for this circuit that:

$v_O = 0$ when both $v_S < 1.4\,V$ and $v_S > -1.4\,V$ ($-1.4 < v_S < 1.4\,V$)

Q: *You know, that dang **Mizzou** grad said we only needed to consider these **three** sets of diode assumptions, yet I am **still** concerned about the other 13. How can we be **sure** that we have analyzed every **possible** set of **valid** diode assumptions?*

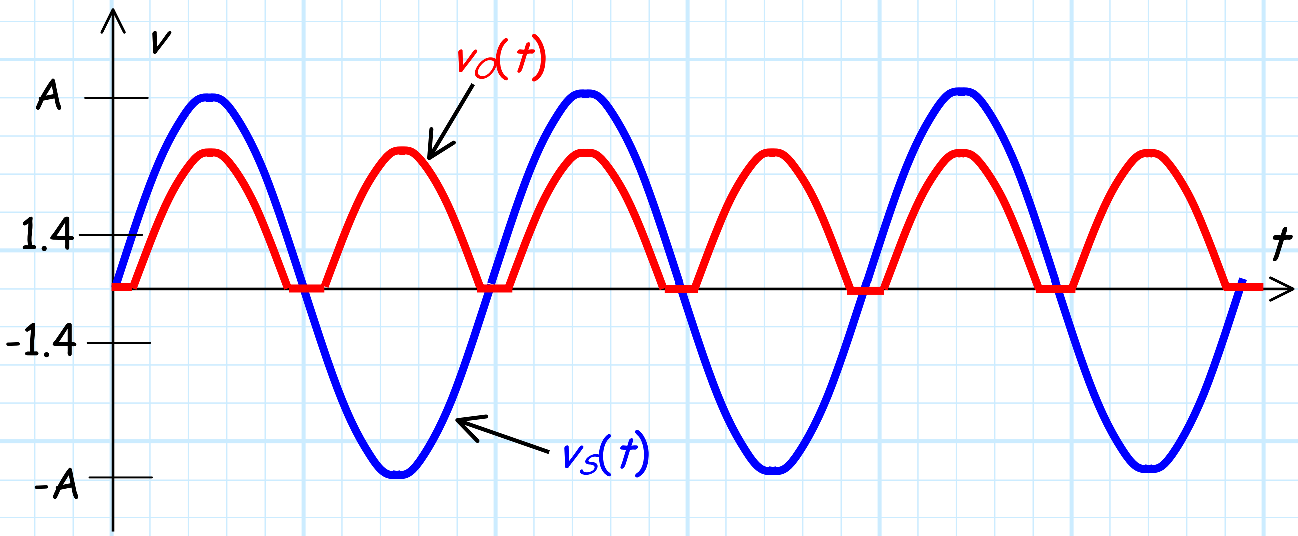
A: We know that we have considered **every** possible case, because when we combine the three results we find that we have a piece-wise linear **function**! I.E.,:

$$v_O = \begin{cases} -v_S - 1.4\text{ V} & \text{if} \quad v_S < -1.4\text{ V} \\ 0 & \text{if} \quad -1.4 < v_S < 1.4\text{ V} \\ v_S - 1.4\text{ V} & \text{if} \quad v_S > 1.4\text{ V} \end{cases}$$



Note that the **bridge** rectifier is a **full-wave** rectifier!

If the input to this rectifier is a **sine wave**, we find that the **output** is approximately that of an ideal **full-wave rectifier**:



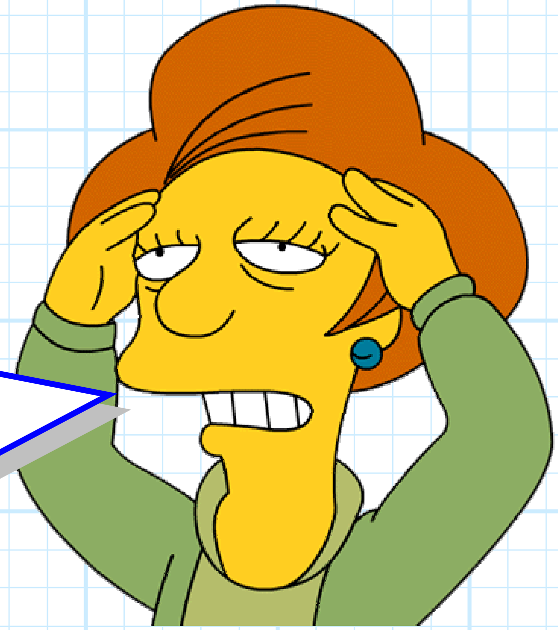
We see that the junction diode bridge rectifier output is **very close** to ideal. In fact, if A>>1.4 V, the **DC component** of this junction diode bridge rectifier is approximately:

$$V_O \approx \frac{2A}{\pi} - 1.4\text{ V}$$

Just 1.4 V less than the **ideal** full-wave rectifier DC component!

Peak Inverse Voltage

Q: *I'm so **confused!** The bridge rectifier and the full-wave rectifier **both** provide full-wave rectification. Yet, the bridge rectifier use **4** junction diodes, whereas the full-wave rectifier only uses **2**. **Why** would we **ever** want to use the **bridge rectifier**?*



A: First, a slight **confession**—the results we derived for the bridge and full-wave rectifiers are **not** precisely correct!

Recall that we used the junction diode **CVD model** to determine the transfer function of each rectifier circuit. The problem is that the CVD model does **not** predict junction diode **breakdown**!

If the **source** voltage v_S becomes too **large**, the junction diodes can in fact **breakdown**—but the transfer functions we derived do **not** reflect this fact!

Q: *Yikes! You mean that we need to **rework** our analysis and find **new** transfer functions!*

A: Fortunately **no**. Breakdown is an **undesirable** mode for circuit rectification. Our job as engineers is to design a rectifier that **avoids** it—that why the **bridge** rectifier is helpful!

To see why, consider the voltage across a **reversed biased** junction diode in **each** of our rectifier circuit designs.

Recall that the voltage across a **reverse biased ideal diode** in the **full-wave rectifier** design was:

$$v_{D2}^{i} = -2v_S$$

so that the voltage across the **junction** diode is approximately:

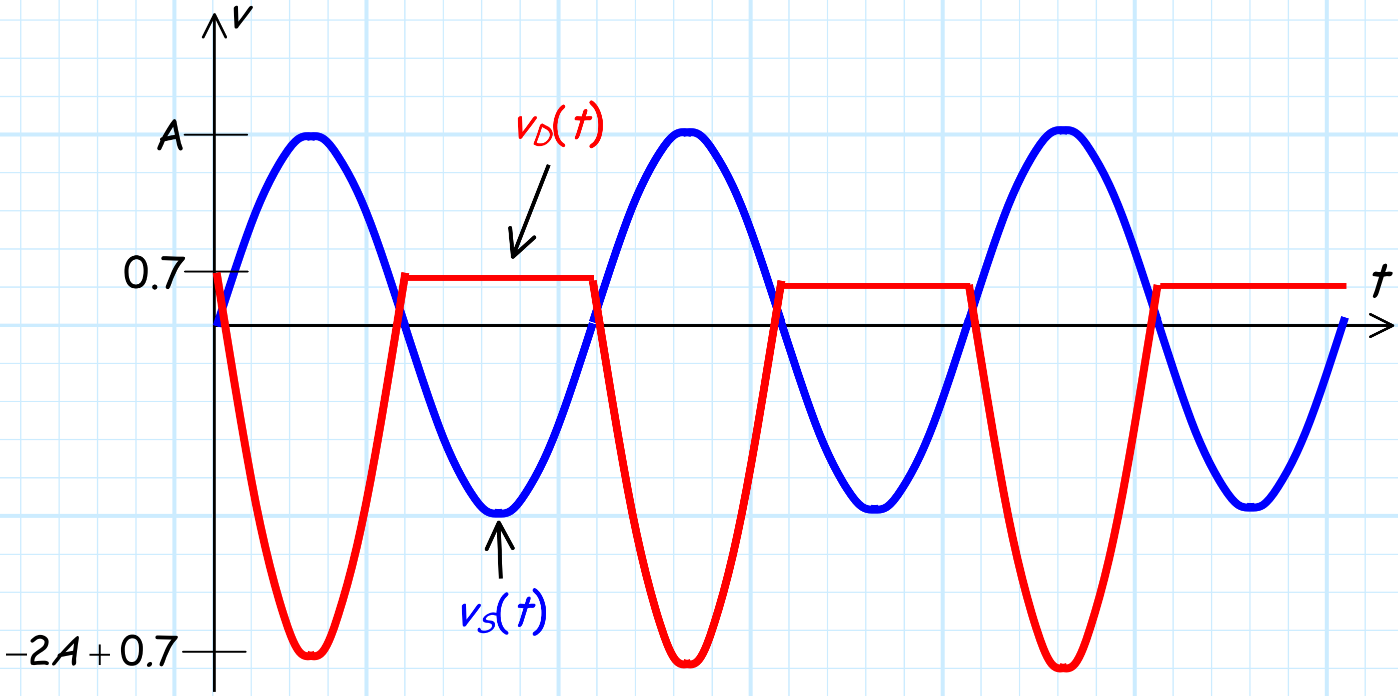
$$\begin{aligned} v_D &= v_D^{i} + 0.7 \\ &= -2v_S + 0.7 \end{aligned}$$

Now, assuming that the **source** voltage is a **sine wave** $v_S = A \sin \omega t$, we find that diode voltage is at it **most negative** (i.e., breakdown danger!) when the **source** voltage is at its **maximum** value A. I.E.,:

$$v_D^{min} = -2A + 0.7$$

Of course, the **largest** junction diode voltage occurs when in **forward** bias:

$$v_D^{max} = 0.7 \text{ V}$$

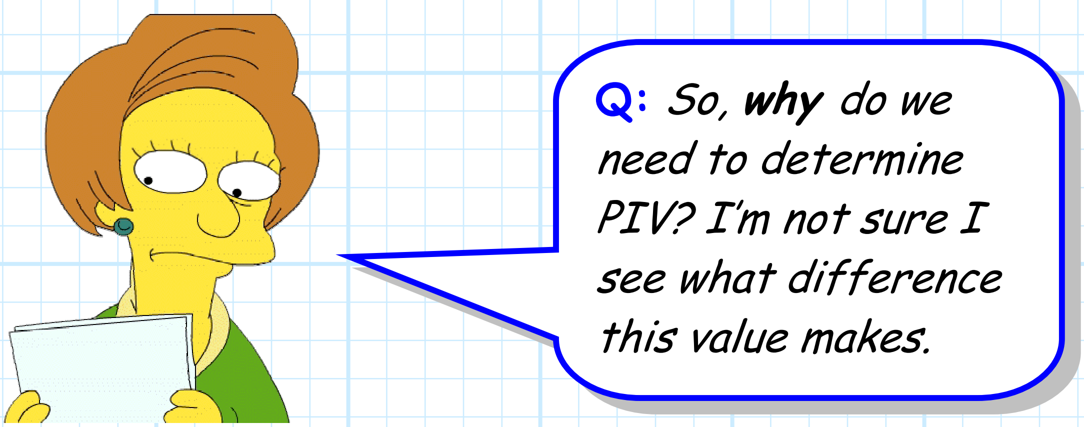


Note that this **minimum** diode voltage v_D is **very negative**, with an absolute value ($\left|v_D^{min}\right| = 2A - 0.7$) nearly **twice** as large as the source magnitude A.

We call the absolute value of the minimum diode voltage the **Peak Inverse Voltage** (PIV):

$$PIV = \left|v_D^{min}\right|$$

Note that this value is dependent on **both** the rectifier design **and** the magnitude of the source voltage v_S.



A: The Peak Inverse Voltage **answers** one important question—**will** the junction diodes in our rectifier **breakdown**?

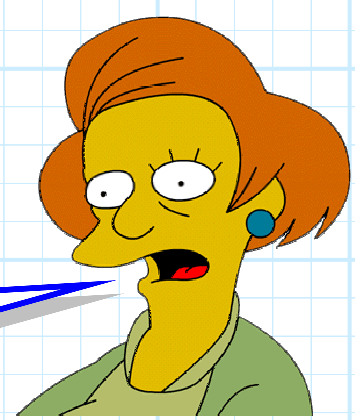
→ **If** the PIV is **less** than the Zener breakdown voltage of our rectifier diodes (i.e., if $PIV < V_{ZK}$), then we know that our junction diodes will **remain** in either forward or reverse bias for all time t. The rectifier will operate "properly"!

→ However, **if** the PIV is **greater** than the Zener breakdown voltage of our rectifier diodes (i.e., if $PIV > V_{ZK}$), then we know that our junction diodes will **breakdown** for at least **some** small amount of time t. The rectifier will **NOT** operate properly!

Q: *So what do we do if PIV **is** greater than V_{ZK}? How do we **fix** this problem?*

A: We have **two** possible **solutions**:

1. Use junction diodes with **larger** values of V_{ZK} (if they exist!).
2. Use the **bridge** rectifier design.



Q: *The **bridge** rectifier! How would that solve our **breakdown** problem?*

A: To see how a **bridge** rectifier can be **useful**, let's determine its Peak Inverse Voltage **PIV**.

First, we recall that the voltage across the **reverse biased ideal diodes** was:

$$v_D^i = -v_S$$

so that the voltage across the **junction** diode is approximately:

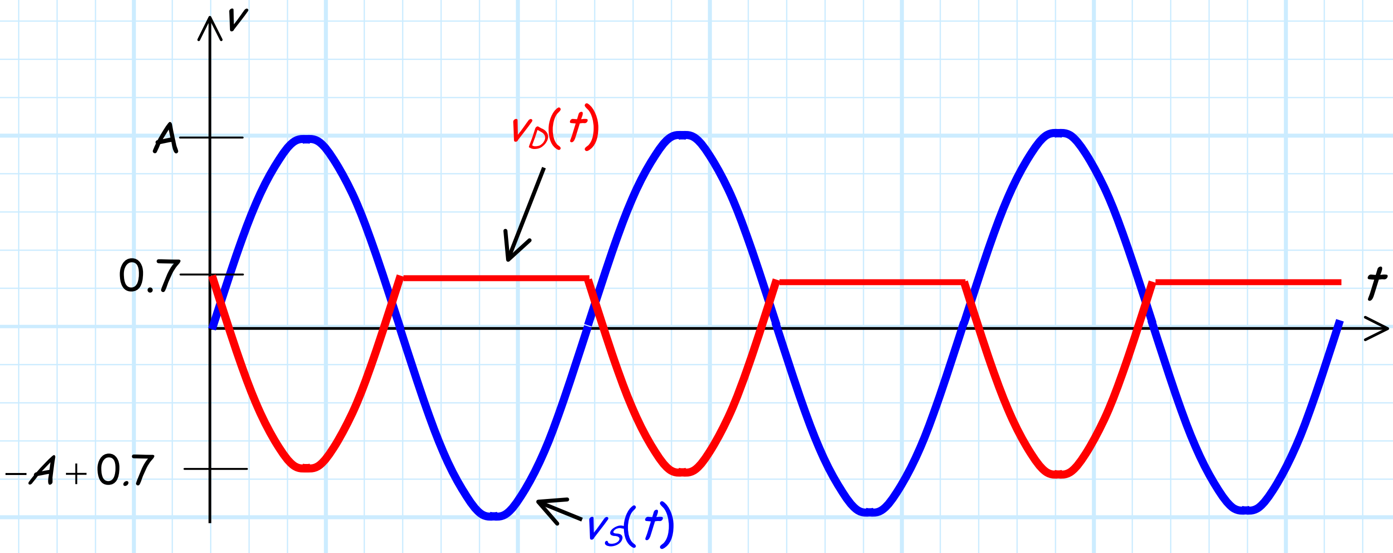
$$\begin{aligned} v_D &= v_D^i + 0.7 \\ &= -v_S + 0.7 \end{aligned}$$

Now, assuming that the **source** voltage is a **sine wave** $v_S = A \sin \omega t$, we find that diode voltage is at it **most negative** (i.e., breakdown danger!) when the **source** voltage is at its **maximum** value A. I.E.,:

$$v_D^{min} = -A + 0.7$$

Of course, the **largest** junction diode voltage occurs when in forward bias:

$$v_D^{max} = 0.7 \text{ V}$$



Note that this minimum diode voltage is **very negative**, with an absolute value ($\left|v_D^{min}\right| = A - 0.7$), approximately **equal** to the value of the **source magnitude** A.

Thus, the **PIV** for a **bridge** rectifier with a **sinusoidal source** voltage is:

$$PIV = A - 0.7$$

Note that this **bridge** rectifier value is approximately **half** the PIV we determined for the **full-wave** rectifier design!

Thus, the source voltage (and the output DC component) of a **bridge** rectifier can be **twice** that of the full-wave rectifier design—this is why the **bridge** rectifier is a very **useful** rectifier design!