

Section 5.4 - BJT Circuits at DC

Reading Assignment: pp. 421-436

To analyze a BJT circuit, we follow the **same** boring procedure as always: ASSUME, ENFORCE, ANALYZE and CHECK.

HO: Steps for D.C. Analysis of BJT Circuits

HO: Hints for BJT Circuit Analysis

For example:

Example: D.C. Analysis of a BJT Circuit

Example: An Analysis of a *pnp* BJT Circuit

Example: Another DC Analysis of a BJT Circuit

Example: A BJT Circuit in Saturation

Steps for D.C. Analysis of BJT Circuits

To analyze BJT circuit with D.C. sources, we **must** follow these **five steps**:

1. **ASSUME** an operating mode
2. **ENFORCE** the equality conditions of that mode.
3. **ANALYZE** the circuit with the enforced conditions.
4. **CHECK** the inequality conditions of the mode for consistency with original assumption. If consistent, the analysis is complete; if inconsistent, go to step 5.
5. **MODIFY** your original assumption and repeat all steps.

Let's look at each step in **detail**.

1. **ASSUME**

We can **ASSUME** Active, Saturation, or Cutoff!

2. ENFORCE

Active

For **active** region, we must ENFORCE **two equalities**.

a) Since the base-emitter junction is **forward** biased in the active region, we ENFORCE these equalities:

$$V_{BE} = 0.7 \text{ V} \quad (\text{nnp})$$

$$V_{EB} = 0.7 \text{ V} \quad (\text{pnp})$$

b) We likewise know that in the **active** region, the base and collector currents are directly proportional, and thus we ENFORCE the equality:

$$i_C = \beta i_B$$

Note we can **equivalently** ENFORCE this condition with either of the the equalities:

$$i_C = \alpha i_E \quad \text{or} \quad i_E = (\beta + 1) i_B$$

Saturation

For **saturation** region, we must likewise **ENFORCE two equalities**.

a) Since the base-emitter junction is **forward** biased, we again **ENFORCE** these equalities:

$$V_{BE} = 0.7 \text{ V} \quad (\text{npn})$$

$$V_{EB} = 0.7 \text{ V} \quad (\text{pnp})$$

b) Likewise, since the collector base junction is **reverse** biased, we **ENFORCE** these equalities:

$$V_{CB} = -0.5 \text{ V} \quad (\text{npn})$$

$$V_{BC} = -0.5 \text{ V} \quad (\text{pnp})$$

Note that from **KVL**, the above two **ENFORCED** equalities will require that these equalities **likewise** be true:

$$V_{CE} = 0.2 \text{ V} \quad (\text{npn})$$

$$V_{EC} = 0.2 \text{ V} \quad (\text{pnp})$$

Note that for saturation, you need to explicitly ENFORCE any **two** of these **three** equalities—the third will be ENFORCED automatically (via KVL)!!

To avoid **negative** signs (e.g., $V_{CB} = -0.5$), I typically ENFORCE the **first** and **third** equalities (e.g., $V_{BE} = 0.7$ and $V_{CE} = 0.2$).

Cutoff

For a BJT in cutoff, both *pn* junctions are **reverse** biased—no current flows! Therefore we ENFORCE these equalities:

$$i_B = 0$$

$$i_C = 0$$

$$i_E = 0$$

3. ANALYZE

Active

The task in D.C. analysis of a BJT in **active** mode is to find **one** unknown **current** and **one** additional unknown **voltage**!

a) In addition the relationship $i_C = \beta i_B$, we have a **second** useful relationship:

$$i_E = i_C + i_B$$

This of course is a consequence of KCL, and is true **regardless** of the BJT mode.

But think about what this means! We have **two** current equations and **three** currents (i.e., i_E, i_C, i_B)—we only need to determine **one** current and we can then immediately find the other two!

Q: *Which current do we need to find?*

A: Doesn't matter! For a BJT operating in the active region, if we know **one** current, we know them **all**!

b) In addition to $V_{BE} = 0.7$ ($V_{EB} = 0.7$), we have a **second** useful relationship:

$$V_{CE} = V_{CB} + V_{BE} \quad (\text{nnp})$$

$$V_{EC} = V_{EB} + V_{BC} \quad (\text{pnp})$$

This of course is a consequence of KVL, and is true **regardless** of the BJT mode.

Combining these results, we find:

$$V_{CE} = V_{CB} + 0.7 \quad (\text{nnp})$$

$$V_{EC} = 0.7 + V_{BC} \quad (\text{pnp})$$

But think about what **this** means! If we find **one** unknown voltage, we can immediately determine the **other**.

Therefore, a D.C. analysis problem for a BJT operating in the active region reduces to:

find one of these values

$$i_B, i_C, \text{ or } i_E$$

and find one of these values

$$V_{CE} \text{ or } V_{CB} \quad (V_{EC} \text{ or } V_{BC})$$

Saturation

For the saturation mode, we know **all** the BJT **voltages**, but know nothing about BJT **currents**!

Thus, for an analysis of circuit with a BJT in saturation, we need to find any **two** of the **three** quantities:

$$i_B, i_C, i_E$$

We can then use **KCL** to find the third.

Cutoff

Cutoff is a bit of the **opposite** of saturation—we know **all** the BJT **currents** (they're all **zero**!), but we know **nothing** about BJT **voltages** !

Thus, for an analysis of circuit with a BJT in cutoff, we need to find any **two** of the **three** quantities:

$$V_{BE}, V_{CB}, V_{CE} \quad (\text{npn})$$

$$V_{EB}, V_{BC}, V_{EC} \quad (\text{pnp})$$

We can then use **KVL** to find the third.

4. CHECK

You do not know if your D.C. analysis is correct unless you **CHECK** to see if it is consistent with your original assumption!

WARNING!-Failure to **CHECK** the original assumption will result in a **SIGNIFICANT REDUCTION** in credit on exams, regardless of the accuracy of the analysis !!!

Q: *What exactly do we CHECK?*

A: We **ENFORCED** the mode **equalities**, we **CHECK** the mode **inequalities**.

Active

We must **CHECK two** separate inequalities after analyzing a circuit with a BJT that we **ASSUMED** to be operating in **active** mode. One inequality involves BJT **voltages**, the other BJT **currents**.

a) In the **active** region, the Collector-Base Junction is "off" (i.e., **reverse** biased). Therefore, we must **CHECK** our analysis results to see if they are **consistent** with:

$$V_{CB} > 0 \quad (\text{nnp})$$

$$V_{BC} > 0 \quad (\text{pnp})$$

Since $V_{CE} = V_{CB} + 0.7$, we find that an **equivalent** inequality is:

$$V_{CE} > 0.7 \quad (\text{nnp})$$

$$V_{EC} > 0.7 \quad (\text{pnp})$$

We need to check **only** one of these two inequalities (**not both!**).

b) In the active region, the Base-Emitter Junction is "on" (i.e., **forward** biased). Therefore, we must **CHECK** the results of our analysis to see if they are **consistent** with:

$$i_B > 0$$

Since the active mode constants α and β are **always** positive values, **equivalent** expressions to the one above are:

$$i_C > 0 \quad \text{and} \quad i_E > 0$$

In other words, we need to **CHECK** and see if **any** one of the currents is positive—if one is positive, they are **all** positive!

Saturation

Here we must **CHECK** inequalities involving BJT **currents**.

a) We know that for saturation mode, the ratio of collector current to base current will be **less than beta!** Thus we **CHECK**:

$$i_C < \beta i_B$$

b) We know that **both** *pn* junctions are **forward** biased, hence we **CHECK** to see if all the **currents are positive**:

$$i_B > 0$$

$$i_C > 0$$

$$i_E > 0$$

Cutoff

For **cutoff** we must **CHECK** two BJT voltages.

a) Since the EBJ is **reverse biased**, we **CHECK**:

$$V_{BE} < 0 \quad (\text{nnp})$$

$$V_{EB} < 0 \quad (\text{pnp})$$

b) Likewise, since the CBJ is also **reverse biased**, we **CHECK**:

$$V_{CB} > 0 \quad (\text{nnp})$$

$$V_{BC} > 0 \quad (\text{pnp})$$

If the results of our analysis are consistent with **each** of these inequalities, then we have made the **correct** assumption! The numeric results of our analysis are then likewise correct. We can stop working!

However, if **even one** of the results of our analysis is **inconsistent** with active mode (e.g., currents are negative, or $V_{CE} < 0.7$), then we have made the **wrong** assumption! Time to move to step 5.

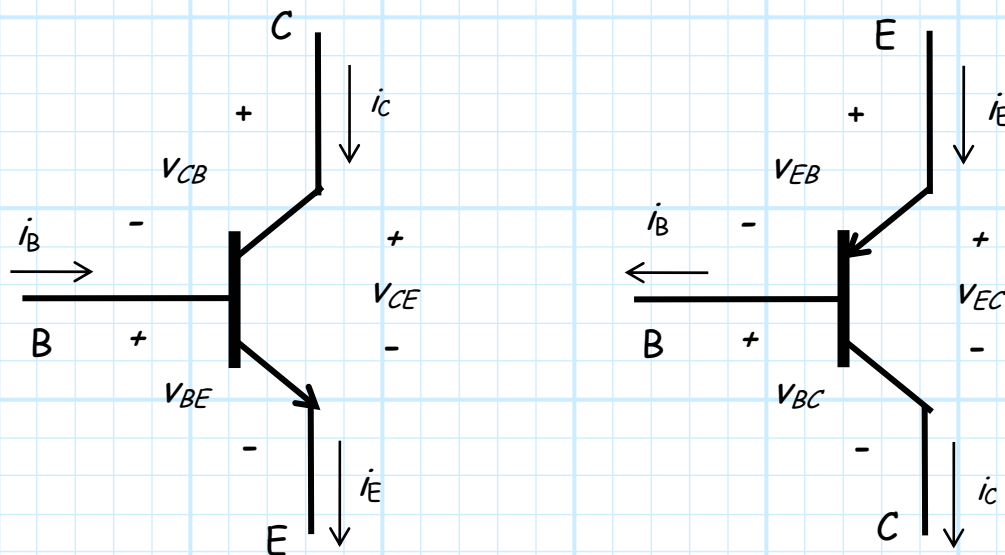
5. *MODIFY*

If one or more of the BJTs are **not** in the active mode, then it must be in either **cutoff** or **saturation**. We must change our assumption and start **completely** over!

In general, **all** of the results of our previous analysis are incorrect, and thus must be **completely** scraped!

Hints for BJT Circuit Analysis

1. Know the BJT symbols and current/voltage definitions!

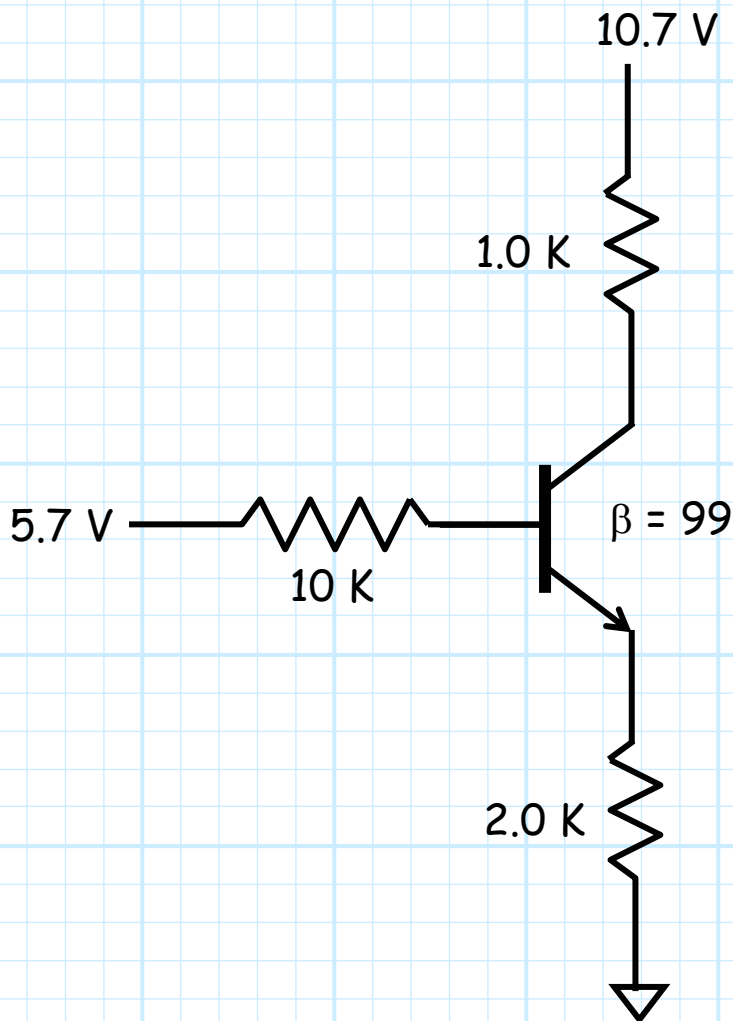


2. Know what **quantities** must be determined for each **assumption** (e.g., for active mode, you must determine one BJT current and one BJT voltage).
3. Write **separate** equations for the BJT (device) and the remainder of the circuit (KVL, KCL, Ohm's Law).
4. Write the KVL equation for the circuit's "**Base-Emitter Leg**". In other words, write a KVL that includes V_{BE} .

5. Forget about what the problem is asking for! Just start by determining **any** and **all** the circuit quantities that you can. If you end up solving the **entire** circuit, the answer will be there somewhere!
6. If you get stuck, try working the problem **backward!** For example, to find a resistor value, you must find the voltage across it and the current through it.
7. Make sure you are using **all** the information provided in the problem!

Example: D.C. Analysis of a BJT Circuit

Consider **again** this circuit from lecture:

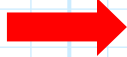


Q: What is I_B , I_C , I_E and also V_{CE} , V_{CB} , V_{BE} ??

A: I don't know! But, we can find out—IF we complete **each** of the five steps **required** for BJT DC analysis.

Step 1 - **ASSUME** an operating mode.

Let's **ASSUME** the BJT is in the **ACTIVE** region !

 Remember, this is just a **guess**; we have no way of knowing for sure what mode the BJT is in at this point.

Step 2 - **ENFORCE** the conditions of the assumed mode.

For **active** region, these are:

$$V_{BE} = 0.7 \text{ V} \quad \text{and} \quad I_C = \beta I_B = 99 I_B$$


Step 3 - **ANALYZE** the circuit.

This is the **BIG** step !

Q: *Where do we even start ?*

A: Recall what the hint sheet says:

"Write KVL equations for the base-emitter "leg"

 I think we should try that !

The **base-emitter KVL** equation is:

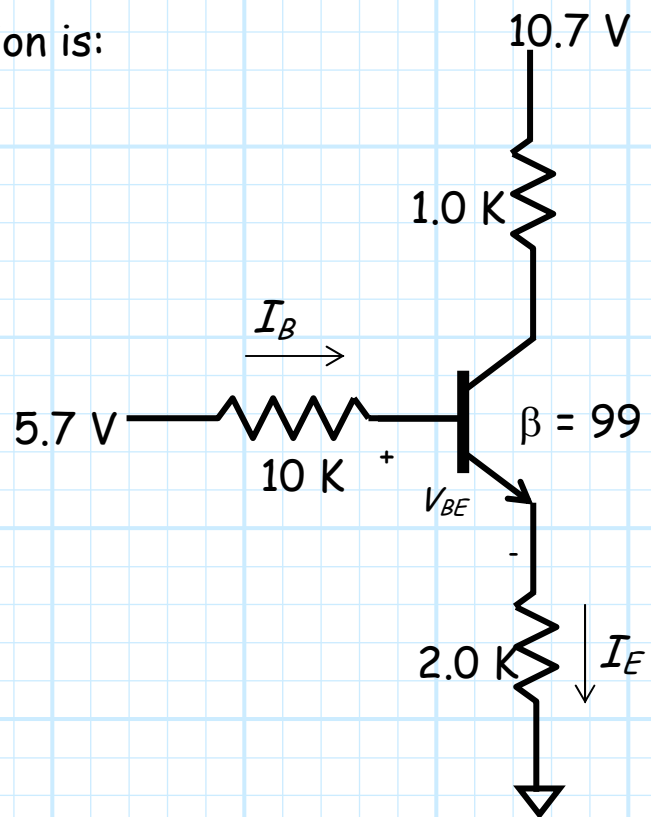
$$5.7 - 10 I_B - V_{BE} - 2 I_E = 0$$

This is the **circuit** equation; note that it contains 3 unknowns (i_B , i_C , and V_{BE}).

Now let's add the relevant **device** equations:

$$V_{BE} = 0.7 \text{ V}$$

$$\begin{aligned} I_E &= (\beta + 1) I_B \\ &= 100 I_B \end{aligned}$$



Look what we now have ! **3** equations and **3** unknowns (this is a **good** thing).

Inserting the device equations into the B-E KVL:

$$5.7 - 10 I_B - 0.7 - 2(99+1)I_B = 0$$

Therefore:

$$5.0 - 210 I_B = 0$$



1 equations and 1 unknown !

Solving, we get:

$$I_B = \frac{5.0}{210} = \underline{23.8 \mu A}$$

Q: Whew ! That was an **awful** lot of work for just one current, and we still have **two more** currents to find.

A: No we don't ! Since we determined **one** current for a BJT in **active** mode, we've determined them **all** !

I.E.,

$$I_C = \beta I_B = \underline{2.356 \text{ mA}}$$

$$I_E = (\beta + 1) I_B = \underline{2.380 \text{ mA}}$$

(Note that $I_C + I_B = I_E$)

Now for the **voltages** !

Since we know the **currents**, we can find the voltages **using KVL**.

For example, let's **determine** V_{CE} . We can do this **either** by finding the voltage at the **collector** V_C (wrt ground) and voltage at the **emitter** V_E (wrt ground) and then subtracting ($V_{CE} = V_C - V_E$).

OR, we can determine V_{CE} **directly** from the **C-E KVL equation**.

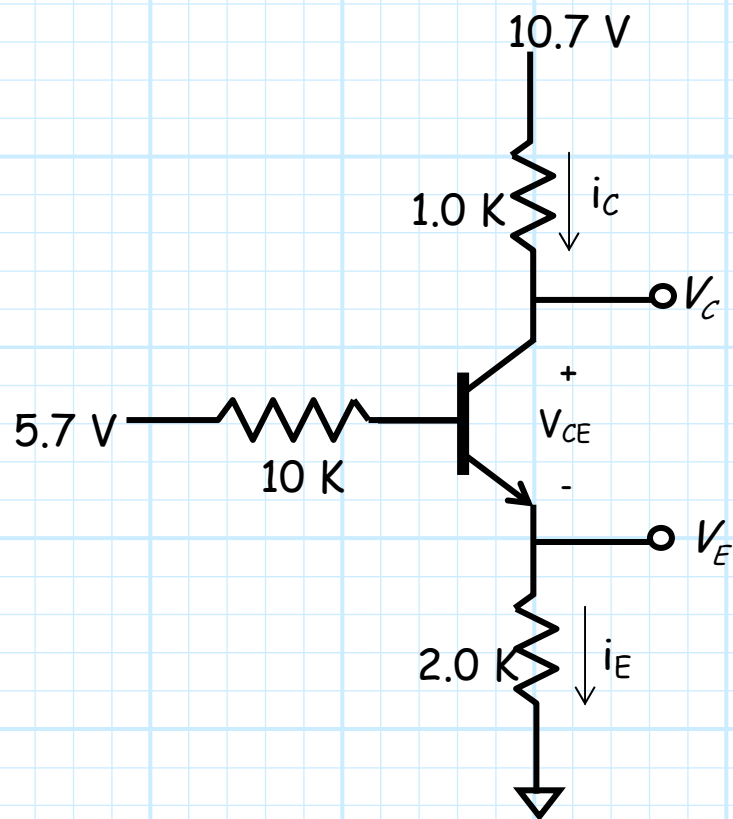
$$\begin{aligned} V_C &= 10.7 - I_C (1) \\ &= 10.7 - 2.36 \\ &= 8.34 \text{ V} \end{aligned}$$

and:

$$\begin{aligned} V_E &= 0 + I_E (2) \\ &= 0 + 4.76 \\ &= 4.76 \text{ V} \end{aligned}$$

Therefore,

$$V_{CE} = V_C - V_E = \underline{3.58 \text{ V}}$$



Note we could have **likewise** written the C-E KVL:

$$10.7 - I_C (1) - V_{CE} - I_E (2) = 0$$

Therefore,

$$V_{CE} = 10.7 - I_C (1) - I_E (2) = 3.58 \text{ V}$$

Q: *So, I guess we write the collector-base KVL to find V_{CB} ?*

A: You can, but a **wiser** choice would be to simply apply **KVL** to the **transistor**!

$$\text{I.E., } V_{CE} = V_{CB} + V_{BE} !!$$

$$\text{Therefore } V_{CB} = V_{CE} - V_{BE} = \underline{2.88 \text{ V}}$$

Q: *This has been hard. I'm glad we're finished!*

A: Finished! We still have **2 more** steps to go!

Step 4 - CHECK to see if your results are **consistent** with your assumption.

For **active** mode:

$$V_{CE} = 3.58 \text{ V} > 0.7 \text{ V} \quad \checkmark$$

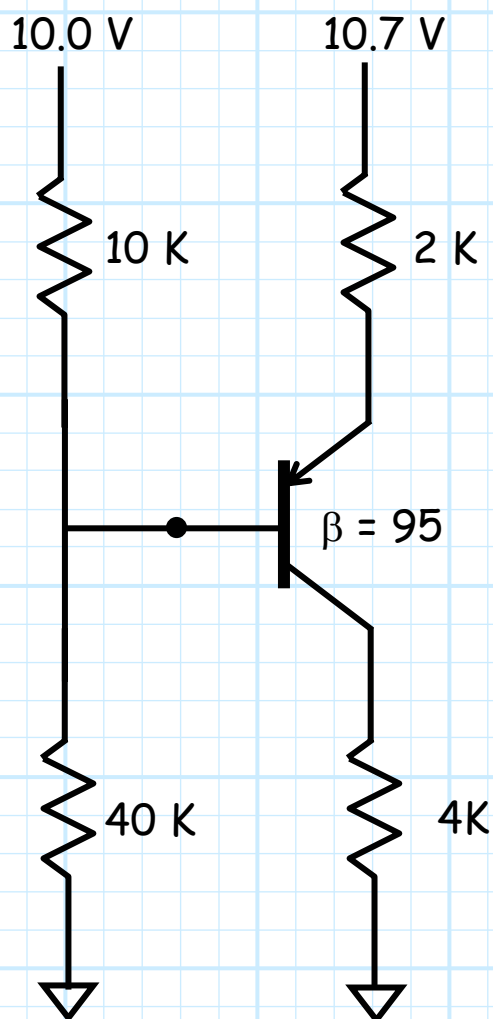
$$I_B = 23.8 \mu\text{A} > 0.0 \quad \checkmark$$

Are assumption was **correct**, and therefore so are our **answers!**

No need to go on to Step 5 .

Example: An Analysis of a pnp BJT Circuit

Determine the collector current and collector voltage of the BJT in the circuit below.



1. ASSUME the BJT is in **active** mode.

2. ENFORCE the conditions:

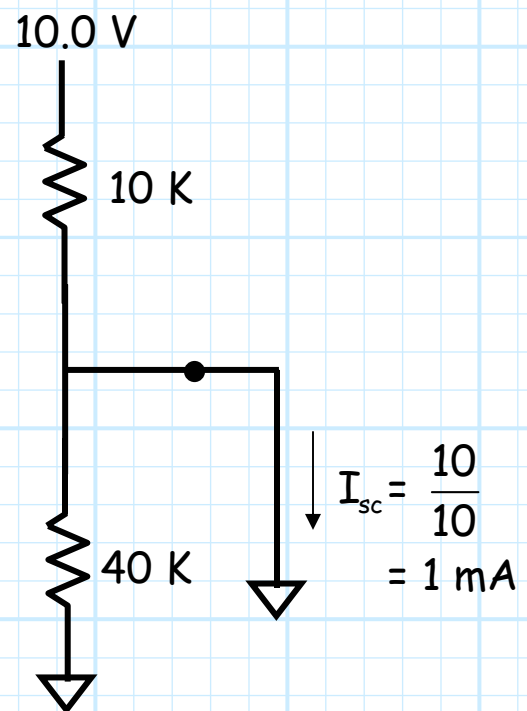
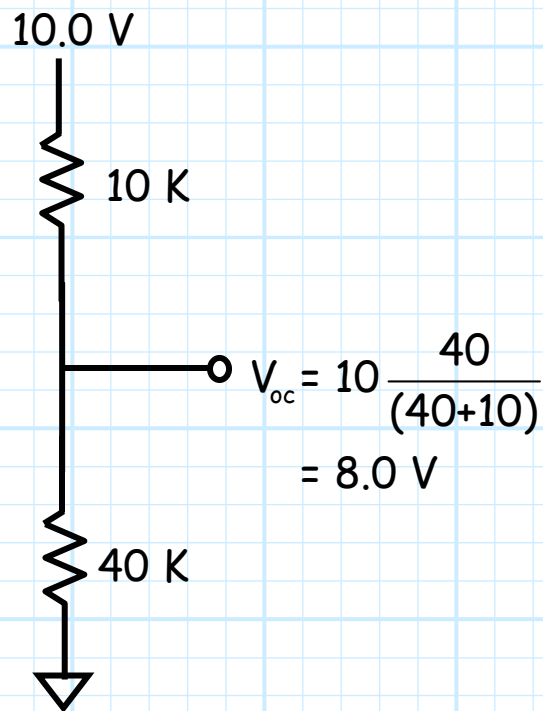
$$V_{EB} = 0.7 \text{ V} \quad \text{and} \quad i_c = \beta i_B$$

3. ANALYZE the circuit.

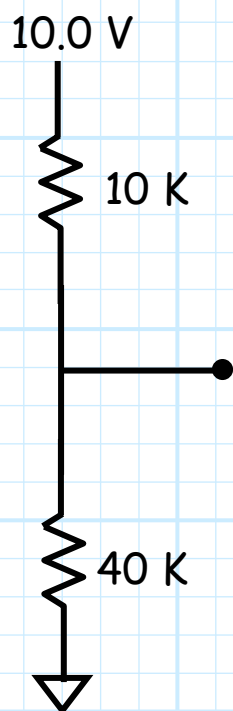
Q: *Yikes ! How do we write the base-emitter KVL ?*

A: This is a perfect opportunity to apply the **Thevenin's** equivalent circuit!

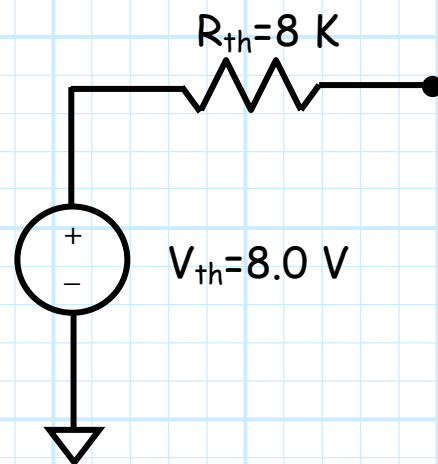
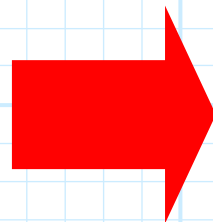
Thevenin's equivalent circuit:



Where $V_{th} = V_{oc} = 8.0 \text{ V}$ and $R_{th} = V_{oc}/I_{sc} = 8/1 = 8 \text{ K}$

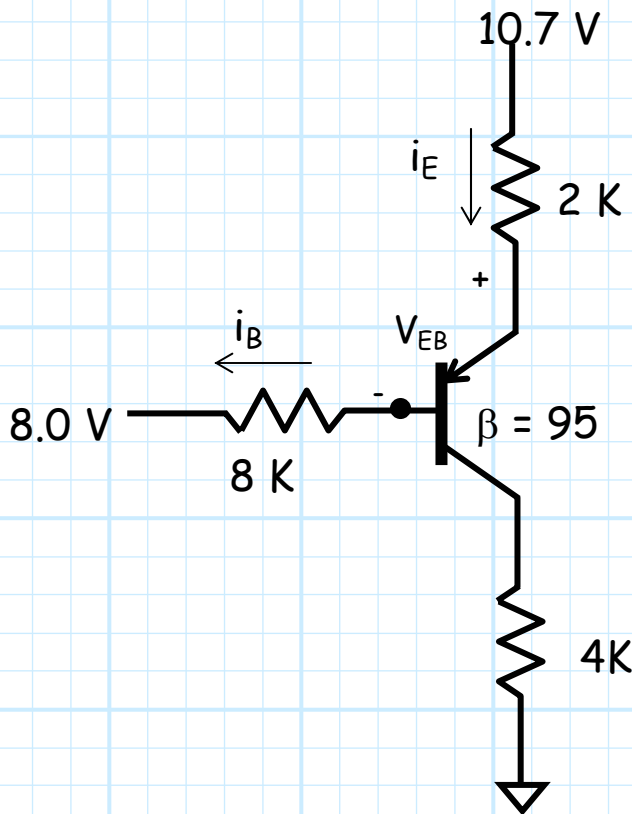


Original Circuit



Equivalent Circuit

Therefore, we can write the BJT circuit as:



NOW we can easily write the emitter-base leg KVL:

$$10.7 - 2i_E - v_{EB} - 8i_B = 8.0$$

Along with our enforced conditions, we now have **three** equations and **three** unknowns!

Combining, we find:

$$10.7 - 2(96)i_B - 0.7 - 8i_B = 8.0$$

Therefore,

$$i_B = \frac{10.7 - 0.7 - 8.0}{2(96) + 8} = \frac{2}{200} = 0.01 \text{ mA}$$

and collector current i_C is:

$$i_C = \beta i_B = 95(0.01) = \underline{0.95 \text{ mA}}$$

Likewise, the collector voltage (wrt ground) V_C is:

$$V_C = 0.0 + 4i_C = \underline{3.8 \text{ V}}$$

But wait ! We're **not** done yet ! We must **CHECK** our assumption.

First, $i_B = 0.01 \text{ mA} > 0$ ✓

But, what is V_{EC} ??

Writing the emitter-collector KVL:

$$10.7 - 2 i_E - V_{CE} - 4 i_C = 0$$

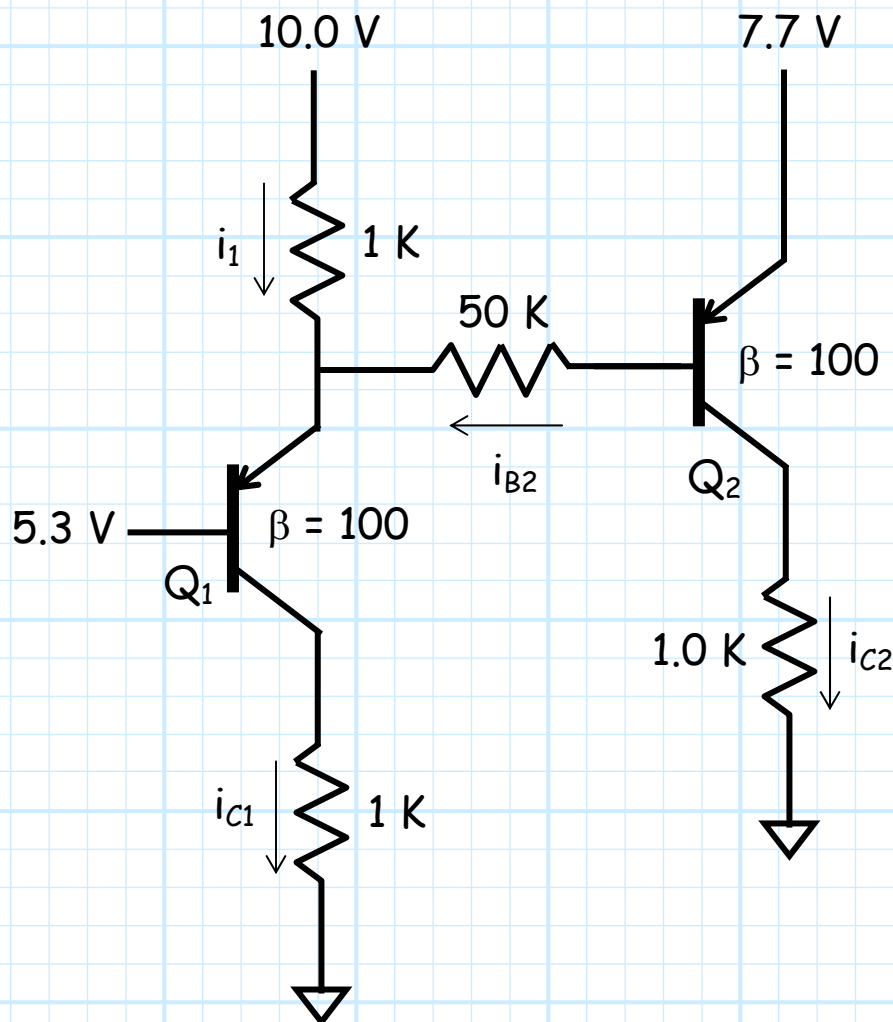
Therefore,

$$V_{EC} = 10.7 - 2(96)(0.01) - 4(0.95) = 4.98 \text{ V} > 0.7 \text{ V} ✓$$

Our assumption was **correct** !

Example: Another DC Analysis of a BJT Circuit

Find the collector voltages of the two BJTs in the circuit below.



ASSUME both BJTs are in **active** mode, therefore ENFORCE

$$V_{EB}^1 = V_{EB}^2 = 0.7 \text{ V}, \quad i_{C1} = 100 i_{B1}, \quad \text{and} \quad i_{C2} = 100 i_{B2}$$

Q: Now, how do we ANALYZE the circuit ??

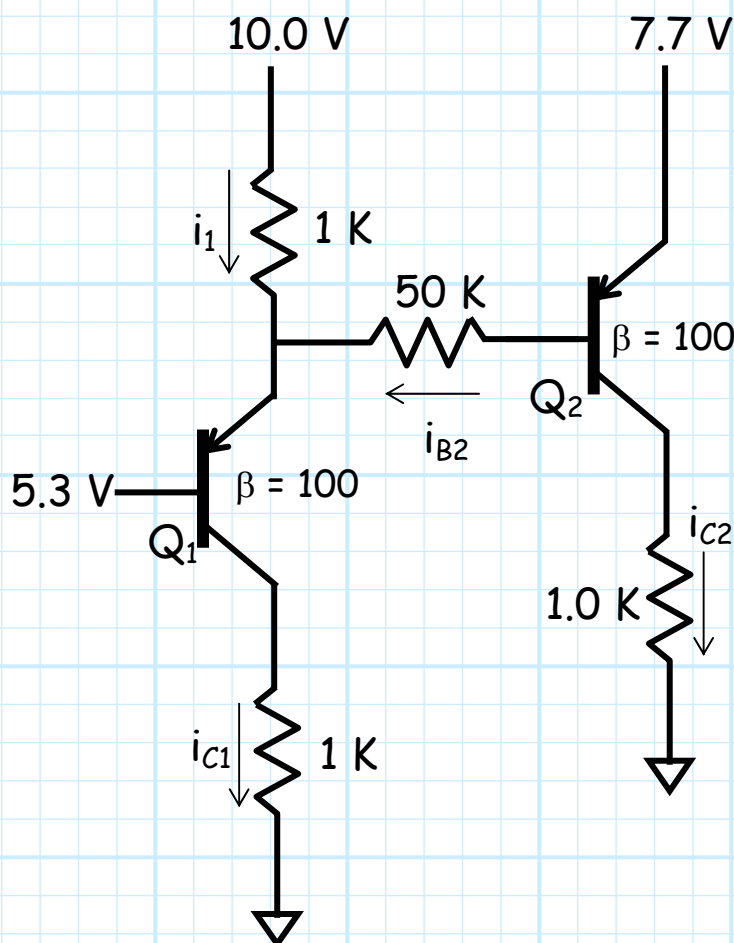
A: This seems to be a problem ! We cannot easily solve the emitter base KVL, as i_1 is NOT EQUAL to i_{E1} (make sure you understand this !). Instead, we find:

$$i_{E1} = i_1 + i_{B2}$$

So, what do we do ?

First, ask the question: **What do we know ??**

Look closely at the circuit, it is apparent that $V_{B1} = 5.3 \text{ V}$ and $V_{E2} = 7.7 \text{ V}$.



Hey! We therefore also know V_{E1} and V_{B2} :

$$V_{E1} = V_{B1} + V_{EB}^1 = 5.3 + 0.7 = 6.0 \text{ V}$$

$$V_{B2} = V_{E2} - V_{EB}^2 = 7.7 - 0.7 = 7.0 \text{ V}$$

Wow ! From these values we get:

$$i_1 = \frac{10 - V_{E1}}{1} = \frac{10 - 6}{1} = 4 \text{ mA}$$

and

$$i_{B2} = \frac{V_{B2} - V_{E1}}{50} = \frac{7 - 6}{50} = 0.02 \text{ mA}$$

This is easy! Since we know i_1 and i_{B2} , we can find i_{E1} :

$$i_{E1} = i_1 + i_{B2} = 4.0 + 0.02 = 4.02 \text{ mA}$$

Since we know **one** current for each BJT, we know **all** currents for each BJT:

$$i_{C1} = \alpha i_{E1} = \frac{\beta}{\beta+1} i_{E1} = \frac{100}{101} 4.02 = 3.98 \text{ mA}$$

$$i_{C2} = \beta i_{B2} = 100(0.02) = 2 \text{ mA}$$

Finally, we can determine the voltages V_{C1} and V_{C2} .

$$V_{C1} = 0.0 + 1 i_{C1} = 0.0 + 1(3.98) = \underline{3.98 \text{ V}}$$

$$V_{C2} = 0.0 + 1 i_{C2} = 0.0 + 1(2.0) = \underline{2.0 \text{ V}}$$

Now, let's CHECK to see if our assumptions were correct:

$$i_{C2} = 2 \text{ mA} > 0 \quad \checkmark$$

$$i_{C1} = 3.98 \text{ mA} > 0 \quad \checkmark$$

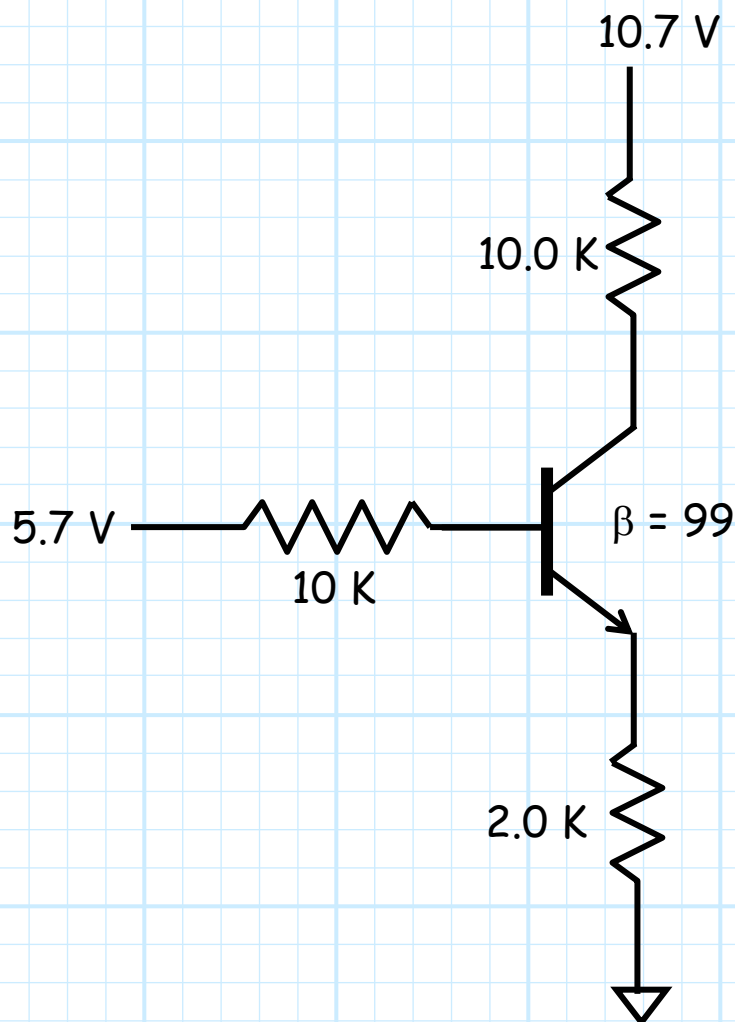
$$V_{EC}^1 = V_{E1} - V_{C1} = 6.0 - 3.98 = 2.02 \text{ V} > 0.7 \text{ V} \quad \checkmark$$

$$V_{BC}^2 = V_{B1} - V_{C1} = 7.0 - 2.0 = 5.0 \text{ V} > 0 \quad \checkmark$$

Assumptions are **correct** !

Example: A BJT Circuit in Saturation

Determine all **currents** for the BJT in the circuit below.



*Hey! I remember this circuit, its just like a **previous example**. The BJT is in **active mode!***

Let's see if you are correct! **ASSUME** it is in active mode and **ENFORCE** $V_{CE} = 0.7 \text{ V}$ and $i_C = \beta i_B$.

The B-E KVL is therefore:

$$5.7 - 10 i_B - 0.7 - 2 (99+1) i_B = 0$$

Therefore $i_B = 23.8 \mu\text{A}$

See! Base current $i_B = 23.8 \mu A$, just like before. Therefore collector current and emitter current are again $i_C = 99i_B = 2.356 \text{ mA}$ and $i_E = 100 i_B = 2.380 \text{ mA}$. Right ?!

Well **maybe**, but we still need to CHECK to see if our assumption is correct!

We know that $i_B = 23.8 \mu A > 0$ ✓ , but what about V_{CE} ?

From collector-emitter KVL we get:

$$10.7 - 10 i_C - V_{CE} - 2 i_E = 0$$

Therefore,

$$V_{CE} = 10.7 - 10(2.36) - 2(2.38) = -17.66 \text{ V} < 0.7 \text{ V} \times$$

Our assumption is **wrong** ! The BJT is **not** in active mode.

In the previous example, the collector resistor was **1K** , whereas in this example the collector resistor is **10K**. Thus, there is 10X the **voltage drop** across the collector resistor, which **lowers** the collector voltage so much that the BJT cannot remain in the active mode.

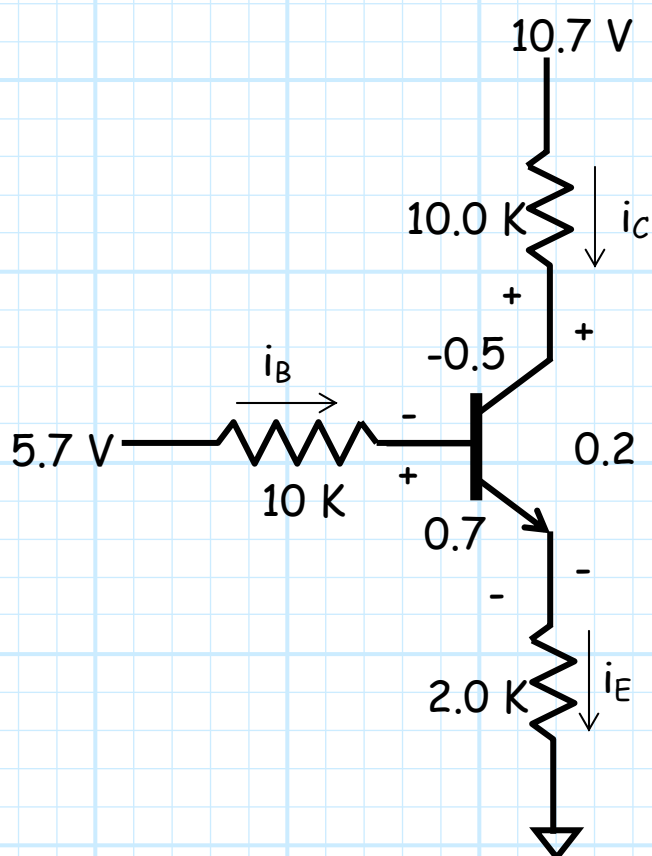
Q: So what do we do now ?

A: Go to **Step 5**; change the assumption and try it again!

Lets ASSUME instead that the BJT is in **saturation**. Thus, we ENFORCE the conditions:

$$V_{CE} = 0.2 \text{ V} \quad V_{BE} = 0.7 \text{ V} \quad V_{CB} = -0.5 \text{ V}$$

Now lets ANALYZE the circuit !



Note that we **cannot** directly determine the currents, as we **do not** know the base voltage, emitter voltage, or collector voltage.

But, we **do** know the **differences** in these voltages!

For example, we know that the collector voltage is 0.2 V **higher** than the emitter voltage, but we **do not** know what the collector or emitter voltages are!

Q: So, how the heck do we ANALYZE this circuit !?

A: Often, circuits with BJTs in **saturation** are somewhat more **difficult** to ANALYZE than circuits with **active** BJTs. There are often **many approaches**, but all result from a logical, systematic application of **Kirchoff's Laws!**

ANALYSIS EXAMPLE 1 - Start with KCL

We know that $i_B + i_C = i_E$ (KCL)

But, what **are** i_B , i_C , and i_E ??

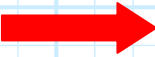
Well, from **Ohm's Law**:

$$i_B = \frac{5.7 - V_B}{10} \quad i_C = \frac{10.7 - V_C}{10} \quad i_E = \frac{V_E - 0}{10}$$

Therefore, combining with KCL:

$$\frac{5.7 - V_B}{10} + \frac{10.7 - V_C}{10} = \frac{V_E}{10}$$

Look what we have, **1 equation** and **3 unknowns**.

 We need **2 more independent equations** involving V_B , V_C , and V_E !

Q: *Two more independent equations !? It looks to me as if we have written all that we can about the circuit using Kirchoff's Laws.*

A: True! There are no more **independent** circuit equations that we can write using KVL or KCL ! But, recall the hint sheet:

*"Make sure you are using **all** available information".*

There is more **information** available to us - the ENFORCED conditions!

$$V_{CE} = V_C - V_E = 0.2 \quad \rightarrow \quad V_C = V_E + 0.2$$

$$V_{BE} = V_B - V_E = 0.7 \quad \rightarrow \quad V_B = V_E + 0.7$$

Two more independent equations! Combining with the earlier equation:

$$\frac{5.7 - (0.7 + V_E)}{10} + \frac{10.7 - (0.2 + V_E)}{10} = \frac{V_E}{10}$$

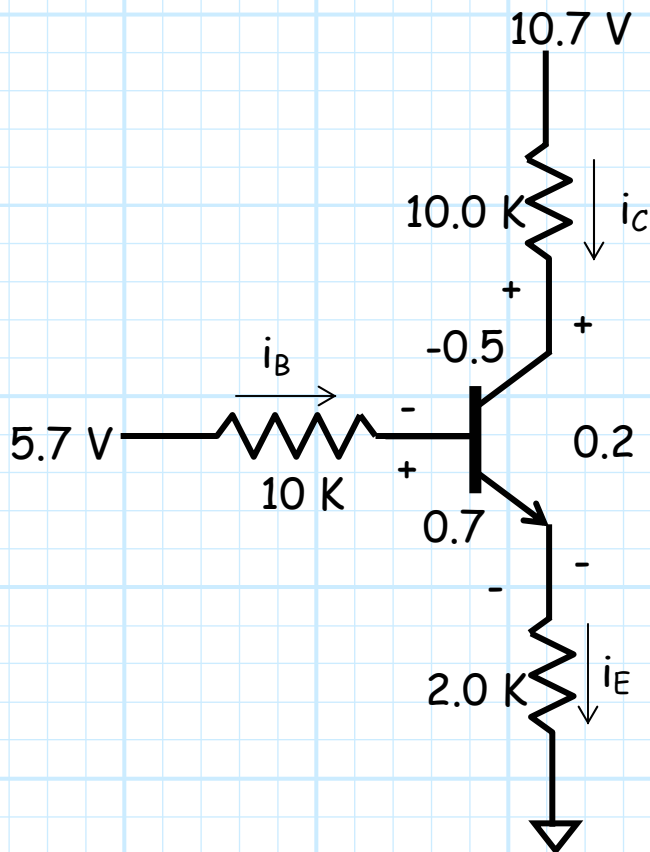
One equation and **one** unknown ! Solving, we get $V_E = 2.2 \text{ V}$.

Inserting this answer into the above equations, we get:

$$V_B = 2.9 \text{ V} \quad V_C = 2.4 \text{ V}$$

$$i_C = 0.83 \text{ mA} \quad i_B = 0.28 \text{ mA} \quad i_E = 1.11 \text{ mA}$$

ANALYSIS EXAMPLE 2 - Start with KVL



We can write the KVL equation for any **two** circuit legs:

B-E KVL:

$$5.7 - 10 i_B - 0.7 - 2 i_E = 0.0$$

C-E KVL:

$$10.7 - 10 i_C - 0.2 - 2 i_E = 0.0$$

Note the ENFORCED conditions are **included** in these KVL equations.

Simplifying, we get these 2 equations with 3 unknowns:

$$5.0 = 10 i_B + 2 i_E$$

$$10.5 = 10 i_C + 2 i_E$$

We need **one more** independent equation involving i_B , i_C , and i_E .

Try KCL!

$$i_B + i_C = i_E$$

Inserting the KCL equation into the 2 KVL equations, we get:

$$5.0 = 12 i_B + 2 i_C$$

$$10.5 = 2 i_B + 12 i_C$$

Solving, we get the **same answers** as in analysis example 1.

Lesson: There are **multiple** strategies for analyzing these circuits; use the ones that you feel most **comfortable** with!

However you **ANALYZE** the circuit, you **must** in the end also **CHECK** your results.

First **CHECK** to see that **all** currents are **positive**:

$$i_C = 0.83 \text{ mA} > 0 \quad \checkmark \quad i_B = 0.28 \text{ mA} > 0 \quad \checkmark \quad i_E = 1.11 \text{ mA} > 0 \quad \checkmark$$

Also **CHECK** collector current:

$$i_C = 0.83 \text{ mA} < \beta i_B = 27.7 \text{ mA} \quad \checkmark$$

Our solution is **correct** !!!