Section 5.4 – BJT Circuits at DC

Reading Assignment: pp. 421-436

To analyze a BJT circuit, we follow the same boring procedure as always: ASSUME, ENFORCE, ANALYZE and CHECK.

**HO: Steps for D.C. Analysis of BJT Circuits**

**HO: Hints for BJT Circuit Analysis**

For example:

**Example: D.C. Analysis of a BJT Circuit**

**Example: An Analysis of a pnp BJT Circuit**

**Example: Another DC Analysis of a BJT Circuit**

**Example: A BJT Circuit in Saturation**
Steps for D.C. Analysis of BJT Circuits

To analyze BJT circuit with D.C. sources, we must follow these five steps:

1. **ASSUME** an operating mode

2. **ENFORCE** the equality conditions of that mode.

3. **ANALYZE** the circuit with the enforced conditions.

4. **CHECK** the inequality conditions of the mode for consistency with original assumption. If consistent, the analysis is complete; if inconsistent, go to step 5.

5. **MODIFY** your original assumption and repeat all steps.

Let's look at each step in detail.

1. **ASSUME**

We can ASSUME Active, Saturation, or Cutoff!
2. **ENFORCE**

**Active**

For active region, we must ENFORCE two equalities.

a) Since the base-emitter junction is forward biased in the active region, we ENFORCE these equalities:

\[
\begin{align*}
V_{BE} &= 0.7 \text{ V} \quad (\text{nnp}) \\
V_{EB} &= 0.7 \text{ V} \quad (\text{pnp})
\end{align*}
\]

b) We likewise know that in the active region, the base and collector currents are directly proportional, and thus we ENFORCE the equality:

\[
i_C = \beta i_B
\]

Note we can equivalently ENFORCE this condition with either of the the equalities:

\[
i_C = \alpha i_E \quad \text{or} \quad i_E = (\beta + 1) i_B
\]
**Saturation**

For saturation region, we must likewise ENFORCE two equalities.

a) Since the base-emitter junction is forward biased, we again ENFORCE these equalities:

\[
V_{BE} = 0.7 \text{ V (nnp)}
\]
\[
V_{EB} = 0.7 \text{ V (pnp)}
\]

b) Likewise, since the collector base junction is reverse biased, we ENFORCE these equalities:

\[
V_{CB} = -0.5 \text{ V (nnp)}
\]
\[
V_{BC} = -0.5 \text{ V (pnp)}
\]

Note that from KVL, the above two ENFORCED equalities will require that these equalities likewise be true:

\[
V_{CE} = 0.2 \text{ V (nnp)}
\]
\[
V_{EC} = 0.2 \text{ V (pnp)}
\]
Note that for saturation, you need to explicitly ENFORCE any two of these three equalities—the third will be ENFORCED automatically (via KVL)!!

To avoid negative signs (e.g., $V_{CB}=-0.5$), I typically ENFORCE the first and third equalities (e.g., $V_{BE}=0.7$ and $V_{CE}=0.2$).

**Cutoff**

For a BJT in cutoff, both pn junctions are reverse biased—no current flows! Therefore we ENFORCE these equalities:

$$
i_B = 0 \\
i_C = 0 \\
i_E = 0$$

3. **ANALYZE**

**Active**

The task in D.C. analysis of a BJT in active mode is to find one unknown current and one additional unknown voltage!

a) In addition the relationship $i_C = \beta i_B$, we have a second useful relationship:

$$i_E = i_C + i_B$$
This of course is a consequence of KCL, and is true **regardless** of the BJT mode.

But think about what this means! We have **two** current equations and **three** currents (i.e., $i_E, i_C, i_B$)—we only need to determine **one** current and we can then immediately find the other two!

**Q:** Which current do we need to find?

**A:** Doesn’t matter! For a BJT operating in the active region, if we know **one** current, we know them **all**!

b) In addition to $V_{BE} = 0.7$ ($V_{EB} = 0.7$), we have a **second** useful relationship:

\[
V_{CE} = V_{CB} + V_{BE} \quad \text{(npn)}
\]

\[
V_{EC} = V_{EB} + V_{BC} \quad \text{(pnp)}
\]

This of course is a consequence of KVL, and is true **regardless** of the BJT mode.

Combining these results, we find:

\[
V_{CE} = V_{CB} + 0.7 \quad \text{(npn)}
\]

\[
V_{EC} = 0.7 + V_{BC} \quad \text{(pnp)}
\]
But think about what this means! If we find one unknown voltage, we can immediately determine the other.

Therefore, a D.C. analysis problem for a BJT operating in the active region reduces to:

find one of these values

\[ i_B, i_C, \text{ or } i_E \]

and find one of these values

\[ V_{CE} \text{ or } V_{CB} \quad (V_{EC} \text{ or } V_{BC}) \]

**Saturation**

For the saturation mode, we know all the BJT voltages, but know nothing about BJT currents!

Thus, for an analysis of circuit with a BJT in saturation, we need to find any two of the three quantities:

\[ i_B, i_C, i_E \]

We can then use KCL to find the third.

**Cutoff**

Cutoff is a bit of the opposite of saturation—we know all the BJT currents (they're all zero!), but we know nothing about BJT voltages!
Thus, for an analysis of circuit with a BJT in cutoff, we need to find any two of the three quantities:

\[ V_{BE}, V_{CB}, V_{CE} \quad (npn) \]

\[ V_{EB}, V_{BC}, V_{EC} \quad (pnp) \]

We can then use KVL to find the third.

4. **CHECK**

You do not know if your D.C. analysis is correct unless you CHECK to see if it is consistent with your original assumption!

**WARNING!**-Failure to CHECK the original assumption will result in a SIGNIFICANT REDUCTION in credit on exams, regardless of the accuracy of the analysis !!!

**Q:** What exactly do we CHECK?

**A:** We ENFORCED the mode equalities, we CHECK the mode inequalities.

*Active*

We must CHECK two separate inequalities after analyzing a circuit with a BJT that we ASSUMED to be operating in active mode. One inequality involves BJT voltages, the other BJT currents.
a) In the *active* region, the Collector-Base Junction is “off” (i.e., *reverse* biased). Therefore, we must CHECK our analysis results to see if they are *consistent* with:

\[
V_{CB} > 0 \quad \text{(nnp)} \\
V_{BC} > 0 \quad \text{(pnp)}
\]

Since \(V_{CE} = V_{CB} + 0.7\), we find that an *equivalent* inequality is:

\[
V_{CE} > 0.7 \quad \text{(nnp)} \\
V_{EC} > 0.7 \quad \text{(pnp)}
\]

We need to check only one of these two inequalities (*not both!*).

b) In the active region, the Base-Emitter Junction is “on” (i.e., *forward* biased). Therefore, we must CHECK the results of our analysis to see if they are *consistent* with:

\[
i_B > 0
\]
Since the active mode constants $\alpha$ and $\beta$ are always positive values, equivalent expressions to the one above are:

$$i_C > 0 \quad \text{and} \quad i_E > 0$$

In other words, we need to CHECK and see if any one of the currents is positive—if one is positive, they are all positive!

**Saturation**

Here we must CHECK inequalities involving BJT currents.

a) We know that for saturation mode, the ratio of collector current to base current will be less than beta! Thus we CHECK:

$$i_C < \beta i_B$$

b) We know that both pn junctions are forward biased, hence we CHECK to see if all the currents are positive:

$$i_B > 0$$
$$i_C > 0$$
$$i_E > 0$$
For cutoff we must CHECK two BJT voltages.

a) Since the EBJ is reverse biased, we CHECK:

\[
V_{BE} < 0 \quad (npn)
\]
\[
V_{EB} < 0 \quad (pnp)
\]

b) Likewise, since the CBJ is also reverse biased, we CHECK:

\[
V_{CB} > 0 \quad (npn)
\]
\[
V_{BC} > 0 \quad (pnp)
\]

If the results of our analysis are consistent with each of these inequalities, then we have made the correct assumption! The numeric results of our analysis are then likewise correct. We can stop working!

However, if even one of the results of our analysis is inconsistent with active mode (e.g., currents are negative, or \( V_{CE} < 0.7 \)), then we have made the wrong assumption! Time to move to step 5.
5. **MODIFY**

If one or more of the BJT\s are *not* in the active mode, then it must be in either **cutoff** or **saturation**. We must change our assumption and start **completely** over!

In general, all of the results of our previous analysis are incorrect, and thus must be **completely** scraped!
**Hints for BJT Circuit Analysis**

1. Know the BJT **symbols** and **current/voltage** definitions!

2. Know what **quantities** must be determined for each assumption (e.g., for active mode, you must determine one BJT current and one BJT voltage).

3. Write **separate** equations for the BJT (device) and the remainder of the circuit (KVL, KCL, Ohm’s Law).

4. Write the KVL equation for the circuit’s “**Base-Emitter Leg**”. In other words, write a KVL that includes \( v_{BE} \).
5. Forget about what the problem is asking for! Just start by determining any and all the circuit quantities that you can. If you end up solving the entire circuit, the answer will be there somewhere!

6. If you get stuck, try working the problem backward! For example, to find a resistor value, you must find the voltage across it and the current through it.

7. Make sure you are using all the information provided in the problem!
Example: D.C. Analysis of a BJT Circuit

Consider again this circuit from lecture:

\[\beta = 99\]

**Q:** What is \(I_B\), \(I_C\), \(I_E\) and also \(V_{CE}\), \(V_{CB}\), \(V_{BE}\)??

**A:** I don't know! But, we can find out—IF we complete each of the five steps **required** for BJT DC analysis.
**Step 1** - **ASSUME** an operating mode.

Let's **ASSUME** the BJT is in the **ACTIVE** region!

Remember, this is just a **guess**; we have no way of knowing for sure what mode the BJT is in at this point.

**Step 2** - **ENFORCE** the conditions of the assumed mode.

For **active** region, these are:

\[
V_{BE} = 0.7 \, V \quad \text{and} \quad I_C = \beta I_B = 99 \, I_B
\]

**Step 3** - **ANALYZE** the circuit.

This is the **BIG** step!

**Q:** Where do we even start?

**A:** Recall what the hint sheet says:

“Write KVL equations for the base-emitter “leg”

I think we should try that!
The base-emitter KVL equation is:

\[ 5.7 - 10 I_B - V_{BE} - 2 I_E = 0 \]

This is the circuit equation; note that it contains 3 unknowns \((I_B, I_C, \text{and } V_{BE})\).

Now let's add the relevant device equations:

\[ V_{BE} = 0.7 \text{ V} \]

\[ I_E = (\beta + 1) I_B = 100 I_B \]

Look what we now have! 3 equations and 3 unknowns (this is a good thing).

Inserting the device equations into the B-E KVL:

\[ 5.7 - 10 I_B - 0.7 - 2(99+1)I_B = 0 \]

Therefore:

\[ 5.0 - 210 I_B = 0 \] 1 equations and 1 unknown!
Solving, we get:

\[ I_B = \frac{5.0}{210} = 23.8 \mu A \]

**Q:** Whew! That was an awful lot of work for just one current, and we still have two more currents to find.

**A:** No we don't! Since we determined one current for a BJT in active mode, we've determined them all!

I.E.,

\[ I_C = \beta I_B = 2.356 \text{ mA} \]

\[ I_E = (\beta + 1) I_B = 2.380 \text{ mA} \]

(Note that \( I_C + I_B = I_E \))

Now for the voltages!

Since we know the currents, we can find the voltages using KVL.

For example, let's determine \( V_{CE} \). We can do this either by finding the voltage at the collector \( V_C \) (wrt ground) and voltage at the emitter \( V_E \) (wrt ground) and then subtracting \( (V_{CE} = V_C - V_E) \).

OR, we can determine \( V_{CE} \) directly from the C-E KVL equation.
\[ V_C = 10.7 - I_C \quad (1) \]
\[ = 10.7 - 2.36 \]
\[ = 8.34 \text{ V} \]

and:

\[ V_E = 0 + I_E \quad (2) \]
\[ = 0 + 4.76 \]
\[ = 4.76 \text{ V} \]

Therefore,

\[ V_{CE} = V_C - V_E = 3.58 \text{ V} \]

Note we could have likewise written the C-E KVL:

\[ 10.7 - I_C \quad (1) - V_{CE} - I_E \quad (2) = 0 \]

Therefore,

\[ V_{CE} = 10.7 - I_C \quad (1) - I_E \quad (2) = 3.58 \text{ V} \]

Q: So, I guess we write the collector-base KVL to find \( V_{CB} \)?

A: You can, but a wiser choice would be to simply apply KVL to the transistor!
I.E., $V_{CE} = V_{CB} + V_{BE}$ !!

Therefore $V_{CB} = V_{CE} - V_{BE} = 2.88 \, \text{V}$

**Q:** This has been hard. I'm glad we're finished!

**A:** Finished! We still have 2 more steps to go!

**Step 4** - CHECK to see if your results are consistent with your assumption.

For active mode:

$V_{CE} = 3.58 \, \text{V} > 0.7 \, \text{V} \quad \checkmark$

$I_B = 23.8 \, \mu\text{A} > 0.0 \quad \checkmark$

Are assumption was correct, and therefore so are our answers!

No need to go on to Step 5.
Example: An Analysis of a pnp BJT Circuit

Determine the collector current and collector voltage of the BJT in the circuit below.

1. ASSUME the BJT is in active mode.

2. ENFORCE the conditions:
   
   $$V_{EB} = 0.7 \text{ V} \quad \text{and} \quad i_c = \beta i_b$$

3. ANALYZE the circuit.

Q: Yikes! How do we write the base-emitter KVL?

A: This is a perfect opportunity to apply the Thevenin's equivalent circuit!
Thevenin's equivalent circuit:

\[ V_{oc} = \frac{10 \cdot 40}{40+10} = 8.0 \, \text{V} \]

\[ I_{sc} = \frac{10}{10} = 1 \, \text{mA} \]

Where \( V_{th} = V_{oc} = 8.0 \, \text{V} \) and \( R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{8}{1} = 8 \, \text{K} \)
Therefore, we can write the BJT circuit as:

\[
\begin{align*}
&\text{NOW we can easily write the emitter-base leg KVL:} \\
&10.7 - 2i_E - v_{EB} - 8i_B = 8.0
\end{align*}
\]

Along with our enforced conditions, we now have three equations and three unknowns!

Combining, we find:

\[
10.7 - 2(96)i_B - 0.7 - 8i_B = 8.0
\]

Therefore,

\[
i_B = \frac{10.7 - 0.7 - 8.0}{2(96) + 8} = \frac{2}{200} = 0.01 \text{ mA}
\]

and collector current \(i_C\) is:

\[
i_C = \beta i_B = 95(0.01) = 0.95 \text{ mA}
\]

Likewise, the collector voltage (wrt ground) \(V_C\) is:

\[
V_C = 0.0 + 4i_C = 3.8 \text{ V}
\]
But wait! We're not done yet! We must CHECK our assumption.

First, $i_B = 0.01 \text{ mA} > 0$

But, what is $V_{EC}$??

Writing the emitter-collector KVL:

$$10.7 - 2i_E - V_{CE} - 4i_C = 0$$

Therefore,

$$V_{EC} = 10.7 - 2(96)(0.01) - 4(0.95) = 4.98 \text{ V} > 0.7 \text{ V}$$

Our assumption was correct!
Example: Another DC Analysis of a BJT Circuit

Find the collector voltages of the two BJTs in the circuit below.

ASSUME both BJTs are in active mode, therefore ENFORCE

\[ V_{EB}^1 = V_{EB}^2 = 0.7 \, V, \quad i_c^1 = 100 \, i_b^1, \quad \text{and} \quad i_c^2 = 100 \, i_b^2 \]
Q: Now, how do we ANALYZE the circuit??

A: This seems to be a problem! We cannot easily solve the emitter base KVL, as \( i_1 \) is NOT EQUAL to \( i_{E1} \) (make sure you understand this!). Instead, we find:

\[ i_{E1} = i_1 + i_{b2} \]

So, what do we do?

First, ask the question: What do we know??

Look closely at the circuit, it is apparent that \( V_{B1} = 5.3 \, \text{V} \) and \( V_{E2} = 7.7 \, \text{V} \).

Hey! We therefore also know \( V_{E1} \) and \( V_{B2} \):

\[ V_{E1} = V_{B1} + V_{\text{BE}} = 5.3 + 0.7 = 6.0 \, \text{V} \]
\[ V_{B2} = V_{E2} - V_{\text{BE}} = 7.7 - 0.7 = 7.0 \, \text{V} \]

Wow! From these values we get:

\[ i_1 = \frac{10 - V_{E1}}{1} = \frac{10 - 6}{1} = 4 \, \text{mA} \]

and

\[ i_{b2} = \frac{V_{B2} - V_{E1}}{50} = \frac{7 - 6}{50} = 0.02 \, \text{mA} \]
This is easy! Since we know $i_1$ and $i_{b2}$, we can find $i_{e1}$:

$$i_{e1} = i_1 + i_{b2} = 4.0 + 0.02 = 4.02 \text{ mA}$$

Since we know one current for each BJT, we know all currents for each BJT:

$$i_{c1} = \alpha \frac{\beta}{\beta + 1} i_{e1} = \frac{100}{101} \times 4.02 = 3.98 \text{ mA}$$

$$i_{c2} = \beta i_{b2} = 100(0.02) = 2 \text{ mA}$$

Finally, we can determine the voltages $V_{c1}$ and $V_{c2}$.

$$V_{c1} = 0.0 + 1 i_{c1} = 0.0 + 1(3.98) = 3.98 \text{ V}$$

$$V_{c2} = 0.0 + 1 i_{c2} = 0.0 + 1(2.0) = 2.0 \text{ V}$$

Now, let’s CHECK to see if our assumptions were correct:

$$i_{c2} = 2 \text{ mA} > 0 \sbullet \quad i_{c1} = 3.98 \text{ mA} > 0 \sbullet$$

$$V_{ec}^1 = V_{e1} - V_{c1} = 6.0 - 3.98 = 2.02 \text{ V} > 0.7 \text{ V} \sbullet$$

$$V_{bc}^2 = V_{b1} - V_{c1} = 7.0 - 2.0 = 5.0 \text{ V} > 0 \sbullet$$

Assumptions are correct!
Example: A BJT Circuit in Saturation

Determine all currents for the BJT in the circuit below.

Hey! I remember this circuit, it's just like a previous example. The BJT is in active mode!

Let's see if you are correct! Assume it is in active mode and enforce $V_{CE} = 0.7$ V and $i_C = \beta \cdot i_B$.

The B-E KVL is therefore:

$$5.7 - 10 \cdot i_B - 0.7 - 2 \cdot (99+1) \cdot i_B = 0$$

Therefore $i_B = 23.8 \mu A$
See! Base current \( i_B = 23.8 \mu A \), just like before. Therefore collector current and emitter current are again \( i_C = 99i_B = 2.356 \text{ mA} \) and \( i_E = 100i_B = 2.380 \text{ mA} \). Right?! 

Well maybe, but we still need to CHECK to see if our assumption is correct!

We know that \( i_B = 23.8 \mu A > 0 \), but what about \( V_{CE} \)?

From collector-emitter KVL we get:

\[
10.7 - 10i_C - V_{CE} - 2i_E = 0
\]

Therefore,

\[
V_{CE} = 10.7 - 10(2.36) - 2(2.38) = -17.66 \text{ V} < 0.7 \text{ V} \quad \times
\]

Our assumption is wrong! The BJT is not in active mode.

In the previous example, the collector resistor was 1K, whereas in this example the collector resistor is 10K. Thus, there is 10X the voltage drop across the collector resistor, which lowers the collector voltage so much that the BJT cannot remain in the active mode.
Q: So what do we do now?

A: Go to Step 5; change the assumption and try it again!

Let's **assume** instead that the BJT is in **saturation**. Thus, we **enforce** the conditions:

\[
V_{CE} = 0.2 \text{ V} \quad V_{BE} = 0.7 \text{ V} \quad V_{CB} = -0.5 \text{ V}
\]

Now let's **analyze** the circuit!

Note that we **cannot** directly determine the currents, as we **do not** know the base voltage, emitter voltage, or collector voltage.

But, we **do** know the **differences** in these voltages!

For example, we know that the collector voltage is 0.2 V **higher** than the emitter voltage, but we **do not** know what the collector or emitter voltages are!
Q: So, how the heck do we ANALYZE this circuit!?

A: Often, circuits with BJTs in saturation are somewhat more difficult to ANALYZE than circuits with active BJTs. There are often many approaches, but all result from a logical, systematic application of Kirchoff’s Laws!

ANALYSIS EXAMPLE 1 - Start with KCL

We know that \( i_B + i_C = i_E \) (KCL)

But, what are \( i_B, i_C, \) and \( i_E \) ?

Well, from Ohm’s Law:

\[
i_B = \frac{5.7 - V_B}{10} \quad i_c = \frac{10.7 - V_C}{10} \quad i_E = \frac{V_E - 0}{10}
\]

Therefore, combining with KCL:

\[
\frac{5.7 - V_B}{10} + \frac{10.7 - V_C}{10} = \frac{V_E}{10}
\]

Look what we have, 1 equation and 3 unknowns.

We need 2 more independent equations involving \( V_B, V_C, \) and \( V_E \)!
Q: Two more independent equations!? It looks to me as if we have written all that we can about the circuit using Kirchoff’s Laws.

A: True! There are no more independent circuit equations that we can write using KVL or KCL! But, recall the hint sheet:

"Make sure you are using all available information".

There is more information available to us – the ENFORCED conditions!

\[
\begin{align*}
V_{CE} &= V_C - V_E = 0.2 \\ V_C &= V_E + 0.2 \\
V_{BE} &= V_B - V_E = 0.7 \\ V_B &= V_E + 0.7
\end{align*}
\]

Two more independent equations! Combining with the earlier equation:

\[
\frac{5.7 - (0.7 + V_E)}{10} + \frac{10.7 - (0.2 + V_E)}{10} = \frac{V_E}{10}
\]

One equation and one unknown! Solving, we get \( V_E = 2.2 \) V.

Inserting this answer into the above equations, we get:

\[
\begin{align*}
V_B &= 2.9 \text{ V} \\
V_C &= 2.4 \text{ V} \\
i_C &= 0.83 \text{ mA} \\
i_B &= 0.28 \text{ mA} \\
i_E &= 1.11 \text{ mA}
\end{align*}
\]
**ANALYSIS EXAMPLE 2** - Start with KVL

We can write the KVL equation for any two circuit legs:

**B-E KVL:**

\[5.7 - 10i_B - 0.7 - 2i_E = 0.0\]

**C-E KVL:**

\[10.7 - 10i_C - 0.2 - 2i_E = 0.0\]

Note the ENFORCED conditions are included in these KVL equations.

Simplifying, we get these 2 equations with 3 unknowns:

\[5.0 = 10i_B + 2i_E\]

\[10.5 = 10i_C + 2i_E\]

We need one more independent equation involving \(i_B\), \(i_C\), and \(i_E\).
Try KCL! \[ i_B + i_C = i_E \]

Inserting the KCL equation into the 2 KVL equations, we get:

\[ 5.0 = 12 i_B + 2 i_C \]

\[ 10.5 = 2 i_B + 12 i_C \]

Solving, we get the same answers as in analysis example 1.

**Lesson:** There are multiple strategies for analyzing these circuits; use the ones that you feel most comfortable with!

However you ANALYZE the circuit, you **must** in the end also CHECK your results.

First CHECK to see that all currents are positive:

\[ i_C = 0.83 \text{ mA} > 0 \checkmark \quad i_B = 0.28 \text{ mA} > 0 \checkmark \quad i_E = 1.11 \text{ mA} > 0 \checkmark \]

Also CHECK collector current:

\[ i_C = 0.83 \text{ mA} < \beta \cdot i_B = 27.7 \text{ mA} \checkmark \]

Our solution is correct !!!