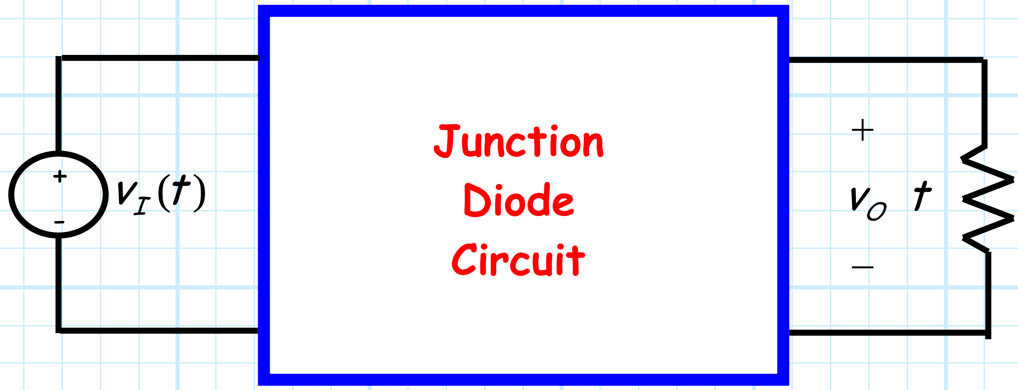
# 4.5 Rectifier Circuits

**Reading Assignment:** *pp. 194-200*

## A. Junction Diode 2-Port Networks

Consider when junction diodes appear in a 2-port network (i.e., a circuit with an **input** and an **output**).




We can characterize a 2-port network with its **transfer function**.

HO: THE TRANSFER FUNCTION OF DIODE CIRCUITS

Finding this transfer function is **similar** to our previous diode circuit analysis—but with a few **very** important **differences**!

HO: STEPS FOR FINDING A JUNCTION DIODE CIRCUIT TRANSFER FUNCTION

EXAMPLE: DIODE CIRCUIT TRANSFER FUNCTION

The **input** voltage is almost always a function of **time**, which means the **output** voltage is as well.

HO: TIME-DOMAIN ANALYSIS OF DIODE CIRCUITS

## B. Diode Rectifiers

Many important **diode circuits** appear in a standard AC to DC **power supply**!

HO: POWER SUPPLIES

The signal **rectifier** is an important component of a power supply—it's what creates the **DC component**.

HO: SIGNAL RECTIFICATION

One standard rectifier design is called the **full-wave rectifier**.

HO: THE FULL-WAVE RECTIFIER

But the **bridge rectifier** also provides full-wave rectification.

HO: THE BRIDGE RECTIFIER

The bridge rectifier is more **complex**, but results in a lower **Peak Inverse Voltage**.

## HO: Peak Inverse Voltage

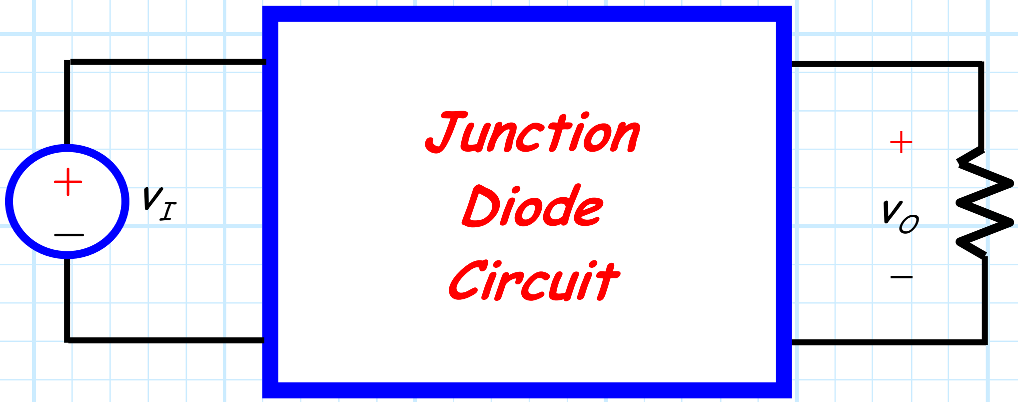
Make sure **you** can determine the **PIV** of a diode circuit. It depends on **both** the specific circuit, **and** the specific input voltage!

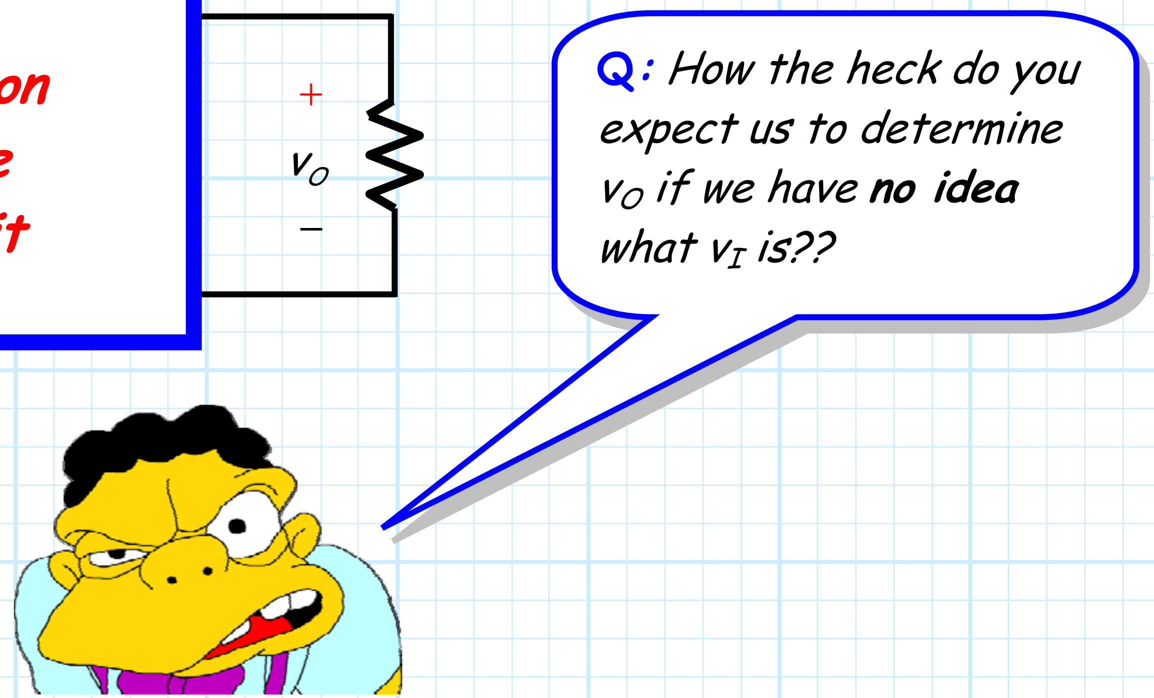
## Example: Peak Inverse Voltage

# The Transfer Function of Diode Circuits

For many junction diode circuits, we find that one of the voltage sources is in fact **unknown**!

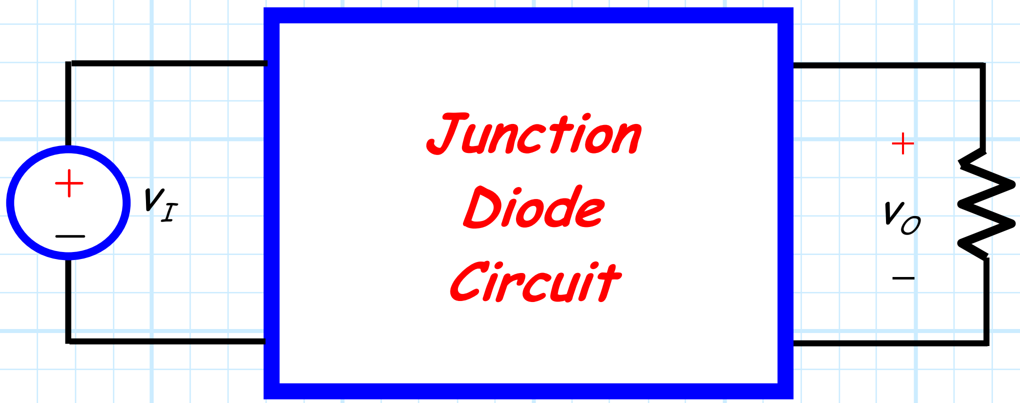
This unknown voltage is typically some **input** signal of the form $v_I$, which results in an output voltage $v_O$.



# The (large-signal) transfer function

**A:** We of course cannot determine an **explicit** value or expression for $v_O$, since it **depends** on the input $v_I$.




Instead, we will attempt to explicitly determine this **dependence** of $v_O$ on $v_I$!

In other words, we seek to find an expression for $v_O$ in **terms** of $v_I$. Mathematically speaking, our goal is to determine the **function**:

$$v_O = f(v_I)$$

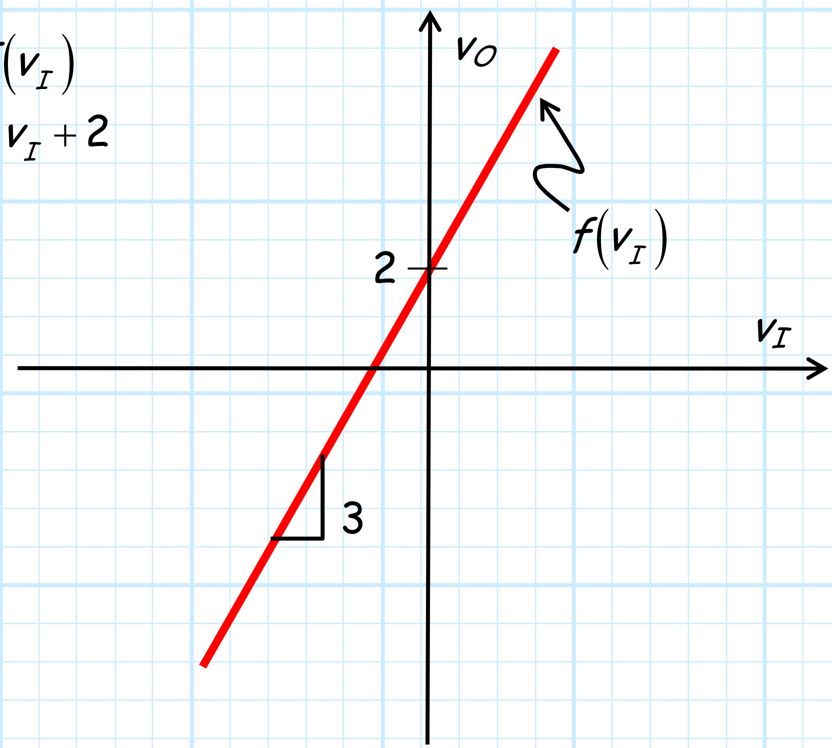
→We refer to this as the (large-signal) circuit **transfer function**.

# Plotting the transfer function

Note that we can **plot** a circuit transfer function on a 2-dimensional plane, just as if the function related values $x$ and $y$ (e.g. $y = f(x)$).

For **example**, say our circuit transfer function is:

$$\begin{aligned} v_O &= f(v_I) \\ &= 3v_I + 2 \end{aligned}$$




Note this is simply the **equation of a line** (e.g., $y = 3x + 2$), with slope $m$ =3 and $y$-intercept $b = 2$.

# Actually, a rare moment when I'm not being annoying and pretentions

**Q:** *A "**function**" eh?*

*Isn't a "**function**" just your **annoyingly pretentious** way of saying we need to find some mathematic **equation** relating $v_O$ and $v_I$?*

**A:** Actually **no**! Although a function **is** a mathematical equation, there are in fact **scads** of equations relating $v_O$ and $v_I$ that are **not** functions!

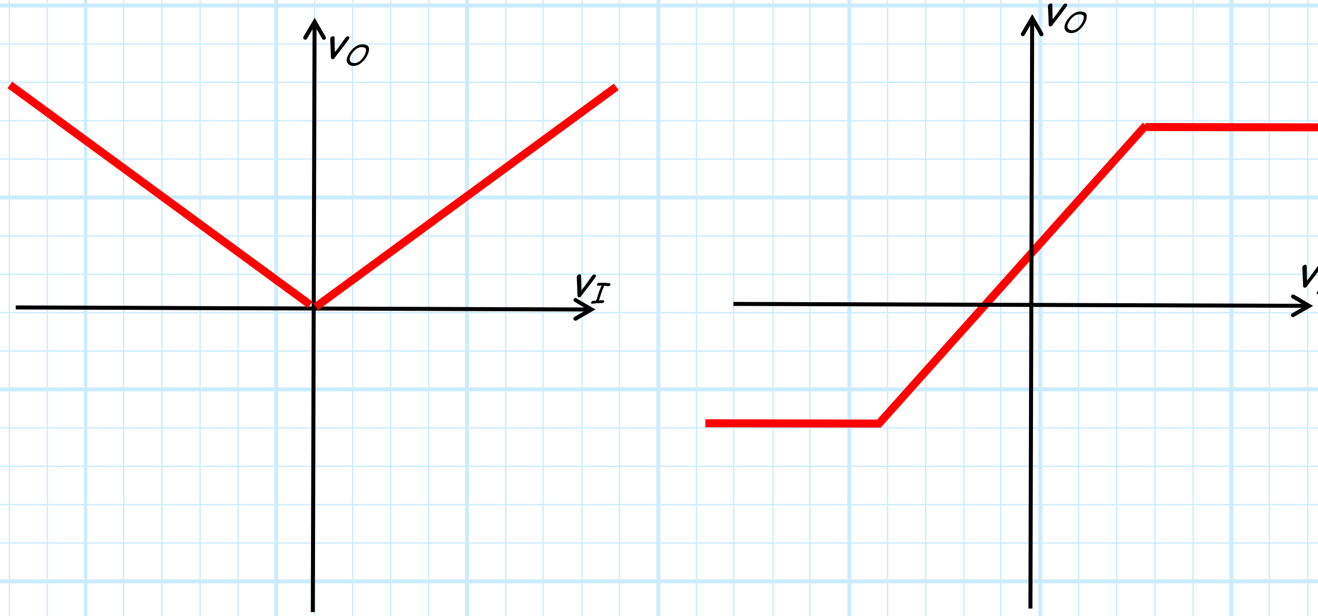
→ The set of all possible functions $y = f(x)$ are a **subset** of the set of all possible equations relating $y$ and $x$.

A **function** $v_O = f(v_I)$ is a mathematical expression such that for **any** value of $v_I$ (i.e., $-\infty < v_I < \infty$), there is **one**, but **only** one, value $v_O$.

# The transfer function must *be* a function!

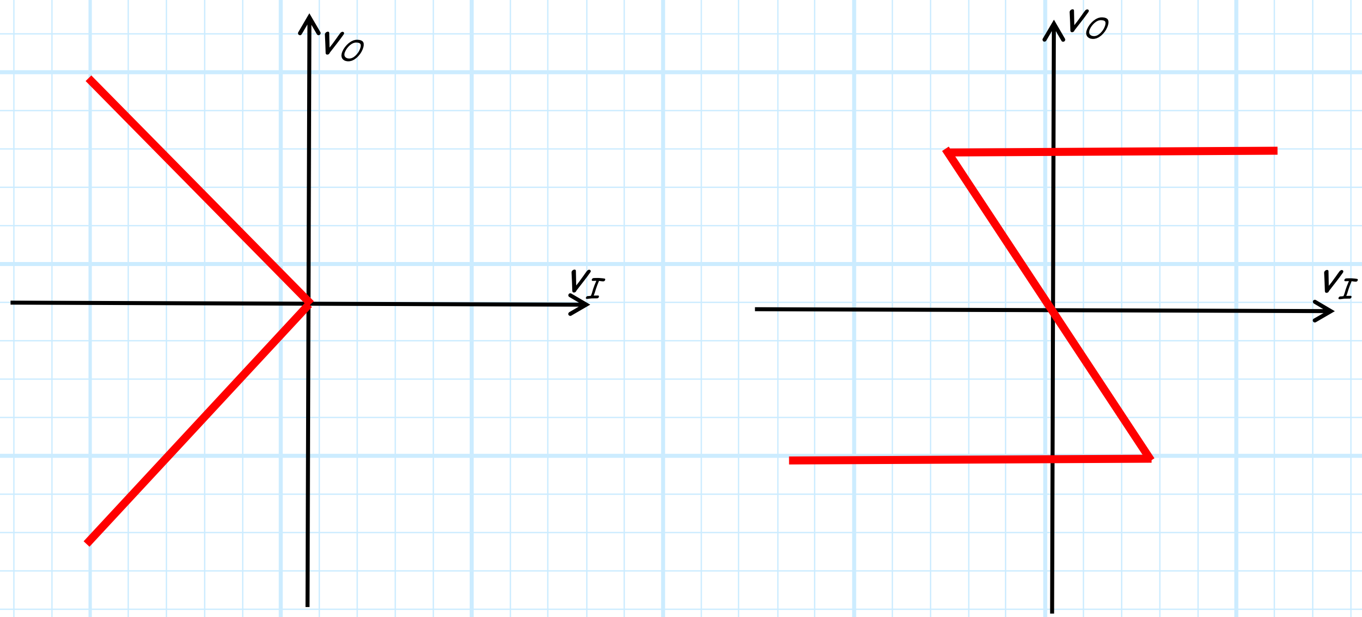
Note this definition of a function is consistent with our **physical** understanding of circuits—we can place **any** voltage on the input that we want (i.e., $-\infty < v_I < \infty$), and the result will be **one** specific voltage value $v_O$ on the output.

Therefore, examples of **valid** circuit transfer **functions** include:



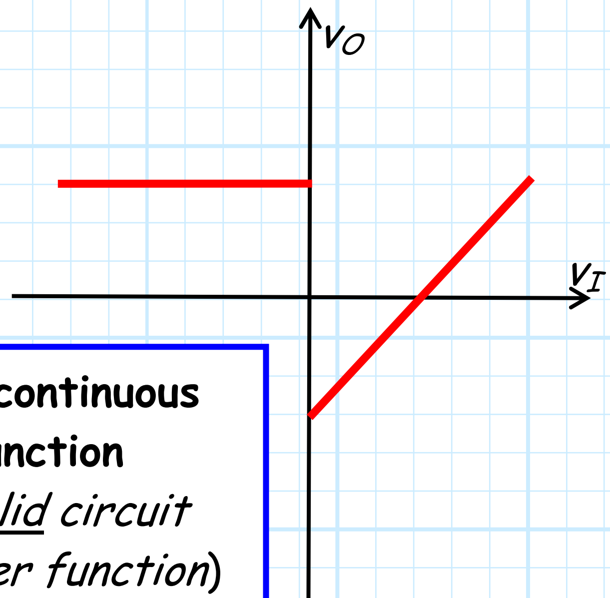
# These are functions—NOT!

Conversely, the transfer "functions" **below** are **invalid**—they **cannot** represent the behavior of circuits, since they **are not** functions!

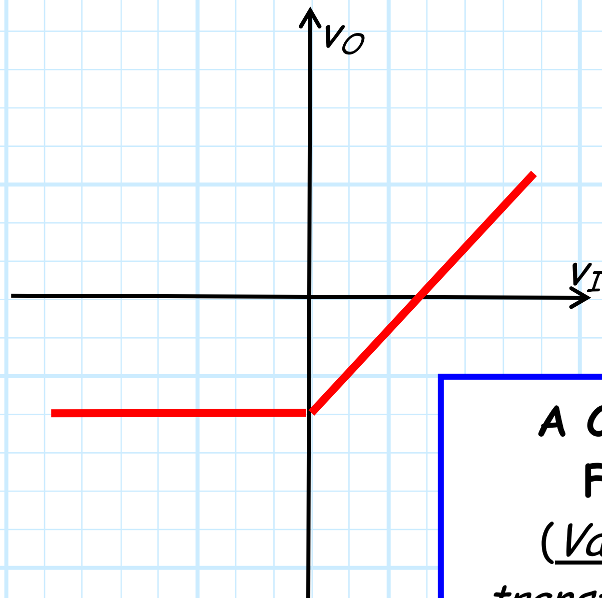


# The transfer function must be continuous

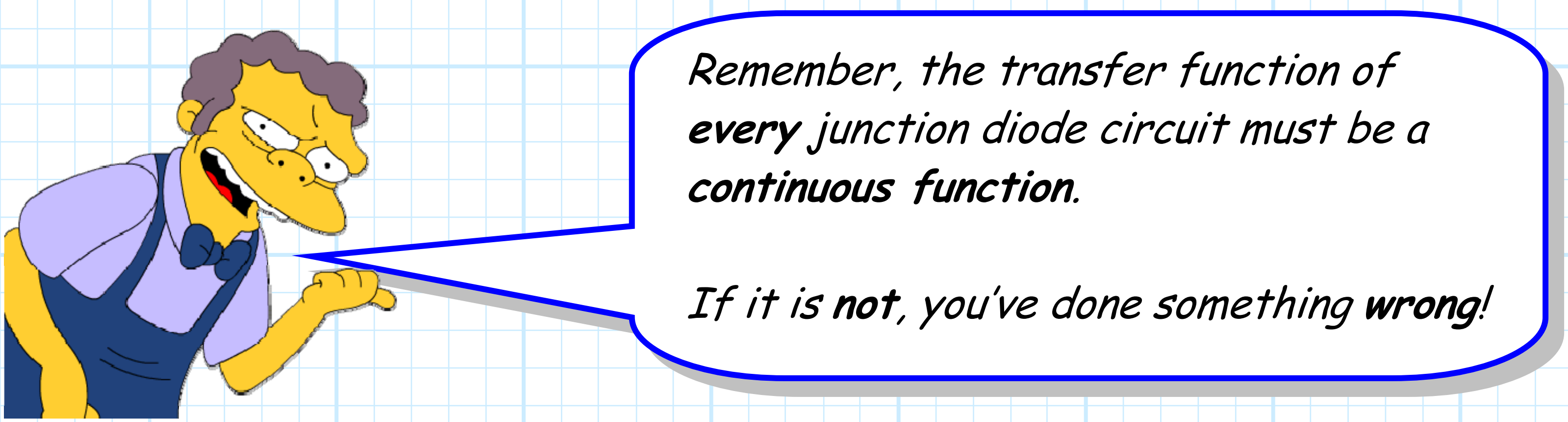
Moreover, we find that **circuit** transfer functions must be **continuous**. That is, $v_O$ **cannot** "instantaneously change" from one value to another as we increase (or decrease) the value $v_I$.




**A Discontinuous Function**
(*<u>Invalid</u> circuit transfer function*)




**A Continuous Function**
(*<u>Valid</u> circuit transfer function*)



# Steps for Finding a Junction Diode Circuit Transfer Function

Determining the **transfer function** of a junction diode circuit is in many ways **very similar** to the analysis steps we followed when analyzing previous junction diode circuits (i.e., circuits where all sources were **explicitly known**).

However, there are also some **important differences** that we must understand completely if we wish to successfully determine the **correct transfer function**!

**Step1:** *Replace all junction diodes with an appropriate junction diode **model**.*

> **Just** like before! We will now have an **IDEAL** diode circuit.

**Step 2:** *ASSUME some mode for all ideal diodes.*

> **Just** like before! An IDEAL diode can be either forward or reverse biased.

**Step 3:** *ENFORCE the bias assumption.*

**Just** like before! ENFORCE the bias assumption by replacing the **ideal** diode with short circuit or open circuit.

**Step 4:** *ANALYZE the remaining circuit.*

**Sort** of, **kind** of, like before!

1. If we assumed an IDEAL diode was forward biased, we must determine $i_D^i$—**just** like before!

However, **instead** of finding the numeric value of $i_D^i$, we determine $i_D^i$ as a **function** of the unknown source (e.g., $i_D^i = f(v_I)$).

2. Or, if we assumed an IDEAL diode was reversed biased, we must determine $v_D^i$—**just** like before!

However, **instead** of finding the numeric value of $v_D^i$, we determine $v_D^i$ as a **function** of the unknown source (e.g., $v_D^i = f(v_I)$).

3. Finally, we must determine all the **other** voltages and/or currents we are interested in (e.g., $v_O$)—**just** like before!

However, **instead** of finding its numeric value, we determine it as a **function** of the unknown source (e.g., $v_O = f(v_I)$).

***Step 5:*** *Determine WHEN the assumption is valid.*

**Q:** *OK, we get the picture. Now we have to **CHECK** to see if our IDEAL diode assumption was correct, right?*

**A:** Actually, **no**! This step is **very different** from what we did before!

We **cannot** determine **IF** $i_D^i > 0$ (forward bias assumption), or **IF** $v_D^i < 0$ (reverse bias assumption), since we **cannot** say for certain what the value of $i_D^i$ or $v_D^i$ is!

Recall that $i_D^i$ and $v_D^i$ are **functions** of the unknown voltage source (e.g., $i_D^i = f(v_I)$ and $v_D^i = f(v_I)$).

Thus, the values of $i_D^i$ or $v_D^i$ are **dependent** on the unknown source ($v_I$, say).

For **some** values of $v_I$, we will find that $i_D^i > 0$ or $v_D^i < 0$, and so our assumption (and thus our solution for $v_O = f(v_I)$) will be! **correct**

**However**, for **other** values of $v_I$, we will find that $i_D^i < 0$ or $v_D^i > 0$, and so our assumption (and thus our solution for $v_O = f(v_I)$) will be **incorrect**!

**Q:** *Yikes! What do we do?*

*How can we determine the circuit transfer function if we can't determine **IF** our ideal diode assumption is correct??*

**A:** Instead of determining **IF** our assumption is correct, we must determine **WHEN** our assumption is correct!

In other words, we must determine for **what values** of $v_I$ is $i_D^i > 0$ (forward bias), or for **what values** of $v_I$ is $v_D^i < 0$ (reverse bias).

We can do this since we earlier (in step 4) determined the function $i_D^i = f(v_I)$ or the function $v_D^i = f(v_I)$.

Perhaps this step is best explained by an **example**. Let's say we assumed that our ideal diode was **forward biased** and, say we determined (in step 4) that $v_O$ is related to $v_I$ as:

$$\begin{aligned} v_O &= f(v_I) \\ &= 2v_I - 3 \end{aligned}$$

Likewise, say that we determined (in step 4) that our ideal diode current is related to $v_I$ as:

$$
\begin{aligned}
i_D^i &= f\left(v_I\right) \\
&> \frac{v_I - 5}{4}
\end{aligned}
$$

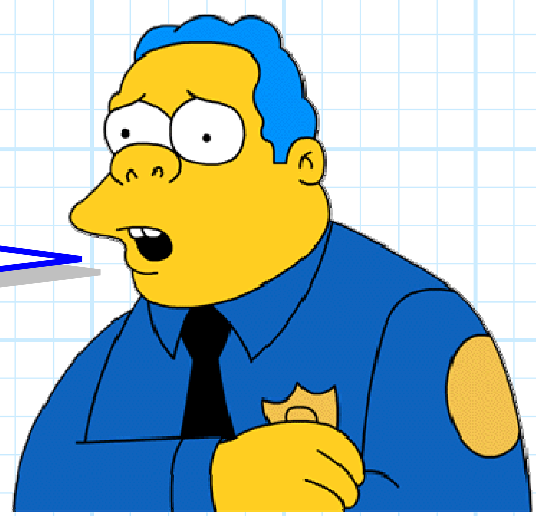
Thus, in order for our forward bias assumption to be **correct**, the function $i_D^i = f\left(v_I\right)$ must be **greater than zero**:

$$
\begin{aligned}
i_D^i &> 0 \\
f\left(v_I\right) &> 0 \\
\frac{v_I - 5}{4} &> 0
\end{aligned}
$$

We can now "solve" this **inequality** for $v_I$:

$$
\begin{aligned}
\frac{v_I - 5}{4} &> 0 \\
v_I - 5 &> 0 \\
v_I &> 5
\end{aligned}
$$

**Q:** *What does **this** mean? Does it mean that $v_I$ **is** some value **greater** than 5.0V ??*



**A:** **NO!** Recall that $v_I$ can be **any** value.

What the inequality above means is that $i_D^i > 0$ (i.e., the ideal diode is forward biased) **WHEN** $v_D^i > 5.0$.

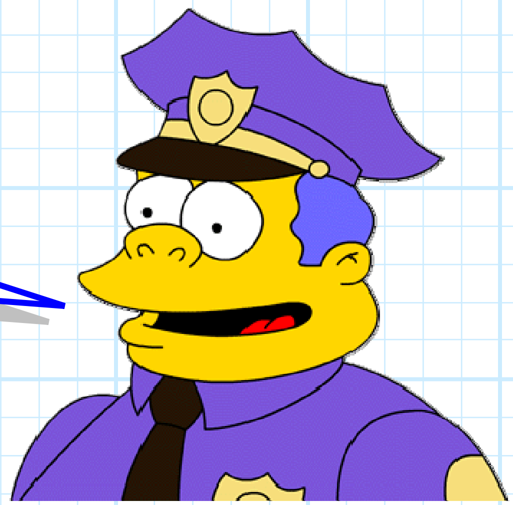
Thus, we know $v_O = 2v_I - 3$ is valid **WHEN** the ideal diode is forward biased, and the ideal diode is forward biased **WHEN** (for this example) $v_D^i > 5.0$.

As a result, we can mathematically state that:

$$v_O = 2v_I - 3 \quad \text{when} \quad v_I > 5.0 \text{ V}$$

**Conversely**, this means that **if** $v_I < 5.0$ V, the **ideal** diode will be **reverse biased**—our forward bias assumption would **not** be valid, and thus our expression $v_O = 2v_I - 3$ is **not** correct ($v_O \neq 2v_I - 3$ for $v_I < 5.0\text{V}$)!

**Q:** *So how **do** we determine $v_O$ for values of $v_I < 5.0$ V ?*



**A:** Time to move to the **last** step !

**Step 6:** ***Change** assumption and **repeat** steps 2 through 5 !*

For our **example**, we would change our bias assumption and now ASSUME reverse bias.

We then ENFORCE $i_D^i = 0$, and then ANALYZE the circuit to find both $v_D^i = f(v_I)$ and a **new** expression $v_O = f(v_I)$ (it will **no longer** be $v_O = 2v_I - 3$ !).

We then determine **WHEN** our reverse bias assumption is valid, by solving the **inequality** $v_D^i = f(v_I) > 0$ for $v_I$.

For the example used here, we would find that the **IDEAL** diode is reverse biased **WHEN** $v_I < 5.0 \text{ V}$.

For junction diode circuits with **multiple** diodes, we may have to repeat this entire process **multiple** times, until **all possible** bias conditions are analyzed.

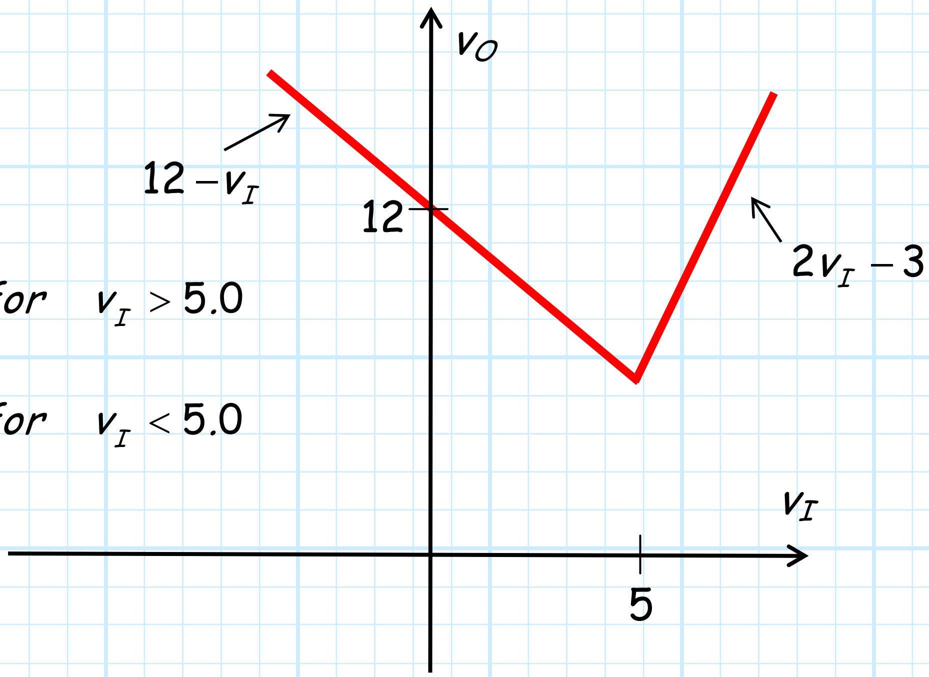
If we have done our analysis **properly**, the result will be a valid **continuous function**!

That is, we will have an expression (but only **one** expression) relating $v_O$ to **all** possible values of $v_I$.

This transfer function will typically be **piecewise linear**.

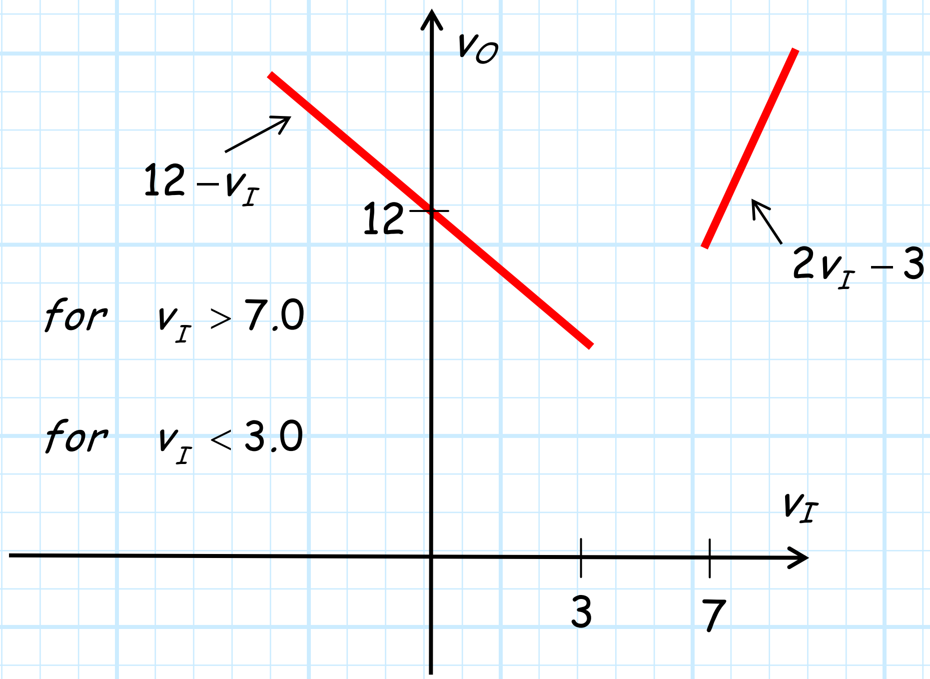
An **example** of a piece-wise linear transfer function is:

$$v_O = \begin{cases} 2v_I - 3 & \text{for} \quad v_I > 5.0 \\ 12 - v_I & \text{for} \quad v_I < 5.0 \end{cases}$$



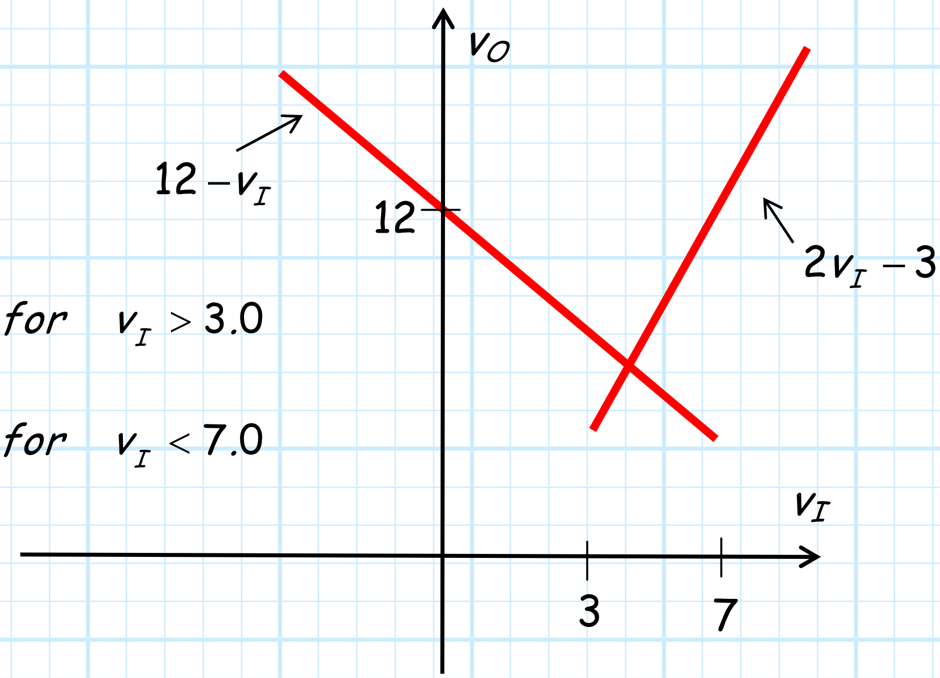

Just to make **sure** that we understand what a function is, note that the following expression is **not** a function:

$$v_O = \begin{cases} 2v_I - 3 & \text{for} \quad v_I > 7.0 \\ 12 - v_I & \text{for} \quad v_I < 3.0 \end{cases}$$



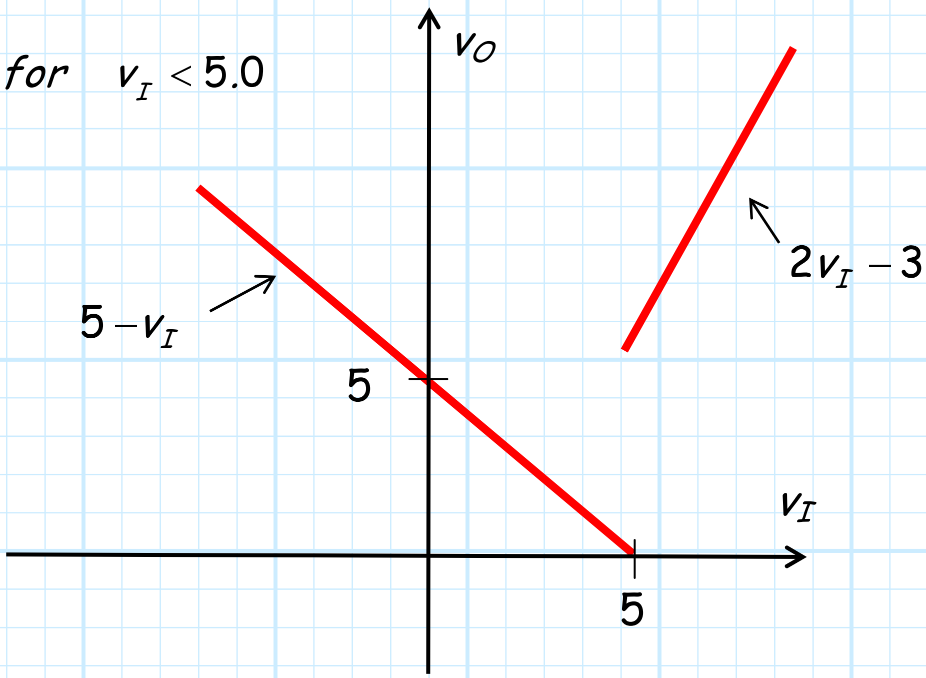
**Nor** is this expression a function:

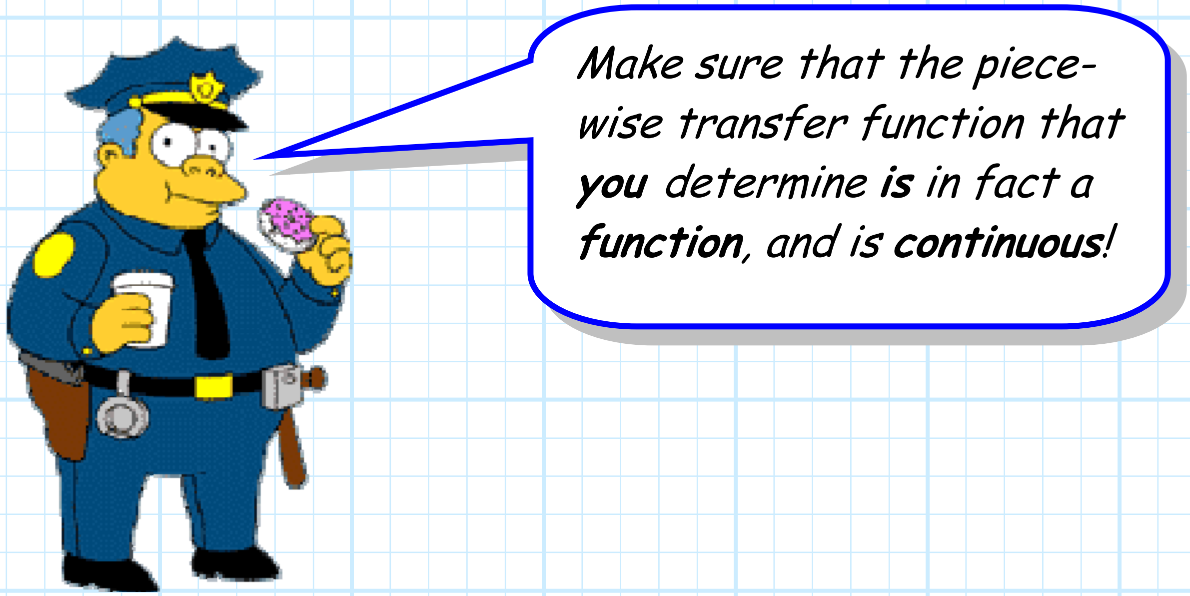
$$v_O = \begin{cases} 2v_I - 3 & \text{for} \quad v_I > 3.0 \\ 12 - v_I & \text{for} \quad v_I < 7.0 \end{cases}$$




Finally, note that the following expression **is** a function, but it is **not continuous**:

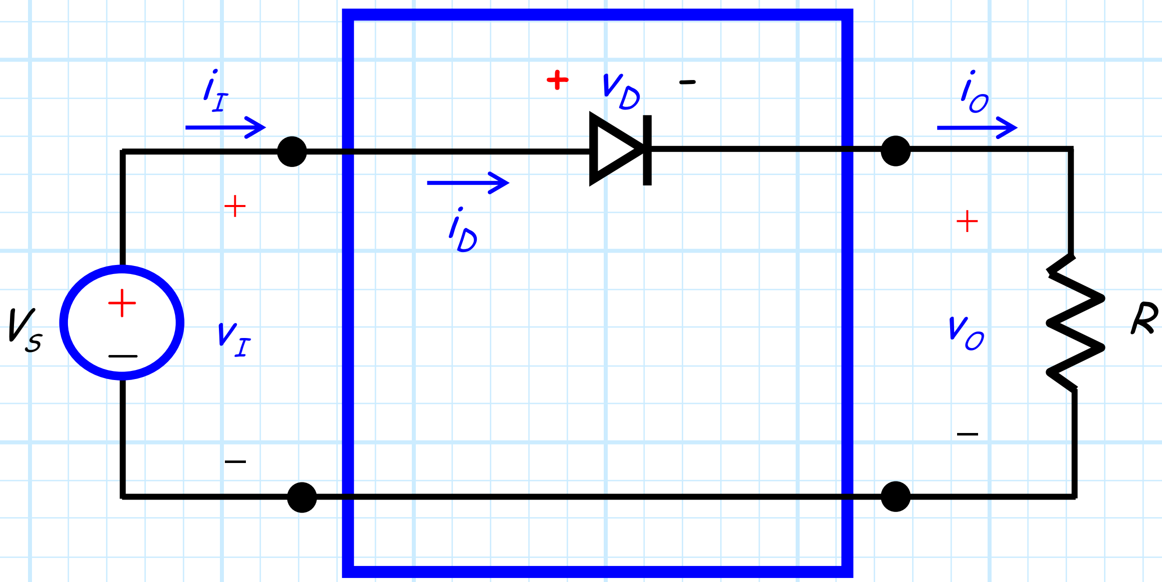
$$v_O = \begin{cases} 2v_I - 3 & \text{for} \quad v_I > 5.0 \\ 5 - v_I & \text{for} \quad v_I < 5.0 \end{cases}$$



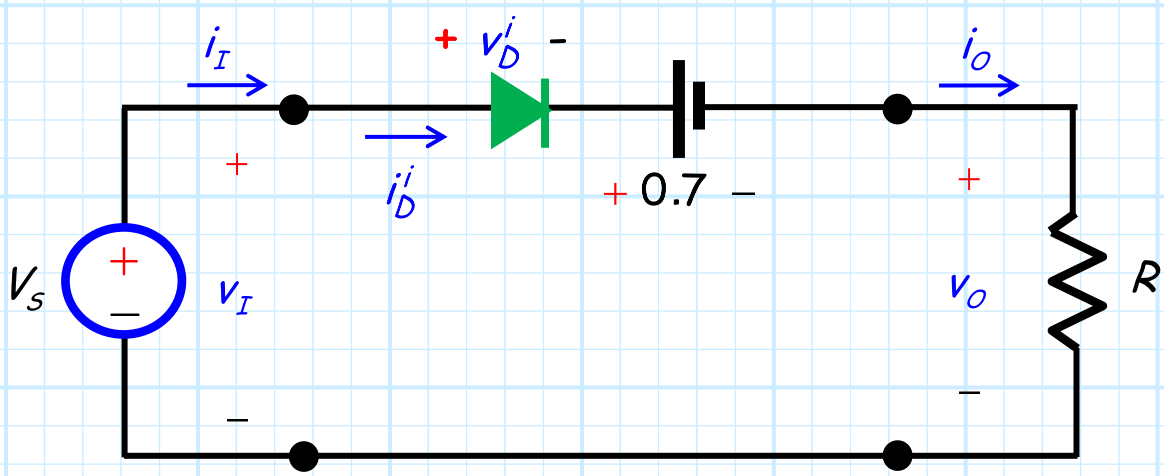

Make sure that the piece-wise transfer function that **you** determine **is** in fact a **function**, and is **continuous**!

# Example: Diode Circuit Transfer Function

Consider the following circuit, called a **half-wave rectifier**:

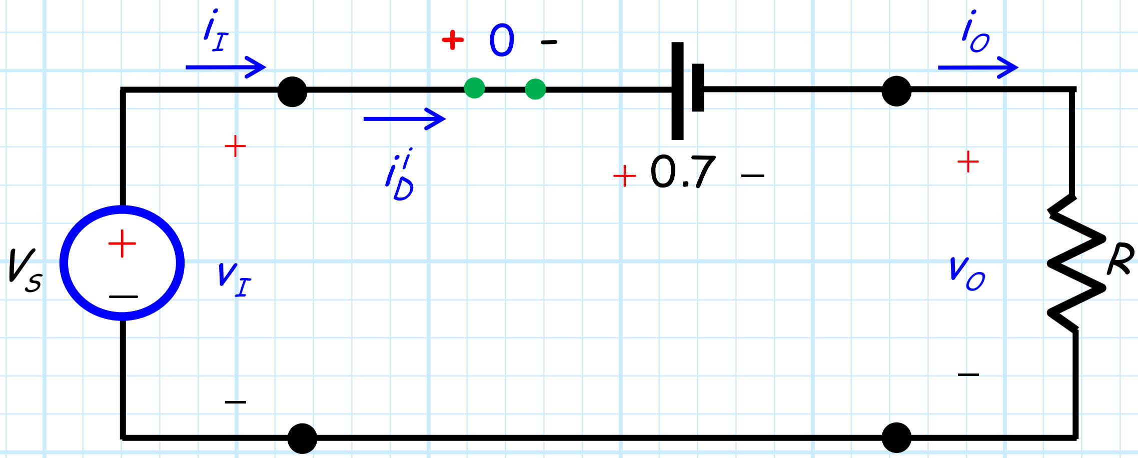



Let's use the **CVD model** to determine the output voltage $v_O$ in terms of the input voltage $v_I$.



In other words, let's determine the diode circuit **transfer function** $v_O = f(v_I)$!

ASSUME the **ideal** diode is **forward** biased, ENFORCE $v_D' = 0$.



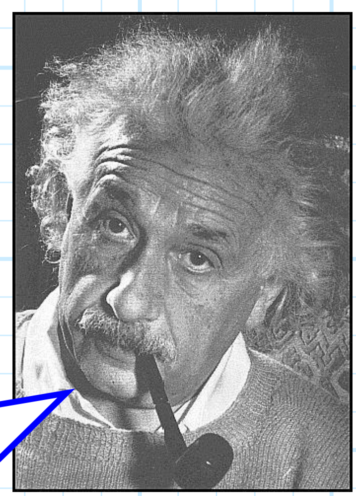

From KVL, we find that:

$$v_I - 0 - 0.7 = v_O \qquad \Rightarrow \qquad \therefore v_O = v_I - 0.7$$

This result is of course true **if** our original assumption is correct—it is valid **if** the ideal diode is forward biased (i.e., $i_D' > 0$)!

From **KCL** and **Ohm's Law**, we find that:

$$i_D' = i_O = \frac{v_O}{R} = \frac{v_I - 0.7}{R}$$



**Q:** *I'm so* ***confused!*** *Is this current* ***greater*** *than zero or* ***less*** *than zero?*

***Is*** *our assumption correct?* ***How*** *can we tell?*

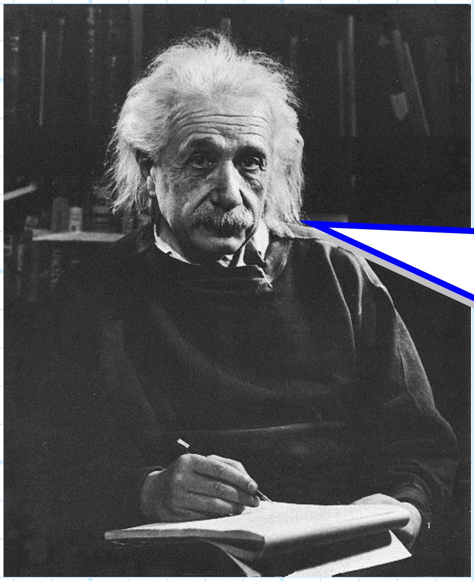
**A:** The ideal diode current is **dependent** on the value of source voltage $v_I$. As such, we **cannot** determine **if** our assumption is correct, we **instead** must find out **when** our assumption is correct!

In other words, we know that the forward bias assumption is correct **when** $i_D^i > 0$. We can rearrage our diode current expression to determine for what values of source voltage $v_S$ this is true:

$$\begin{aligned} i_D^i &> 0 \\ \frac{v_I - 0.7}{R} &> 0 \\ v_I - 0.7 &> 0 \\ v_I &> 0.7 \end{aligned}$$

So, we have found that **when** the source voltage $v_I$ is greater than 0.7 V, the output voltage $v_O$ is:

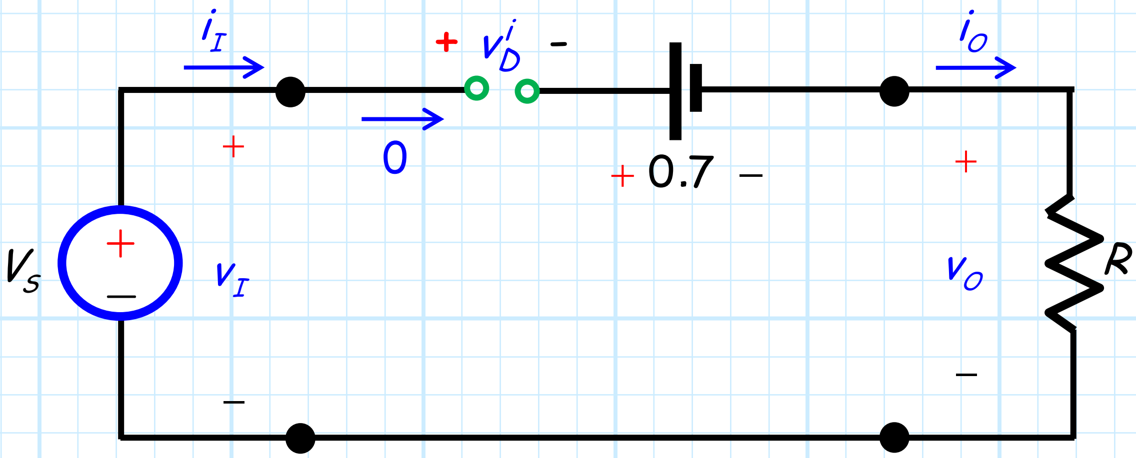
$$v_O = v_I - 0.7$$



**Q:** *OK, I've got this result written down.*

*However, I still don't know what the output voltage $v_O$ is **when** the source voltage $v_I$ is **less** than 0.7V!?!*

Now we **change** our assumption and ASSSUME the ideal diode in the CVD model is **reverse** biased, an assumption ENFORCEd with the condition that $i_D^i = 0$ (i.e., an open circuit).

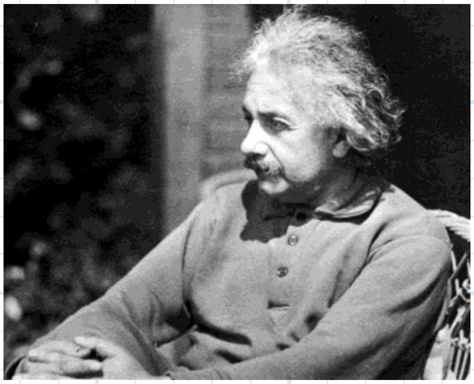



From KCL:

$$i_O = i_D^i = 0$$

From Ohm's Law, we find that the output voltage is:

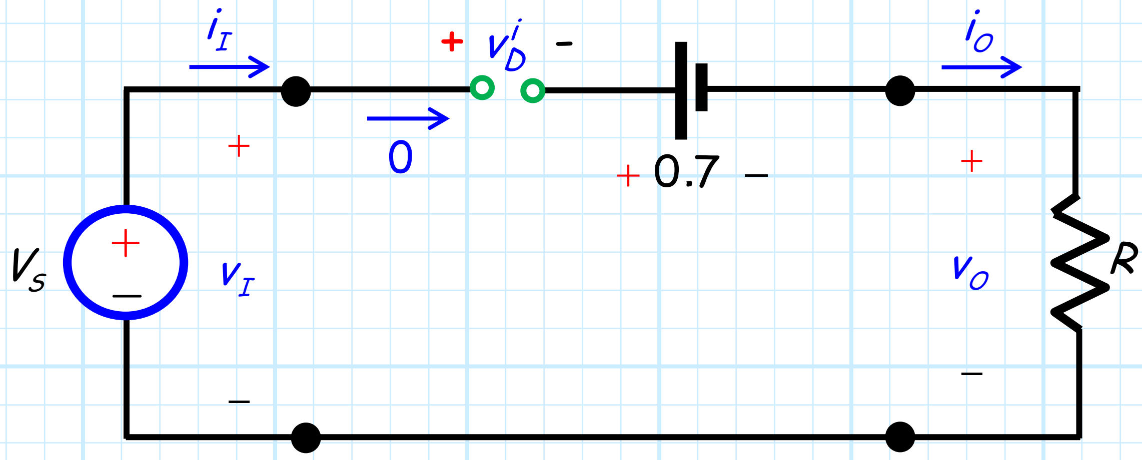
$$\begin{aligned} v_O &= R\, i_O \\ &= R(0) \\ &= 0 \text{ V} \quad !!! \end{aligned}$$



**Q:** *Fascinating! The output voltage is* ***zero*** *when the ideal diode is* ***reverse biased.***

*But, precisely* ***when is*** *the ideal diode reverse biased? For* ***what*** *values of* $v_I$ *does this occur ?*

**A:** To answer these questions, we must determine the **ideal** diode voltage in **terms** of $v_S$ (i.e., $v_D^i = f(v_I)$).




From KVL:

$$v_I - v_D^i - 0.7 = v_O$$

Therefore:

$$\begin{aligned} v_D^i &= v_I - 0.7 - v_O \\ &= v_I - 0.7 - 0.0 \\ &= v_I - 0.7 \end{aligned}$$

Thus, the ideal diode is in reverse bias **when:**

$$\begin{aligned} v_D^i &< 0 \\ v_I - 0.7 &< 0 \end{aligned}$$

Solving for $v_I$, we find:

$$\begin{aligned} v_I - 0.7 &< 0 \\ v_I &< 0.7 \text{ V} \end{aligned}$$

In other words, we have determined that the **ideal** diode will be reverse biased **when** $v_I < 0.7$ V, and that the output voltage will be $v_O = 0$.

**A:** That's right! The **transfer function** for this circuit is therefore:

$$v_O = \begin{cases} v_S - 0.7 & \textit{for} \quad v_S > 0.7 \\ 0 & \textit{for} \quad v_S < 0.7 \end{cases}$$

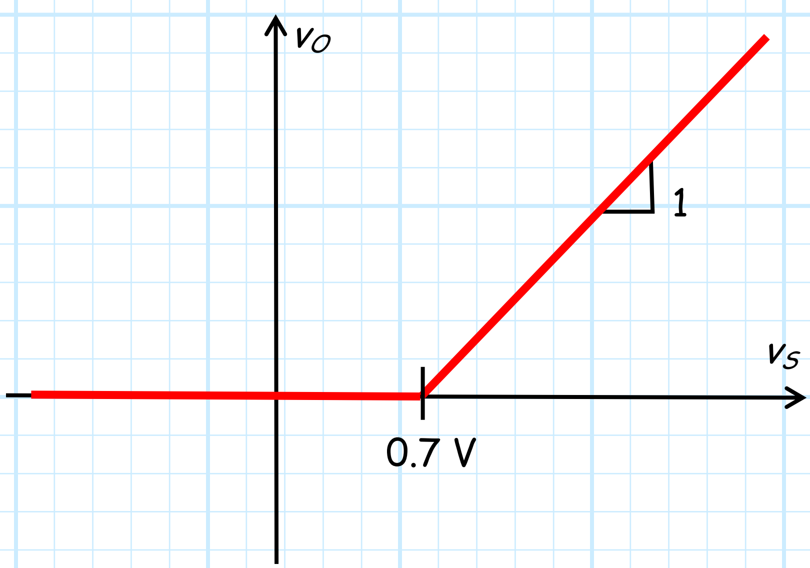
**Q:** *So, I see we have found that:*

$$v_O = v_I - 0.7 \quad \textit{when} \quad v_I > 0.7 \text{ V}$$

*and,*

$$v_O = 0.0 \quad \textit{when} \quad v_I < 0.7 \text{ V}$$

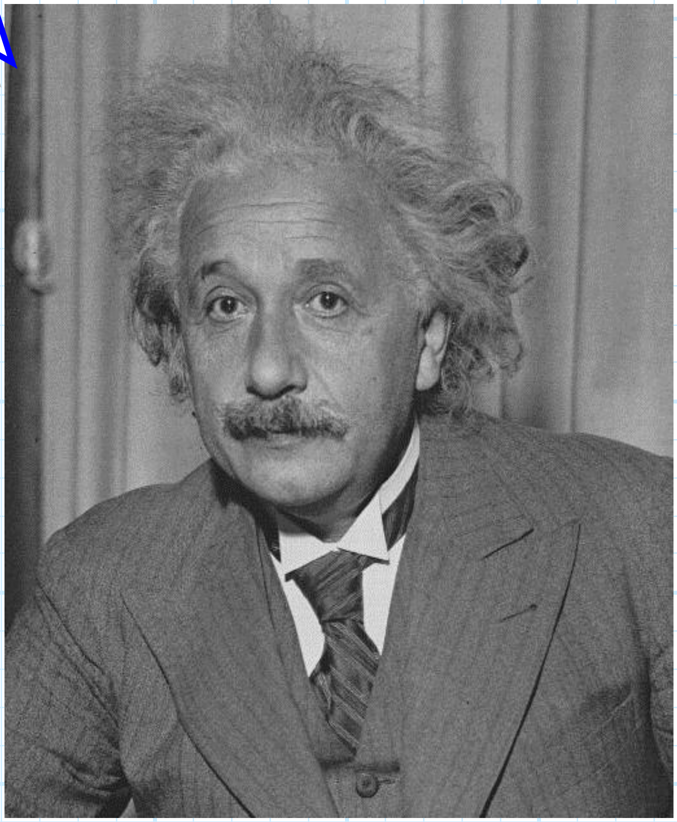
*It* ***appears*** *we have a* ***valid, continuous, function****!*



*Like $E = mc^2$, the circuit in this example may **seem** trivial, but it is actually **very important**!*

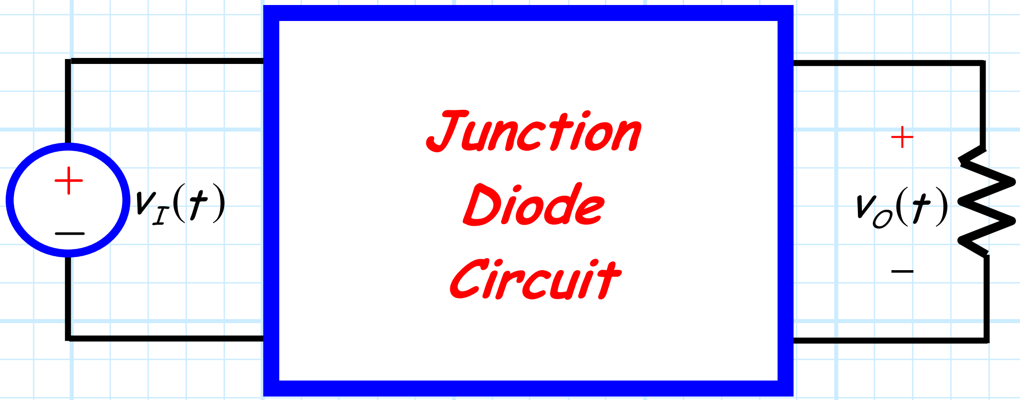
*This circuit is called a **half-wave rectifier**, and provides signal **rectification**.*

*Rectifiers are an **essential** part of every AC to DC **power supply**!*



# Time-domain Analysis of Diode Circuits

Let's consider what happens when the input to a junction diode circuit **varies with time**!




The **output** will **likewise** vary with time!

If we know the large signal **transfer function** of diode circuit:

$$v_O = f(v_I)$$

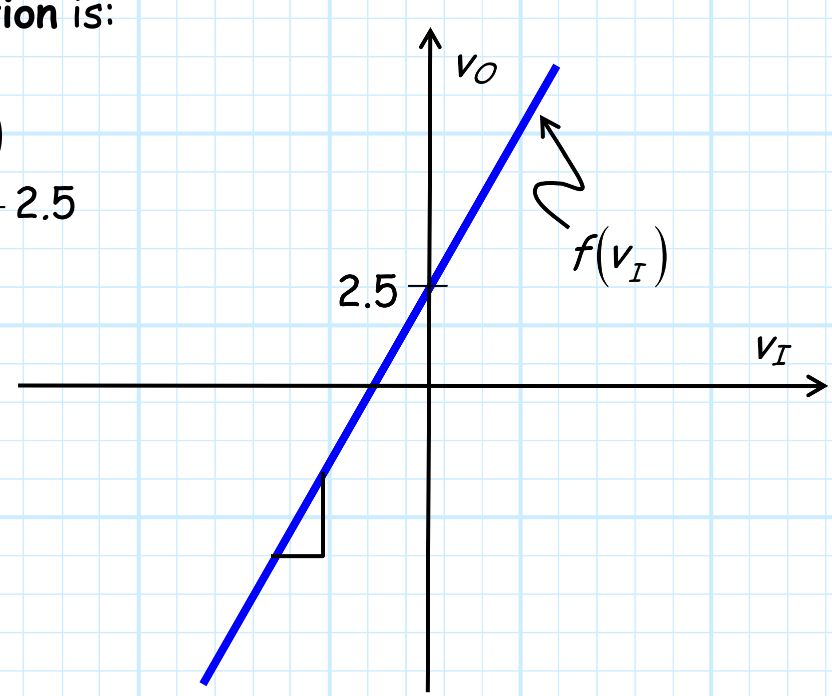
Then the **time-varying output** (assuming the input is not changing **too fast**!) is expressed as:

$$v_O(t) = f(v_I(t))$$

# For example…

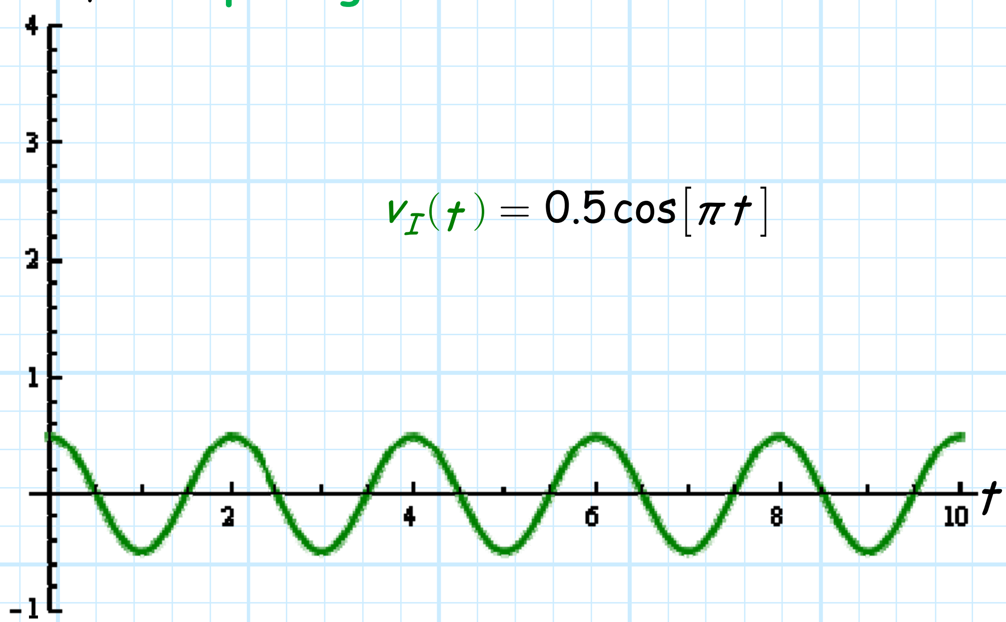
For **example**, say our circuit **transfer function** is:

$$v_O = f(v_I) = 2v_I + 2.5$$




And, the **input signal** **varies with time** as:

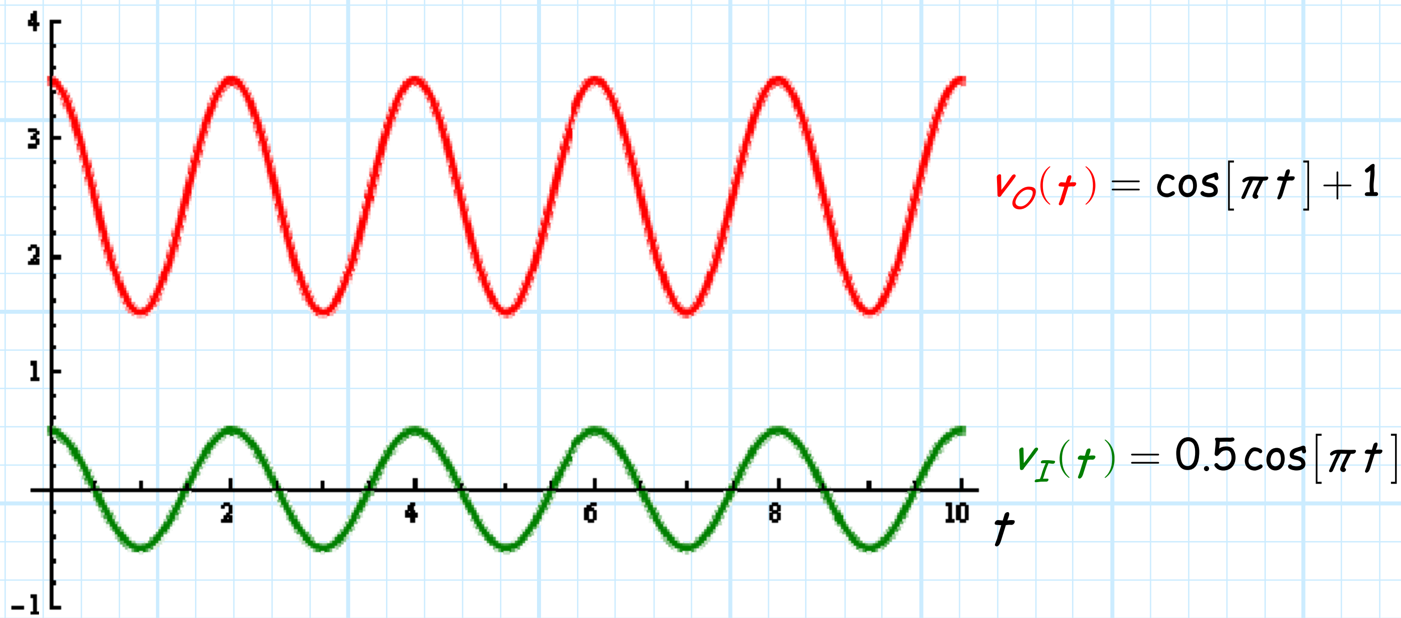
$$v_I(t) = 0.5\cos[\pi t]$$

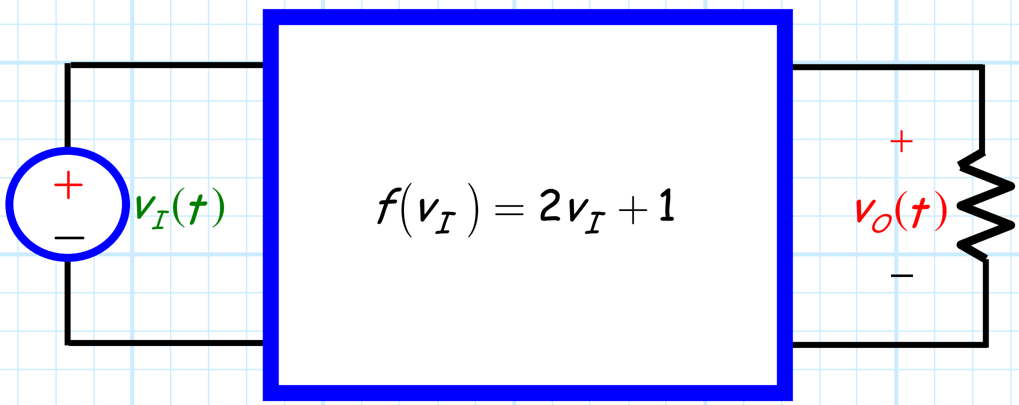


# Make this make sense to you

The **output signal** is thus:

$$
\begin{aligned}
v_O(t) &= 2\,v_I(t) + 1 \\
&= 2\left(0.5\cos[\pi t]\right) + 1 \\
&= \cos[\pi t] + 1
\end{aligned}
$$

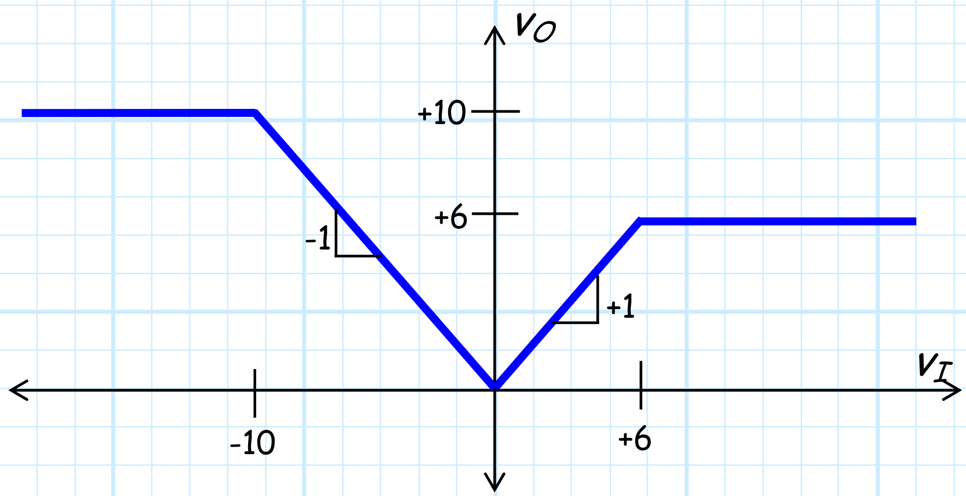


# A trickier example…

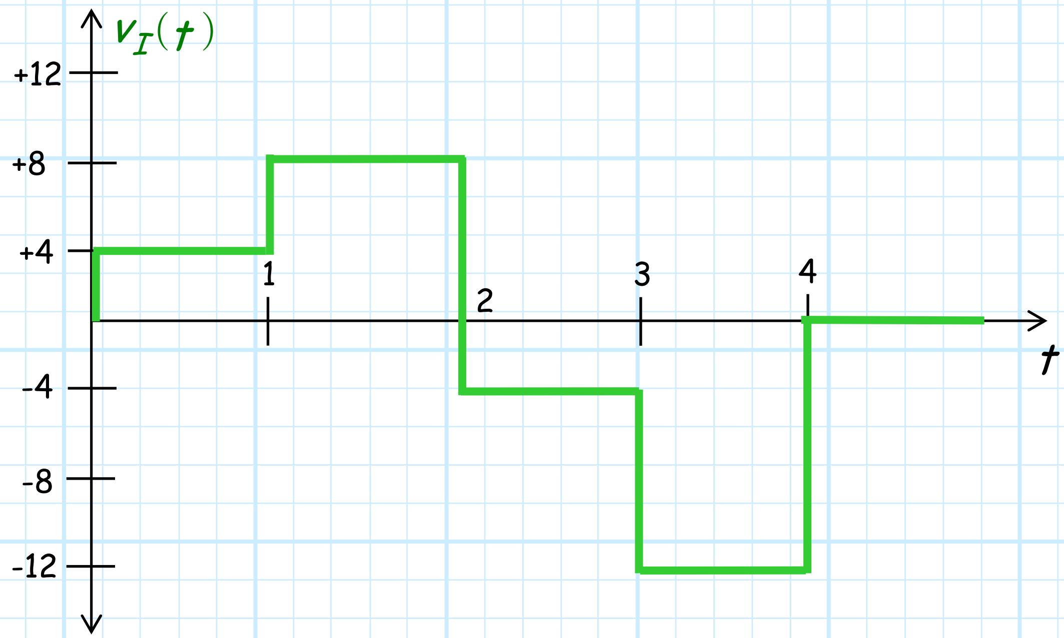
Or, consider a diode circuit with **this** transfer function:

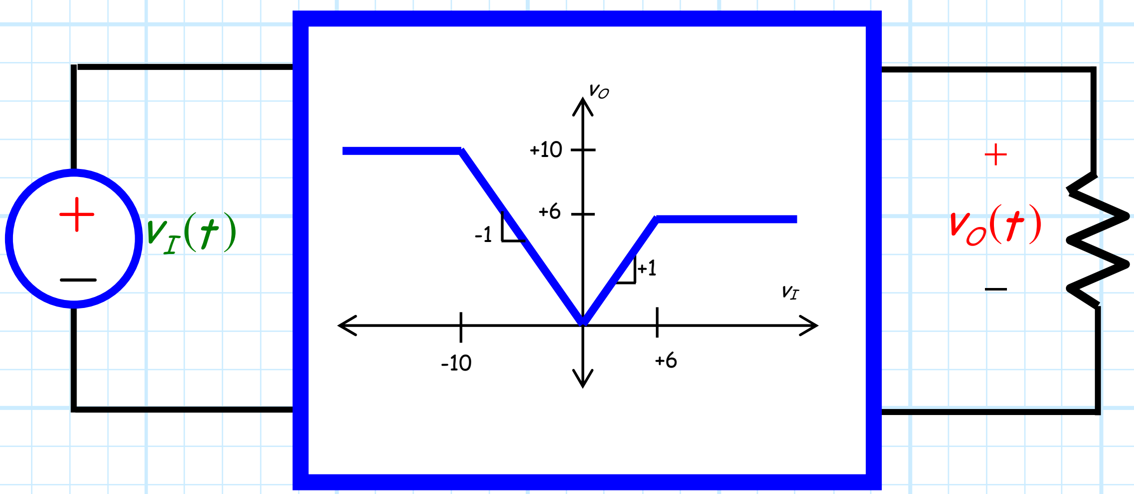
$$v_O = \begin{cases} 6.0 & \text{for} \quad v_I > 6.0 \\ v_I & \text{for} \quad 0 < v_I < 6.0 \\ -v_I & \text{for} \quad -10.0 < v_I < 0 \\ 10.0 & \text{for} \quad v_I < -10.0 \end{cases}$$



# What is the output?

Say that **this** time-varying signal appears at its **input**:

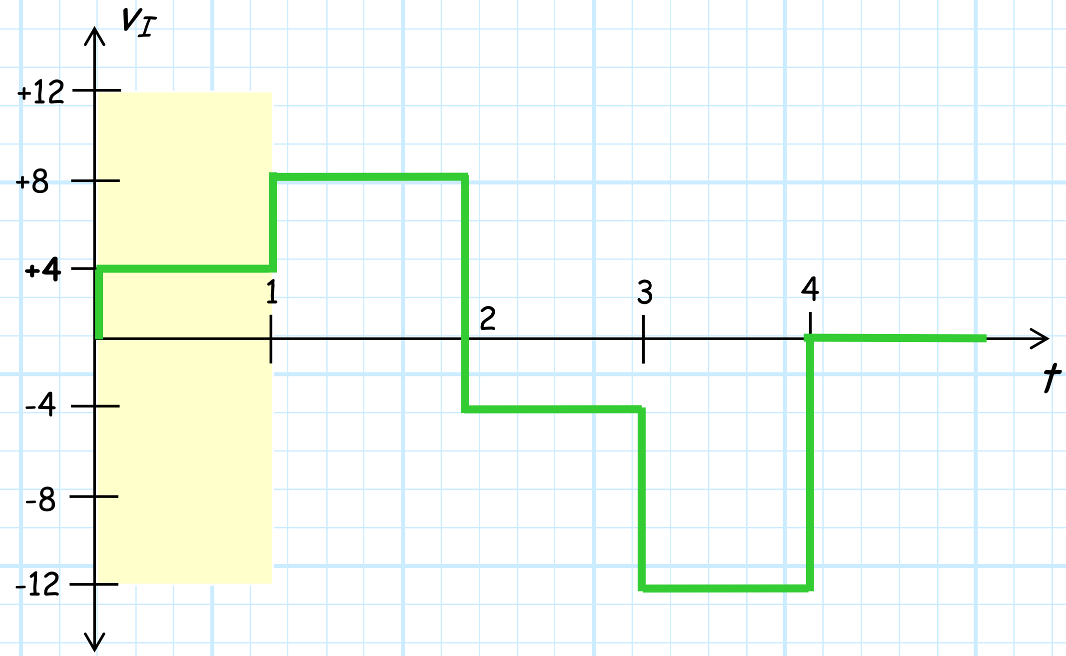


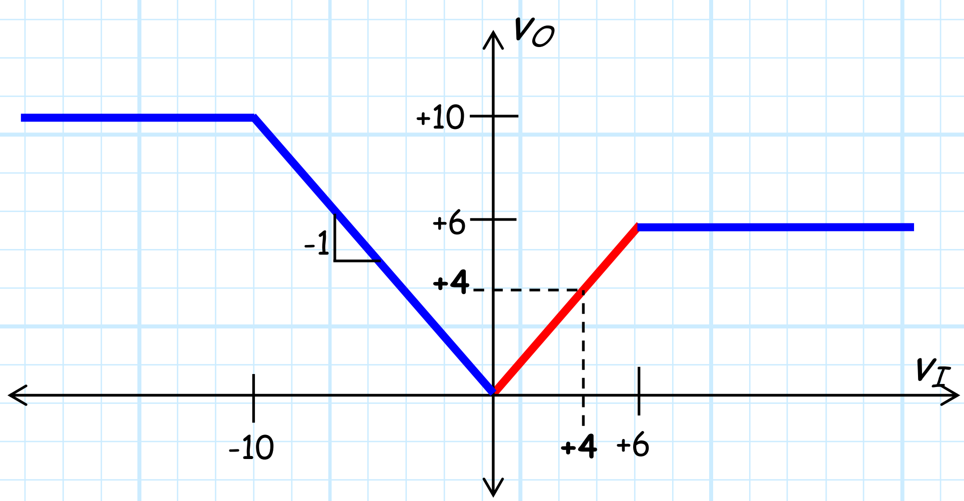

What then is this **output voltage**?

# The answer is simple—*if* you think

Note that for time $0 < t < 1$, the input voltage is **4.0 V**



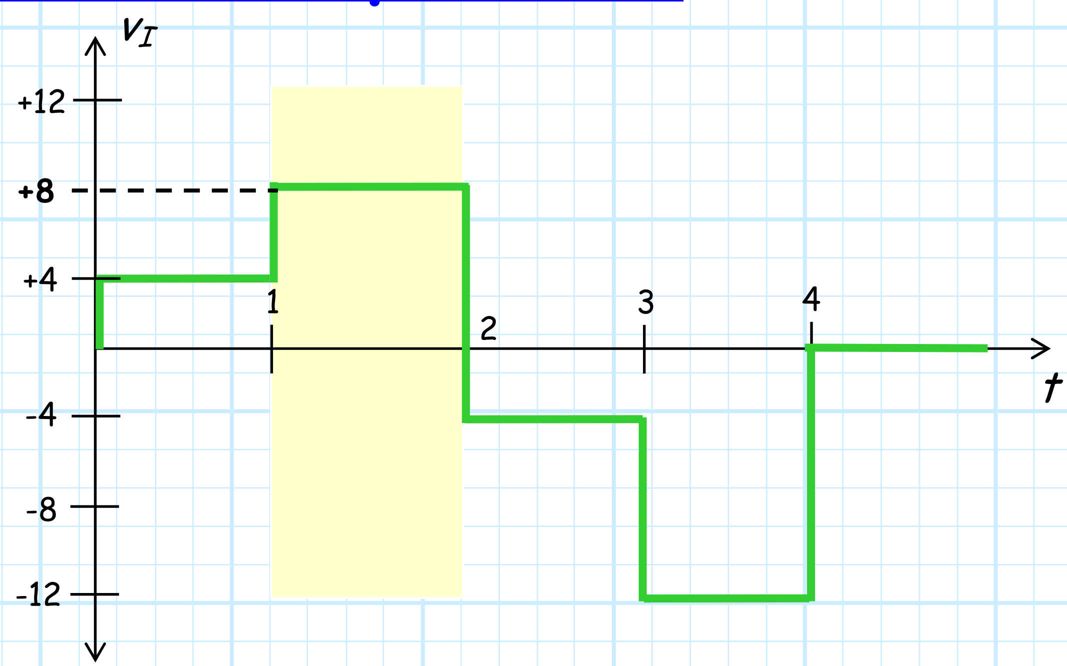

Since $v_I = 4.0$ volts is greater than zero, but less than 6.0 V, $(0 < v_I < 6.0)$ the output voltage during this time period is $v_O = v_I = 4.0\ V$:



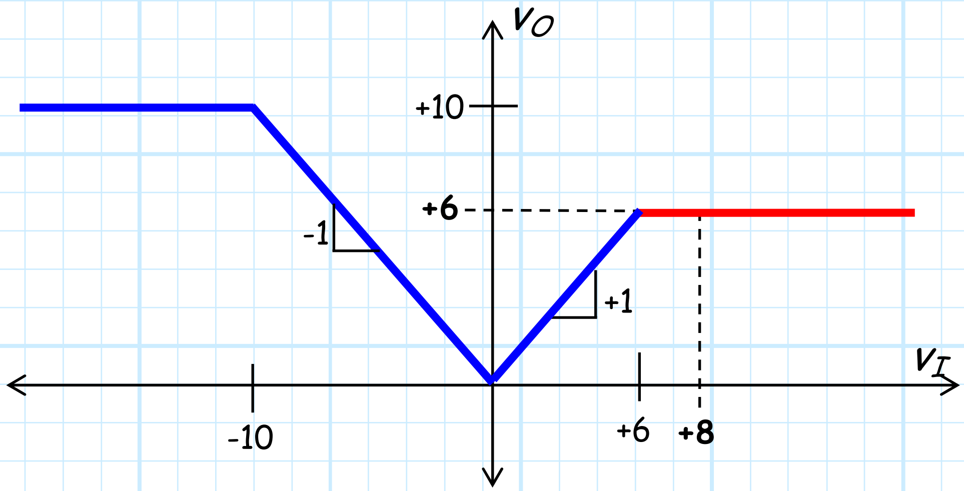

$$v_O = 4.0 = \begin{cases} 6.0 & \text{for} \quad v_I > 6.0 \\ v_I & \text{for} \quad 0 < v_I < 6.0 \\ -v_I & \text{for} \quad -10.0 < v_I < 0 \\ 10.0 & \text{for} \quad v_I < -10.0 \end{cases}$$

# What is the output now?

Now for time $1 < t < 2$, the input voltage is 8.0 V



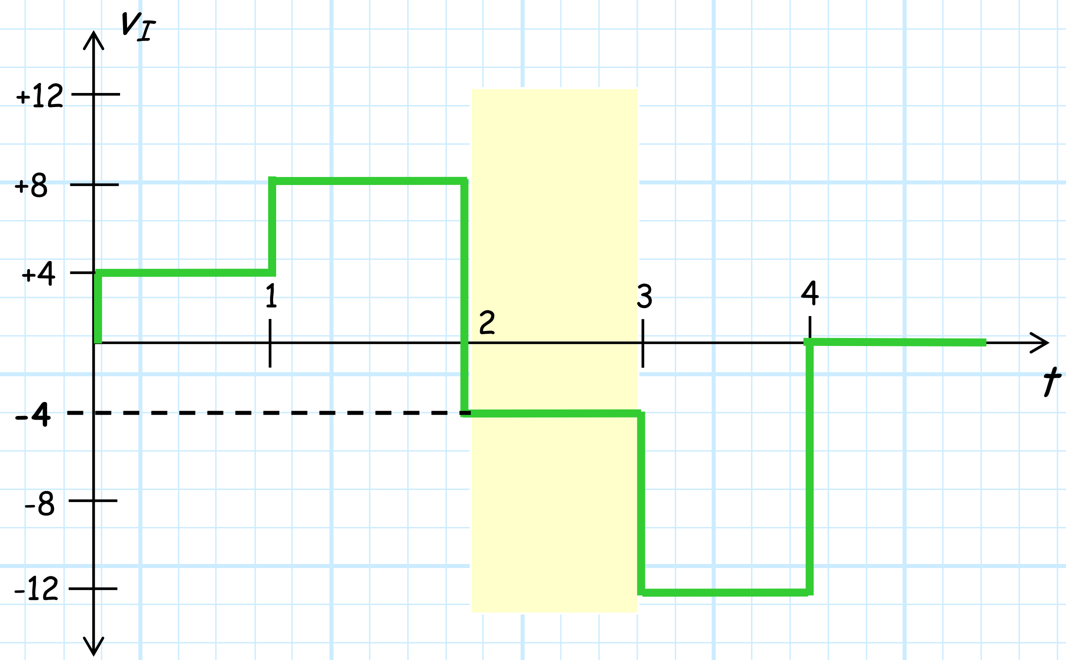

Since $v_I = 8.0$ volts is greater than 6.0 V, ($v_I > 6.0$) the output voltage during this time period is $v_O = 6.0\,V$:



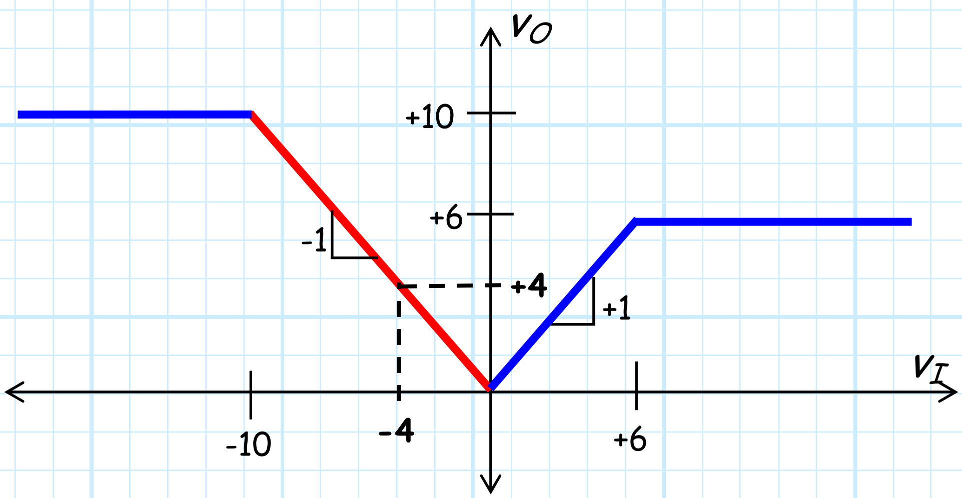

$$v_O = 6.0 = \begin{cases} 6.0 & \text{for} \quad v_I > 6.0 \\ v_I & \text{for} \quad 0 < v_I < 5.0 \\ -v_I & \text{for} \quad -10.0 < v_I < 0 \\ 10.0 & \text{for} \quad v_I < -10.0 \end{cases}$$

# The hardest one yet!

Now for time $2 < t < 3$, the input voltage is **-4.0 V**.



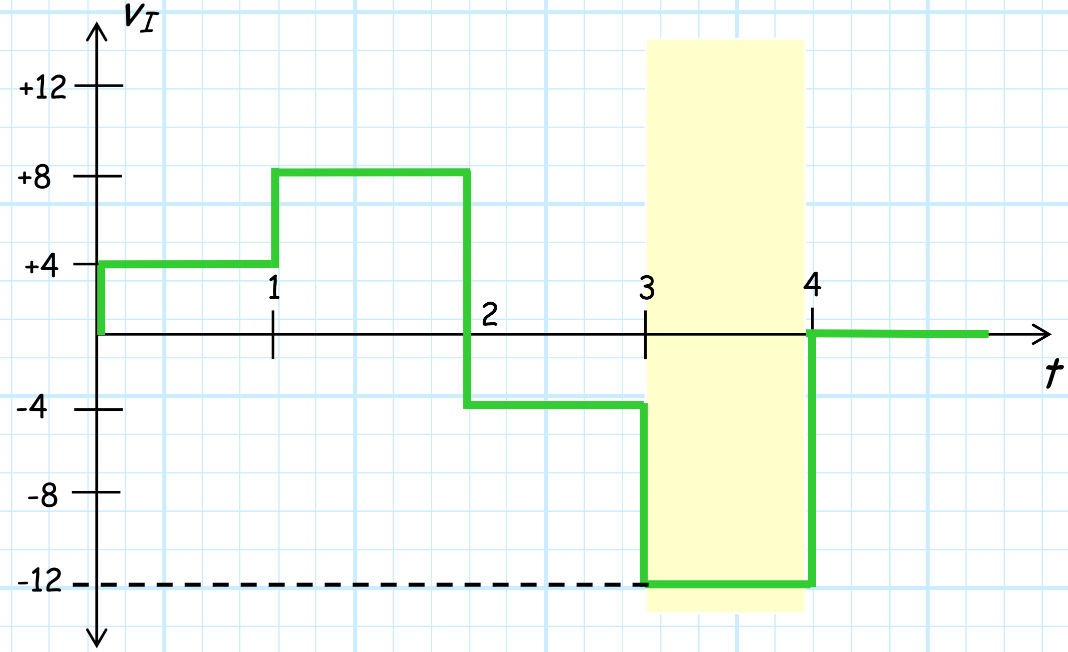

Since $v_I = -4.0$ volts is less than 0 $V$ but greater than -10.0 $V$, $(-10 > v_I > 0)$ the output voltage during this time period is $v_O = -v_I = +4.0\ V$:



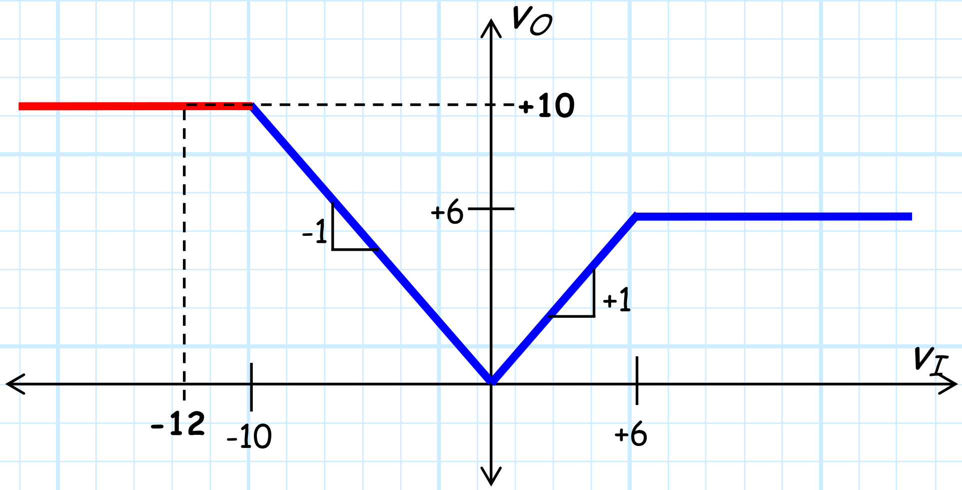

$$v_O = 4.0 = \begin{cases} 6.0 & \text{for} \quad v_I > 6.0 \\ v_I & \text{for} \quad 0 < v_I < 6.0 \\ -v_I & \text{for} \quad -10.0 < v_I < 0 \\ 10.0 & \text{for} \quad v_I < -10.0 \end{cases}$$

# One last bit of scholarly effort

Now for time $3 < t < 4$, the input voltage is **-12.0 V**



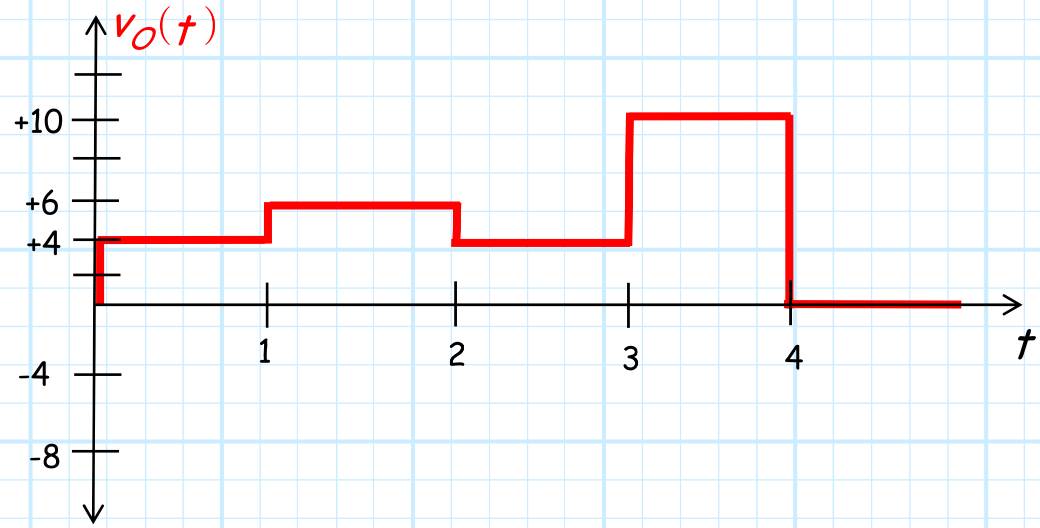

Since $v_I = -12.0$ volts is less than -10.0 V, ($v_I < -10.0$) the output voltage during this time period is $v_O = 10.0\ V$:



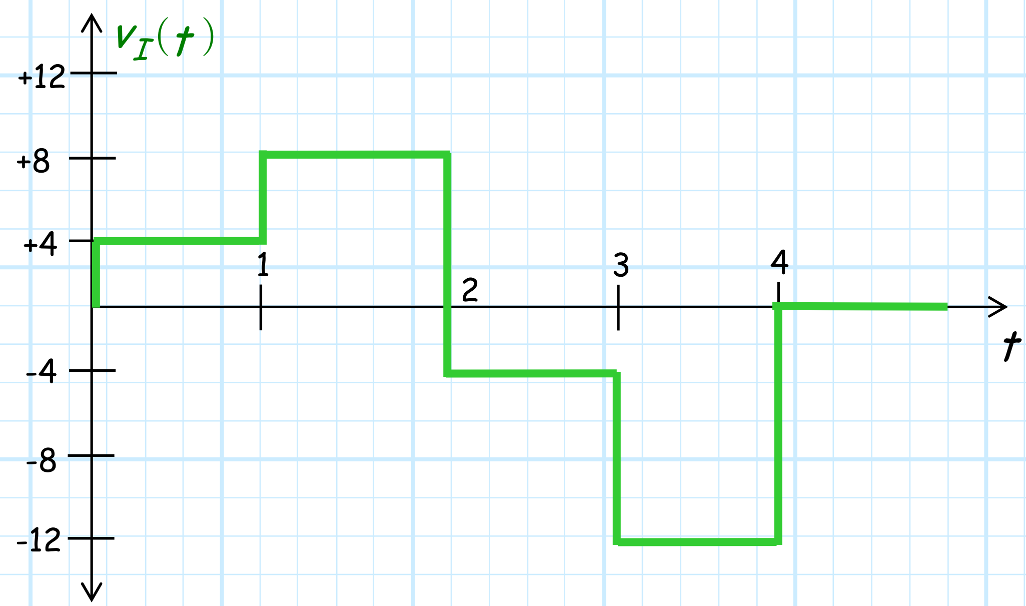

$$v_O = 10.0 = \begin{cases} 6.0 & for \quad v_I > 6.0 \\ v_I & for \quad 0 < v_I < 6.0 \\ -v_I & for \quad -10.0 < v_I < 0 \\ 10.0 & for \quad v_I < -10.0 \end{cases}$$

# Put the pieces together; our output is revealed

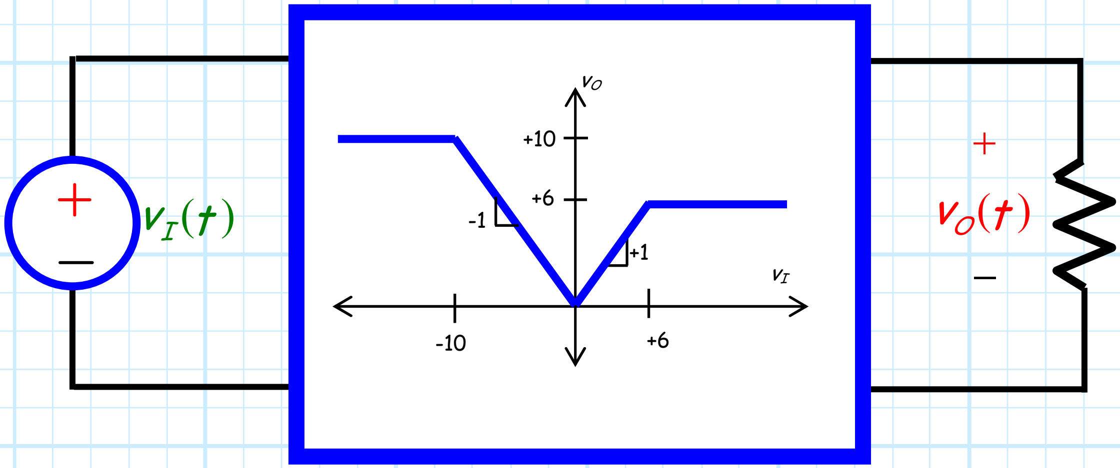
Thus, we find that the **output signal** is:

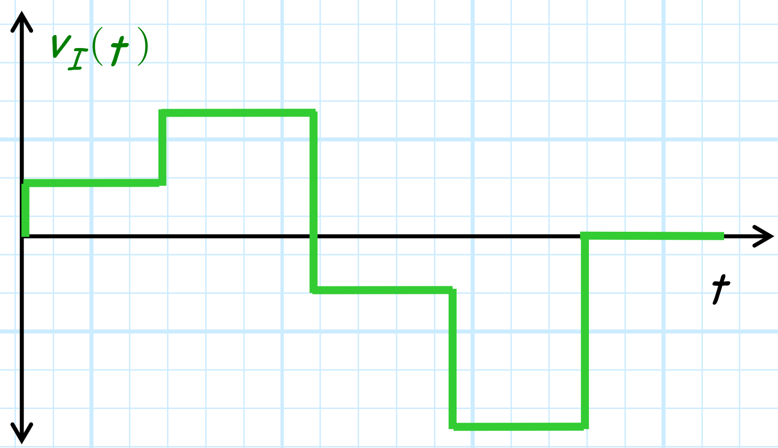


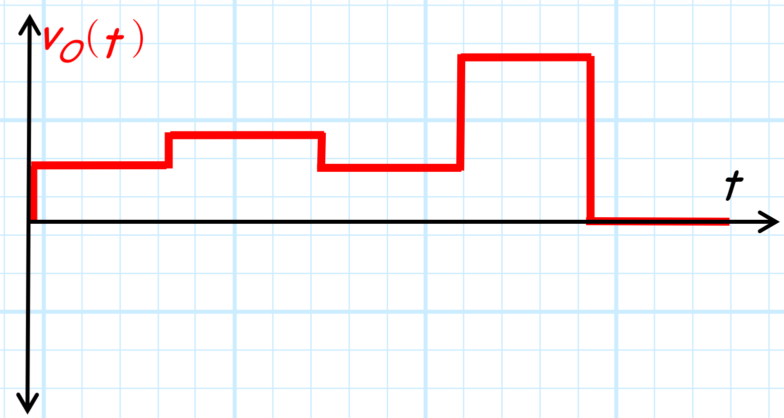

That is, the output when **this** is the **input signal**.



# Make this make sense to you

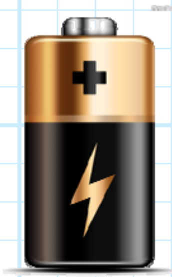


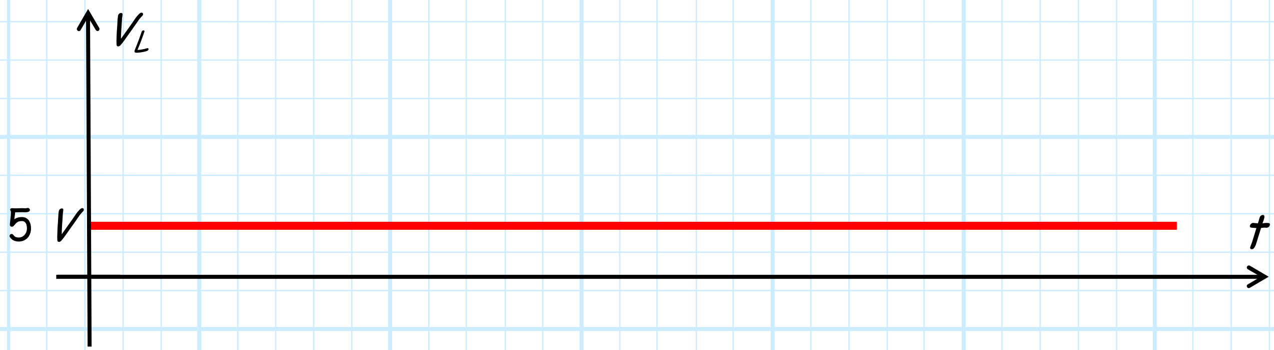



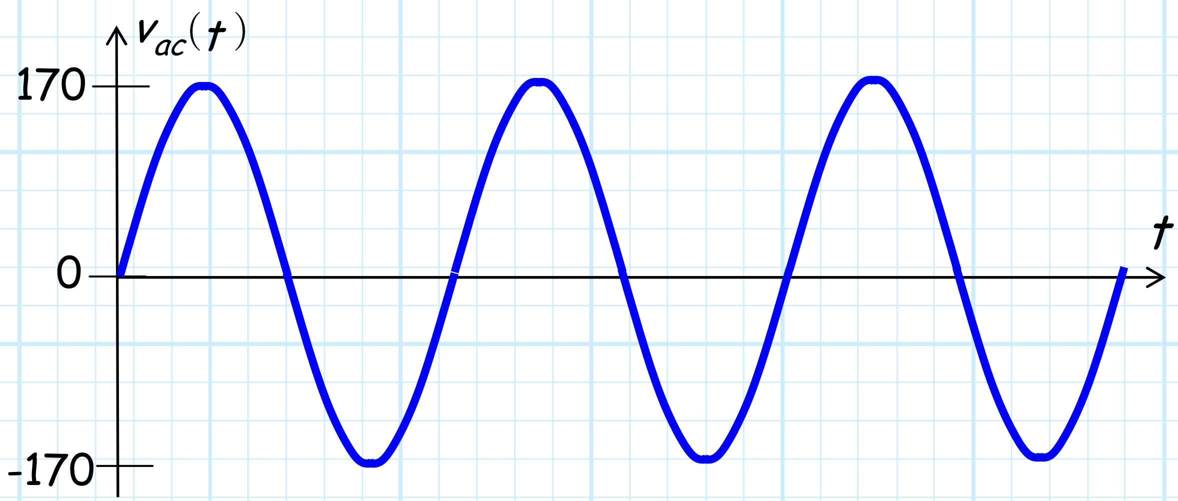
# Power Supplies

Most modern electronic circuits and devices require one or more relatively **low DC voltages** (e.g., 5.0 $V$) for biasing and for supplying power to the circuit.





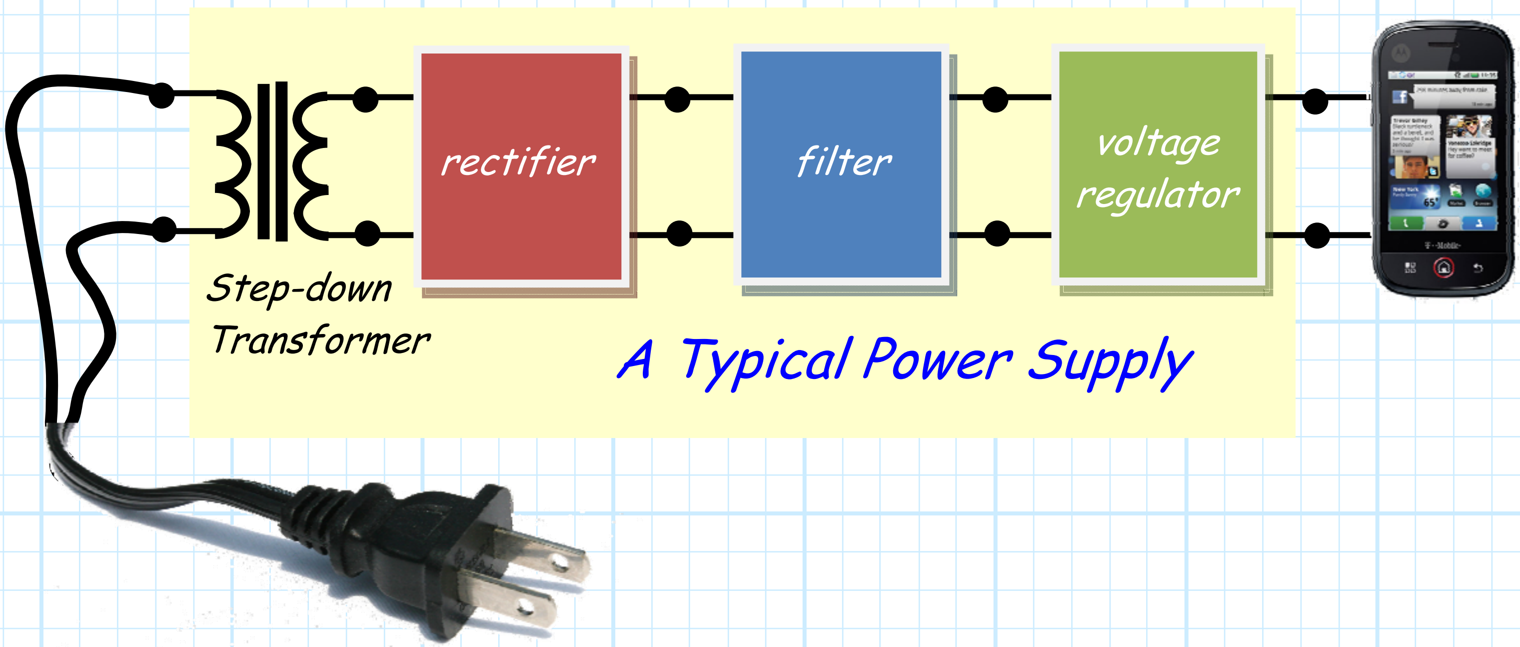

**Big Problem →** Our electrical power distribution system provides a **high-voltage AC output**—a 120 $V_{rms}$, 60Hz sinewave!




# The power supply

Thus, a major component in electronic systems (e.g., computers, televisions, etc.) is the **power supply**.

The purpose of the power supply is simply to **convert high voltage AC** into one or more **low DC voltages**.

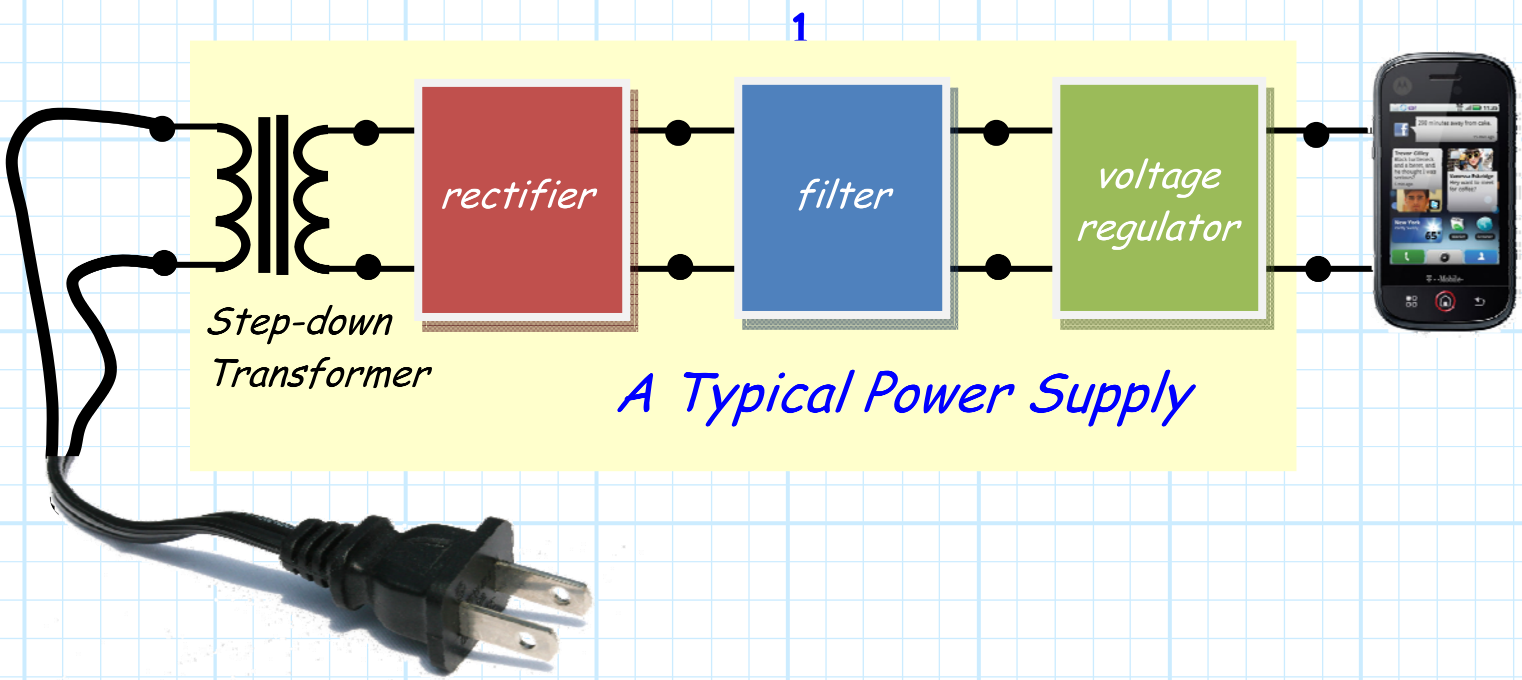


# The functional schematic of a power supply

A typical **power supply** consists of **four majors sections**:

1. Step-Down **Transformer**
2. Signal **Rectifier**
3. **Filter**
4. Voltage **Regulator**

Let's look at each of these devices **individually**!

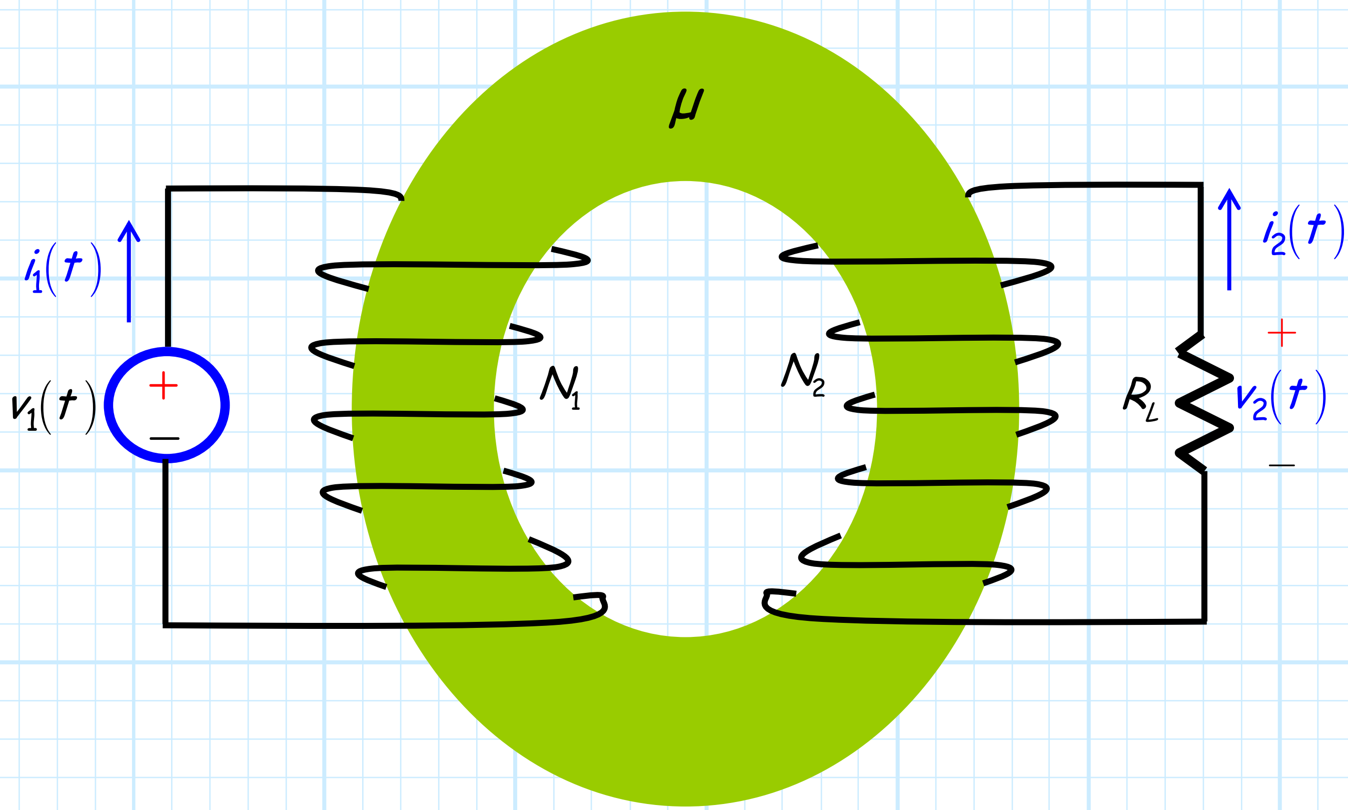


# 1. The Step-Down Transformer


A voltage 120 $V_{rms}$ is simply **too big**!

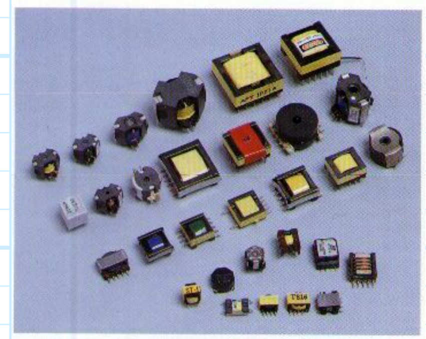
The first thing to be accomplished it to "**step-down**" the AC to a more manageable (e.g., safe) level.

We do this with a **step down transformer**.



# You remember—don't you?

You will recall that the important design parameters of a transformer are the number of **"turns"** on each side of the transformer.



The integer value $N_1$ represents the number of turns on the **primary** side, whereas $N_2$ represents the number of turns on the **secondary** side.

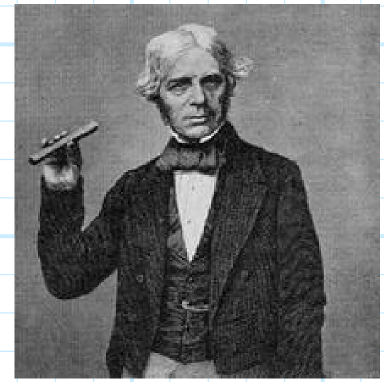
The voltage on the **secondary** side is related to the voltage on the **primary** side as:

$$v_2(t) = \frac{N_2}{N_1} v_1(t)$$

Thus, if we wish to **step-down** the voltage (i.e., make $v_2(t) < v_1(t)$), we need to make the number of primary turns **larger** than the number of secondary turns (i.e., $N_1 > N_2$).

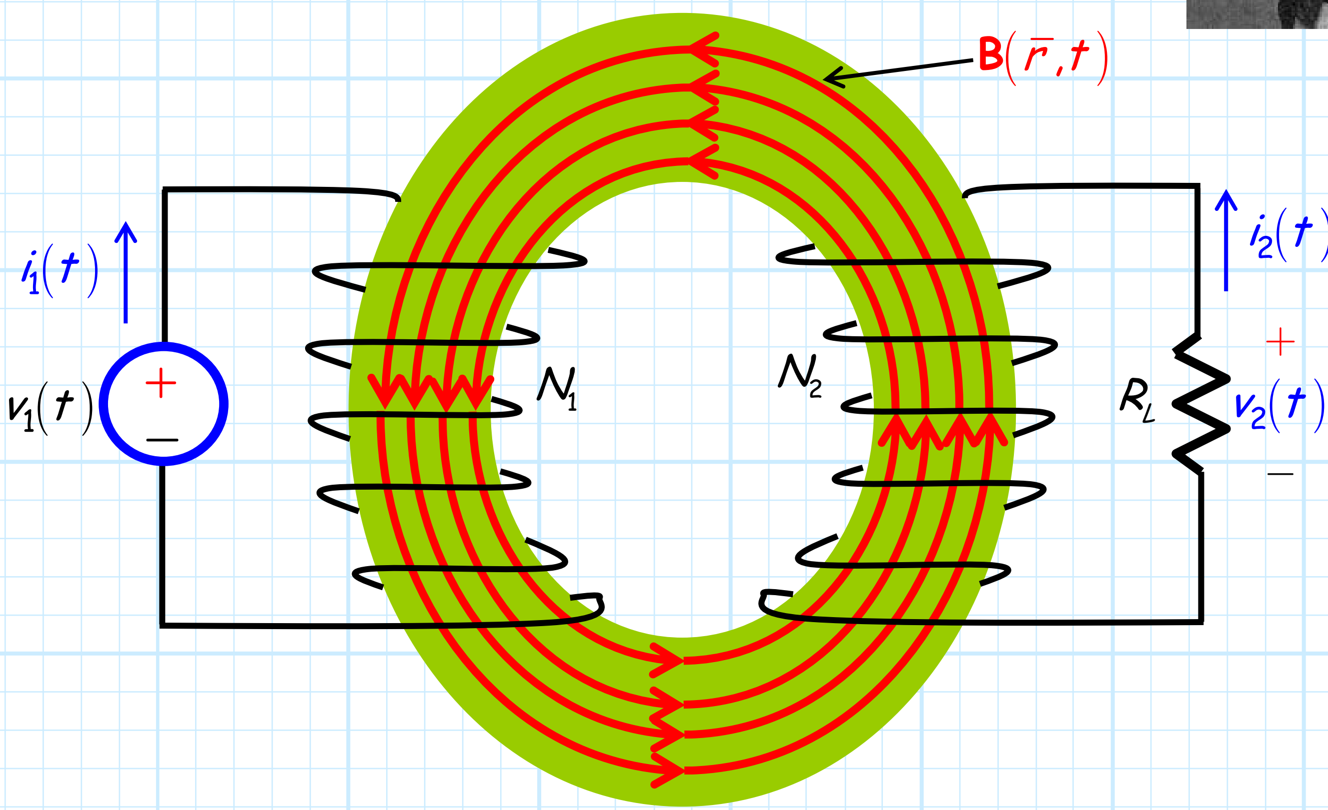
→ **Typically**, a step-down transformer will lower the *AC* voltage to around **30 V**$_{rms}$.

# And Michael Faraday never even attended college—so now what's your excuse?



Remember, a transformer is a fundamental application of **Faraday's Law**—one of Maxwell's equations!

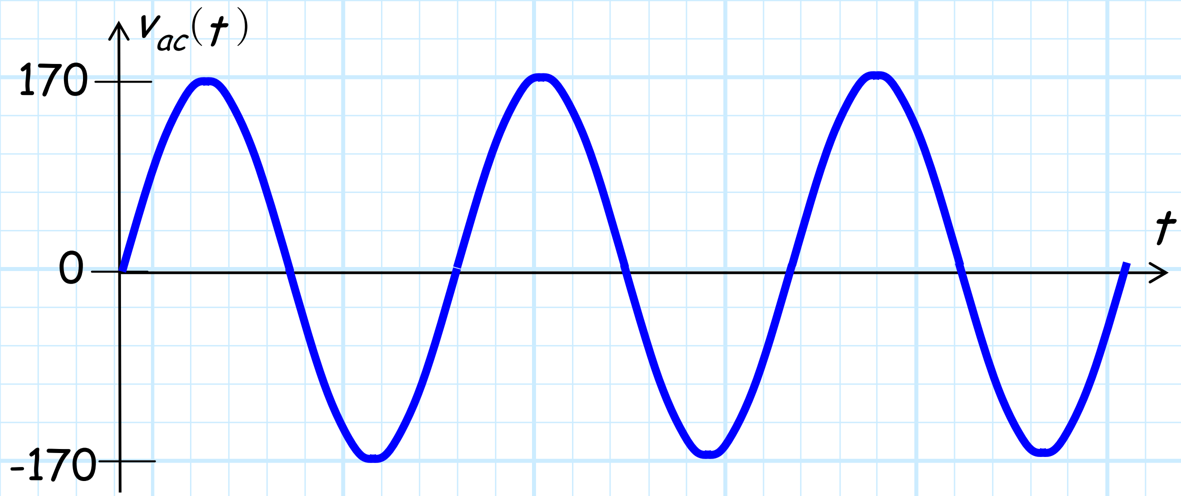
$$\oint_{C_1} \mathbf{E}(\bar{r}) \cdot \overline{d\ell} = -\frac{\partial}{\partial t} \iint_{S_1} \mathbf{B}(\bar{r},t) \cdot \overline{ds}$$

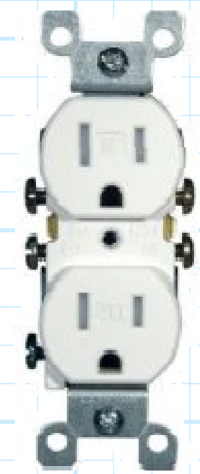


# Pure AC; no DC

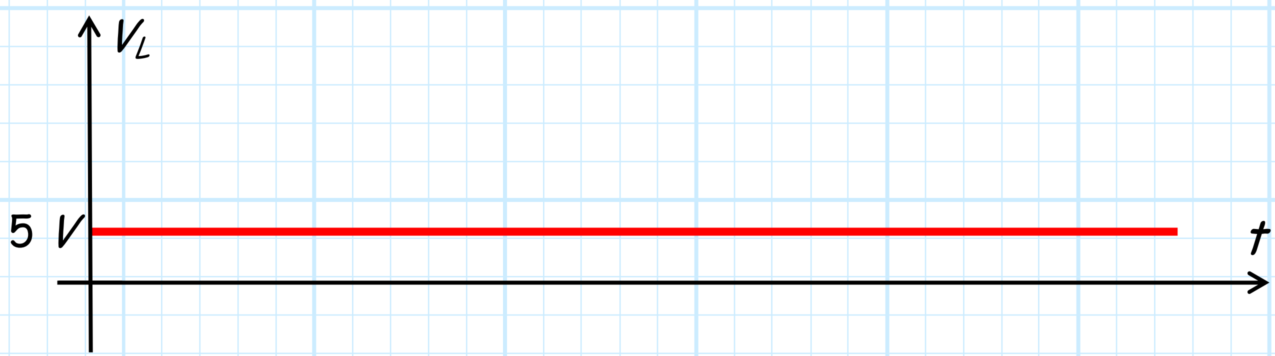
Recall that the output of the step-down transformer is a **sinusoid**—a **"pure" AC signal**.

In other words, the output has **no DC component**.



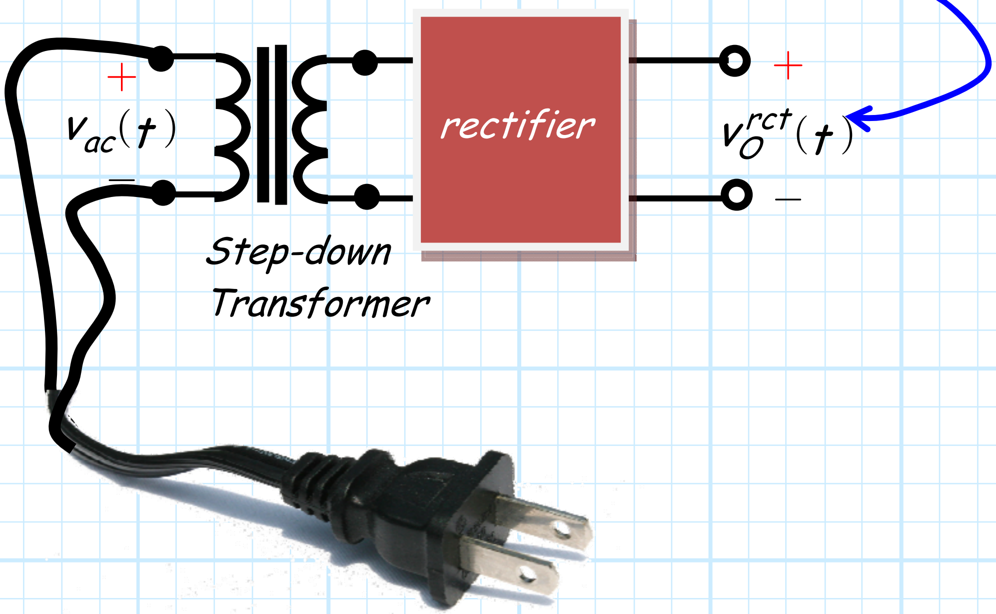



But this is a problem, as we need a **DC signal** at the power supply output!



# 2. The Signal Rectifier

This is where the signal rectifier comes in—its job is to take an AC signal, and produce a **signal with a DC component**.




**Q:** *So, the output of a rectifier* ***is a DC signal?***

**A:** **NO!** I didn't say that! The output of a rectifier has a **DC component**.

However, it **likewise has an AC component**—the signal output is still **time varying**!

# AC/DC


**Q:** *Huh?*

**A:** Most signals are **neither** "purely" AC (a time-varying signal with a **time-averaged** value of **zero**), or "purely" DC (a **constant** with respect to time).

Most signals are a **combination** of AC and DC—they have both an **AC component** and a **DC component**.

I.E., they can be expressed as the **sum** of a DC and AC signal:

$$v(t) = V_{DC} + v_{ac}(t)$$

The **DC component** of a signal $v(t)$ is simply its **time-averaged** value:

$$V_{DC} = \frac{1}{T}\int_0^T v(t)\, dt$$

and the **AC component** of a signal $v(t)$ is simply the signal $v(t)$ with its **DC component removed**:

$$v_{ac}(t) = v(t) - V_{DC}$$

# Most time-varying signals are *not* AC signals

The **important fact** about an **AC component** is that it has a **time-averaged value of zero**!

$$
\begin{aligned}
\frac{1}{T}\int_0^T v_{ac}(t)\,dt &= \frac{1}{T}\int_0^T \left(v(t) - V_{DC}\right)dt \\
&= \frac{1}{T}\int_0^T v(t)\,dt - \frac{1}{T}\int_0^T V_{DC}\,dt \\
&= V_{DC} - V_{DC}\,\frac{1}{T}\int_0^T dt \\
&= V_{DC} - V_{DC}\,\frac{1}{T}(T-0) \\
&= V_{DC} - V_{DC} \\
&= 0
\end{aligned}
$$

**Pay attention:** an **AC signal** is defined as a **time-varying** signal whose **average value is zero**!

# Please tell me this is obvious

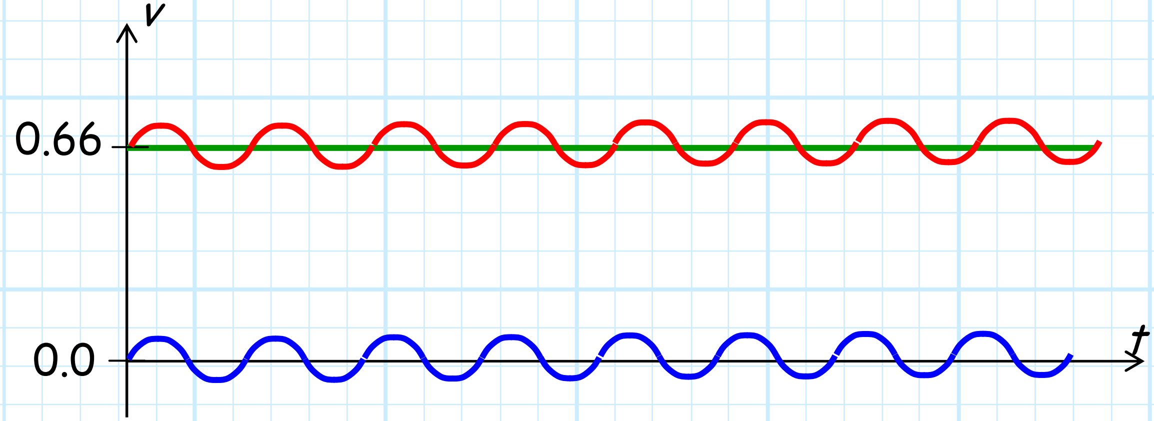
For **example**, a signal **might** have the form: $v(t) = 0.66 + 0.001\cos\omega t$

It is **hopefully evident** that this signal has a **DC component**:

$$
\begin{aligned}
V_{DC} &= \frac{1}{T}\int_0^T v(t)\,dt \\
&= \frac{1}{T}\int_0^T \left(0.66 + 0.001\cos\omega t\right)dt \\
&= \frac{0.66}{T}\int_0^T dt + \frac{0.001}{T}\int_0^T \cos\omega t\,dt \\
&= 0.66 + 0 = 0.66
\end{aligned}
$$

and an ***AC* component**:

$$
\begin{aligned}
v_{ac}(t) &= v(t) - V_{DC} \\
&= \left(0.66 + 0.001\cos\omega t\right) - 0.66 \\
&= 0.001\cos\omega t
\end{aligned}
$$



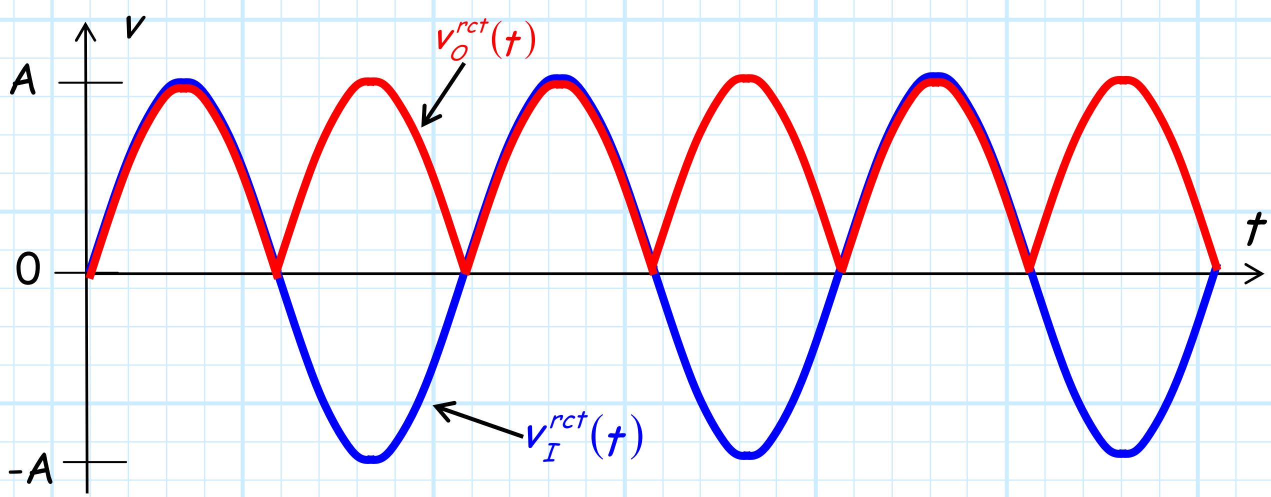
# A time-varying signal with an DC component

Since the **input** to the rectifier is a 60Hz **"sine wave"**, it has **no DC component**—the time averaged value of a sine function is **zero**.

Thus, the input is a **"pure" AC signal**.

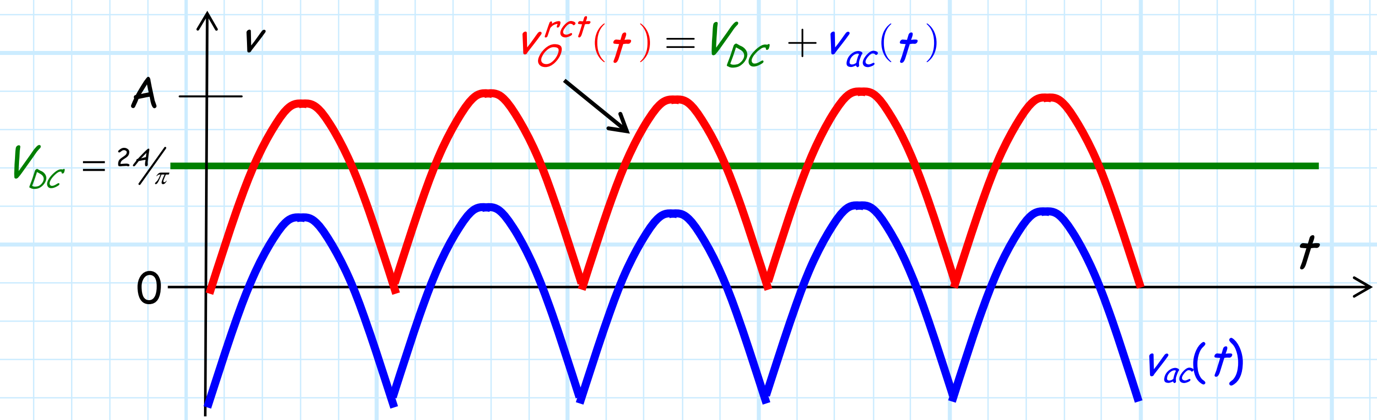
The **output** of a rectifier is likewise time-varying—it has an **AC component**.

However, the time-averaged value of this output is **non-zero**—the **output** also has a **DC component**!



# The Rectifier creates a DC component; but the AC component is still there

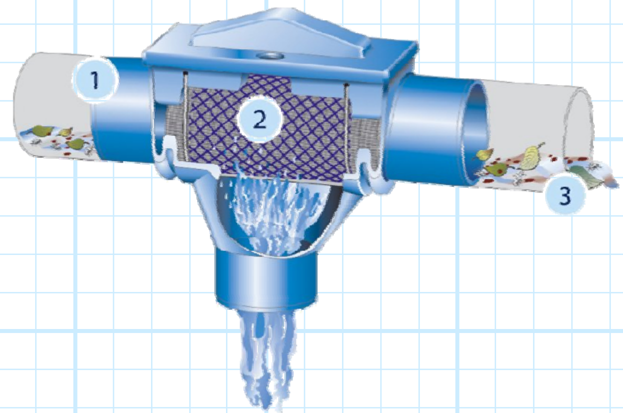
Thus, a rectifier creates a **DC component** on the output, a component that does **not** exist on the **input**!



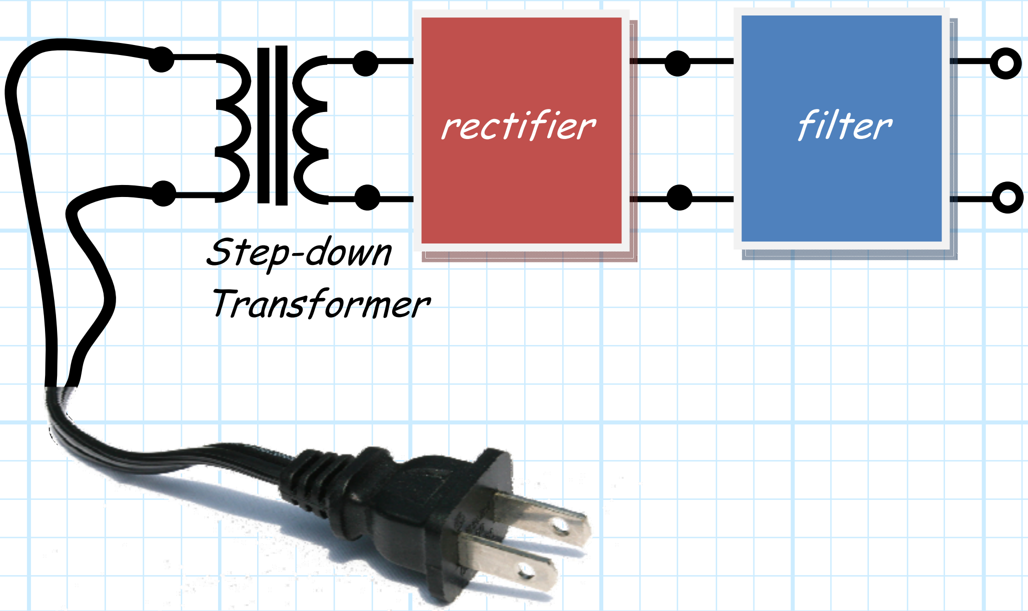
# 3. The Filter

So, the rectifier has **added a DC component** to the signal—but the output of the rectifier is not DC, it has an AC component as well.

The job of any filter is to **remove unwanted components, while allowing the desired components to pass through.**




For this **electrical filter**, the **unwanted** component is the **AC signal**, and the **desired** component is the **DC signal**!



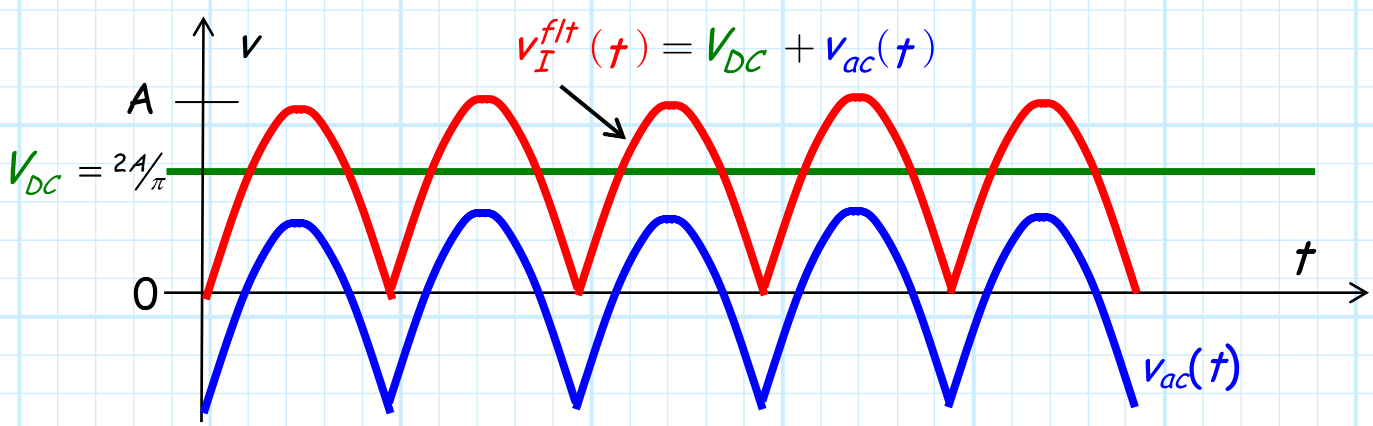

We do this with what is essentially a **low pass filter**.

# The DC component passes through the filter unchanged

Ideally, the low-pass filter would **eliminate** the **AC component**, leaving **only** a constant, **DC signal**.

However, a low-pass filter will typically just **attenuate** the AC signal, leaving a **small AC component** at the filter output.

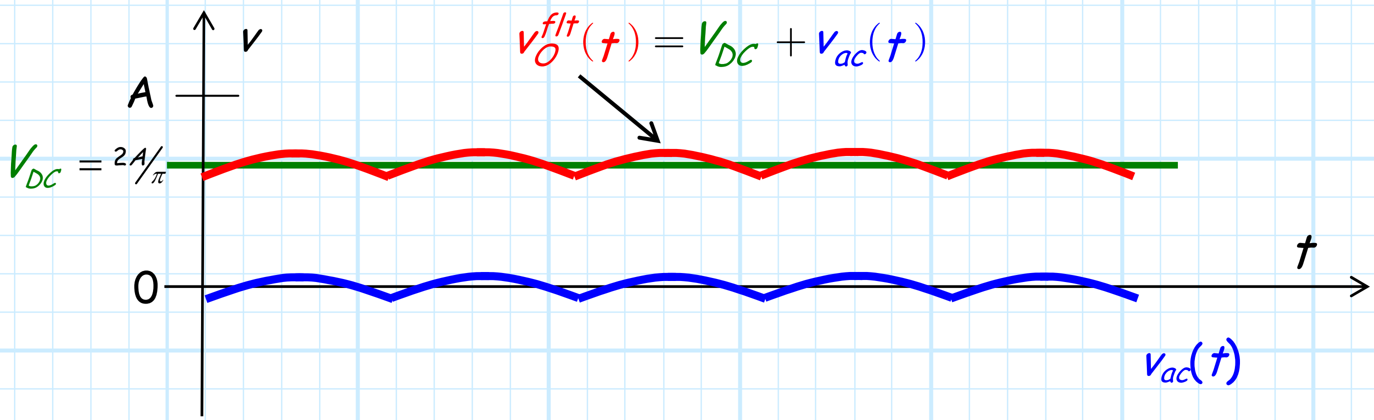
For example, the **ouput** of the rectifier is now the **input to filter**:




→ The **DC component** passes through the low-pass filter **unscathed**—it's the **same** at the output as the input.

# Ripple voltage!

But; the **AC** component is **greatly attenuated**. Thus, the output is **almost** a DC signal—it is a **DC signal** with a **small AC** "ripple voltage".




**Q:** *Yikes! Our load (e.g., computer, HDTV, DVR, DVD, etc.) will **require** a better **regulated voltage** than that!*

*Won't the "ripple voltage" cause all sorts of problems—**if** it is used to supply power to the load!*

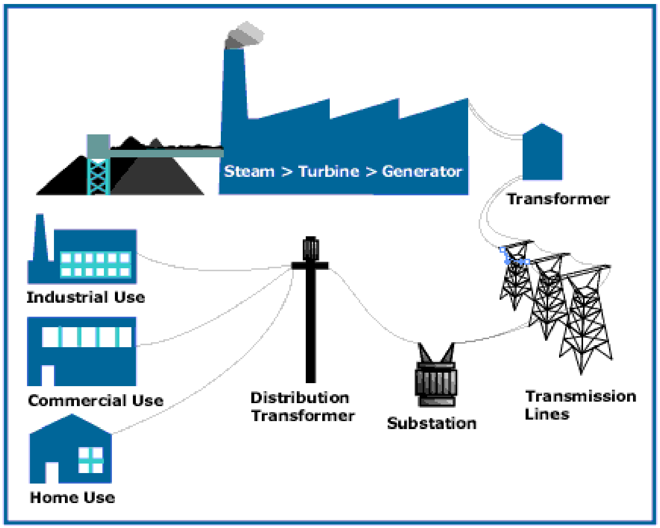
**A:** Yes, and it's **even worse** than that!

→ You see, the 120 $V_{rms}$, 60 Hz signal from our wall socket may **not be exactly** that!

# It will be 60 Hz; it might not be 120 $V_{rms}$

**Q:** *You mean it might not be exactly **60 Hz**?*

**A:** Actually, a 60 Hz sinusoid is the one thing we **can** count on—the AC power signal **will** have a frequency of **exactly**, precisely **60Hz**.




However, the **magnitude** of this AC power signal (nominally 120 $V_{rms}$) is a bit more problematic.

Power lines, transformers—all the **stuff** that the AC power must pass through—exhibit **resistance**.

The more **current** passing through the power system, the more this resistance results in a **voltage drop** (it's just **Ohm's Law** at work!).

→ As a result, the AC voltage at **your** wall plug will be only **approximately** 120 $V_{rms}$.

# When all those air conditioners are "on"

Say after a brutally **hot** July day, it's still 93 degrees *F* at 8:30 p.m.

The whole town is at **home**; lights on, watching TV, playing Wii.

And, most importantly, running their **air conditioners**!

All this stuff requires **energy**—for this case, energy delivered at a **particularly high rate**.

Thus, the *AC* current flowing into all these cool and content homes is **particularly large**, meaning that the "Ohmic Loss"—the **voltage drop** that occurs between the power plant to your home—is likewise **particularly large**.

Therefore, the *AC* voltage at your wall socket on this **sweltering** day may be **significantly less** than 120 $V_{rms}$!

# It's all proportional to the AC power voltage

**Q:** *Is this a problem?*

**A:** Absolutely!

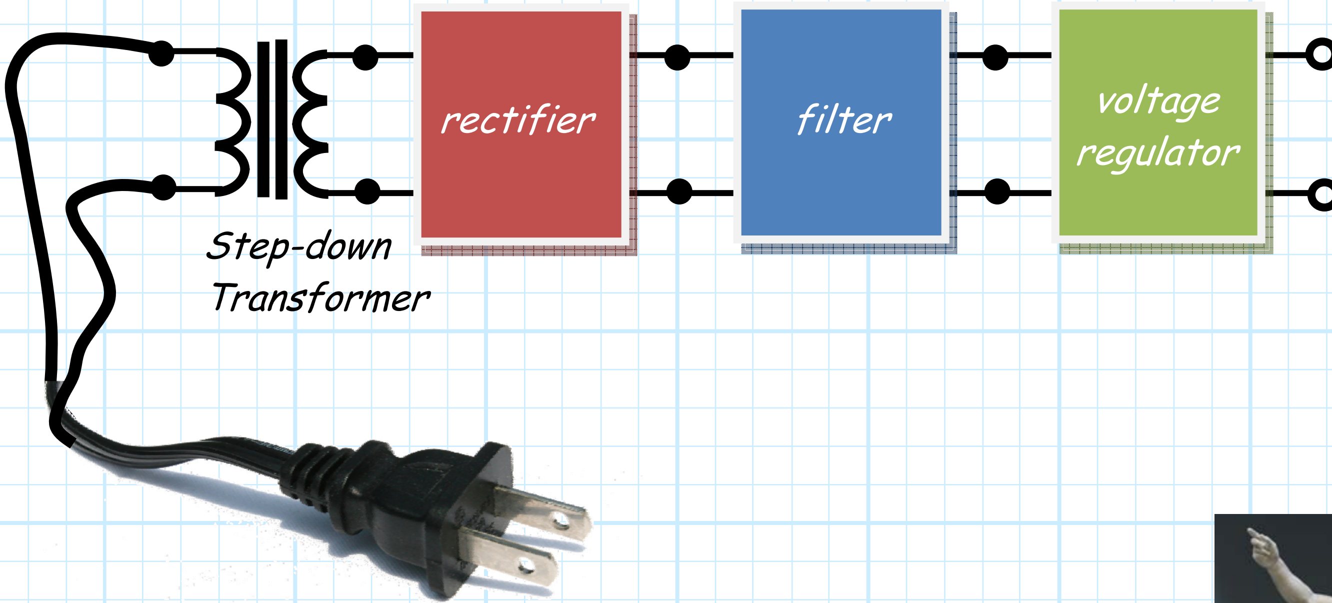
If the voltage of the AC power lessens, a **proportionate reduction** will occur at the output of the step-down transformer.

Therefore the **DC component** at the output of the rectifier (and thus at the output of the filter) will also **reduce proportionately**!

**Q:** *So in* ***addition*** *to the ripple voltage, the* ***DC component*** *of the signal can drift* ***up or down****—isn't this DC supply voltage horribly regulated?*

**A:** That's exactly correct—which is why we need, as the last component of the power supply, a **voltage regulator**!

# 4. The Voltage Regulator




This regulator of course provides at its output a **regulated DC voltage**—a voltage that stays a **rock-solid** constant, regardless of **how many air-conditioners** are running (i.e., great line regulation).

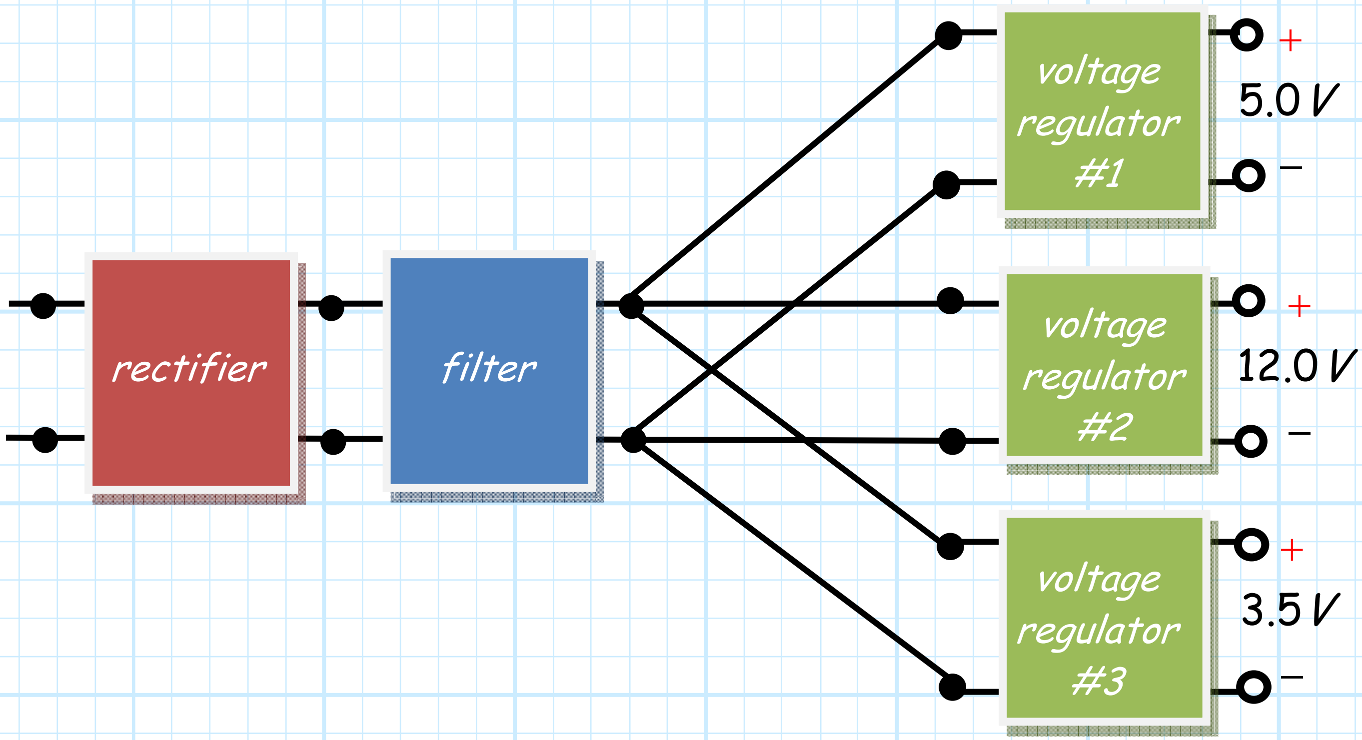
And, regardless of the current drawn by the load (great **load regulation**).

This **voltage regulator** could of course either be a **linear** or **switching** regulator.

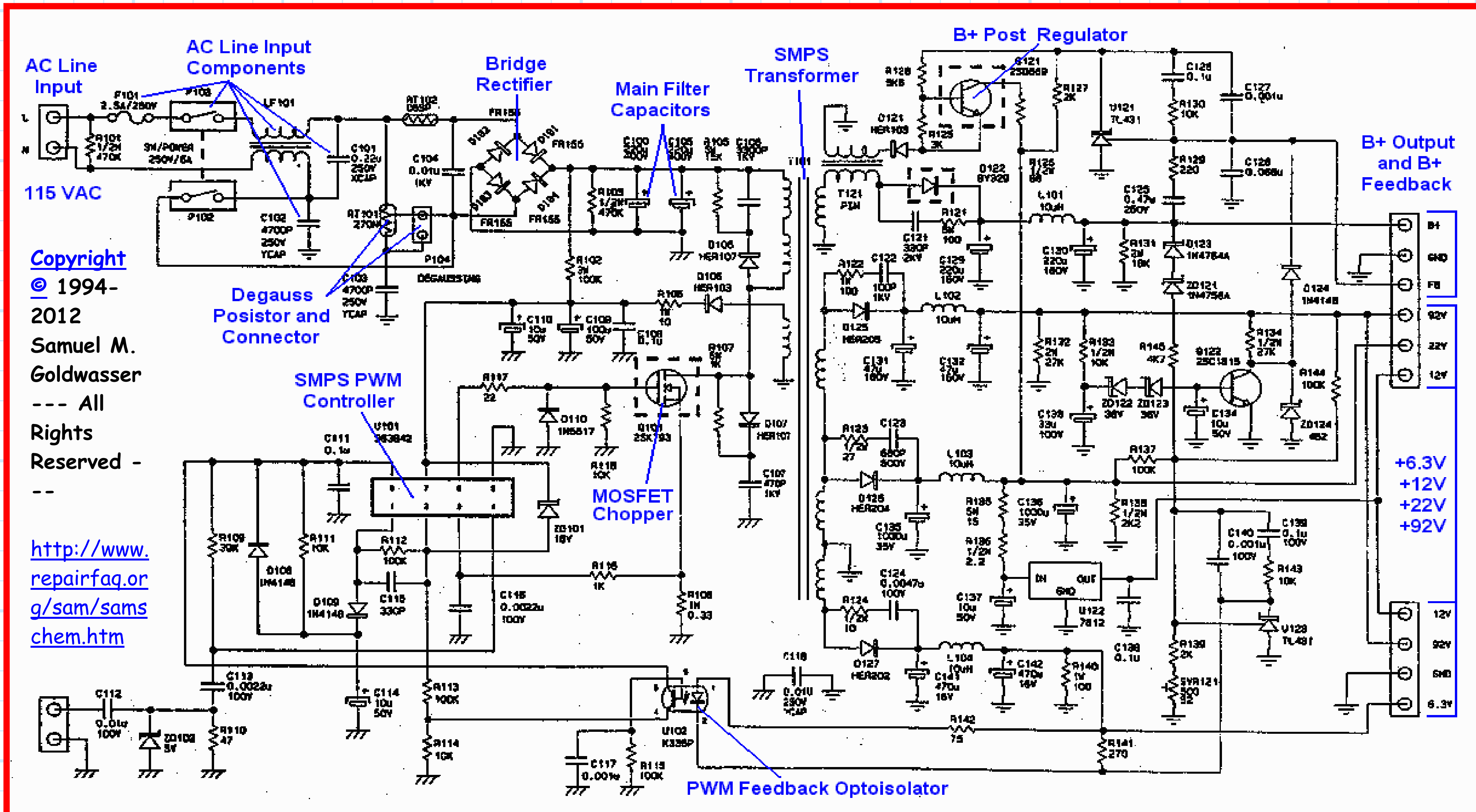
# We often need several regulators

Moreover, we find that the **power supplies** of most complex loads (e.g., computers, TVs, etc.) employ components that require **several different** DC source voltages.

As a result, power supplies often use **many regulators**, each with a **different DC voltage** :



# This circuit will be on the final




Typical Switchmode Power Supply for Small SVGA Color Monitor

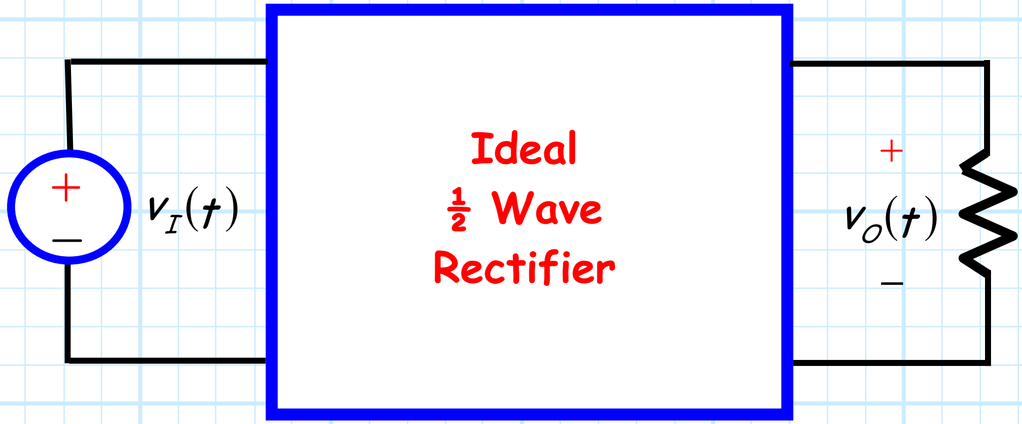
The **power supply**: make sure **you** know **why** it's needed; **what** it does; and **how** it does it!

# Signal Rectification

An important application of junction diodes is **signal rectification**.

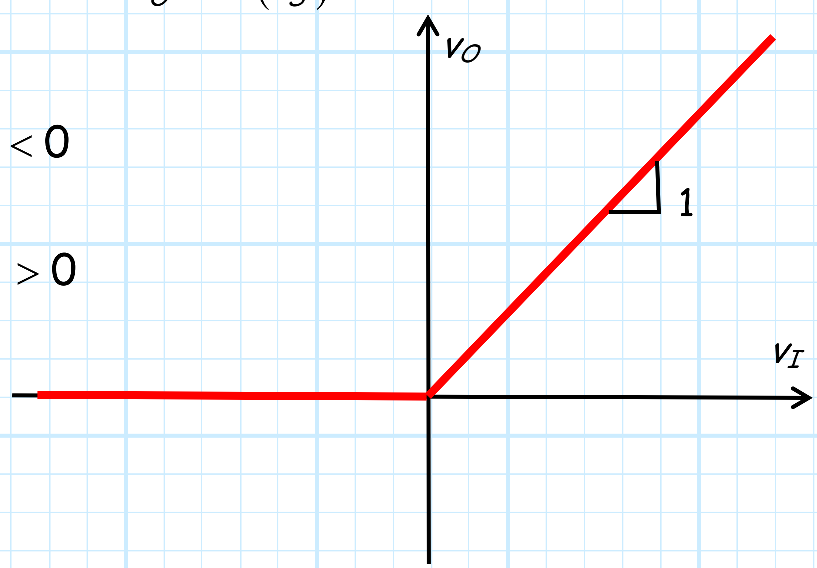
There are **two** types of signal rectifiers, **half-wave** and **full-wave**.

Let's first consider the **ideal half-wave rectifier**.

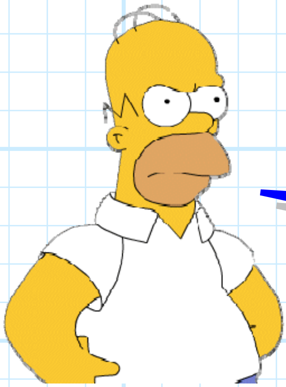



It is a circuit with the transfer function $v_O = f(v_S)$:

$$v_O = \begin{cases} 0 & for \quad v_I < 0 \\ v_I & for \quad v_I > 0 \end{cases}$$



# It's the same—or zero

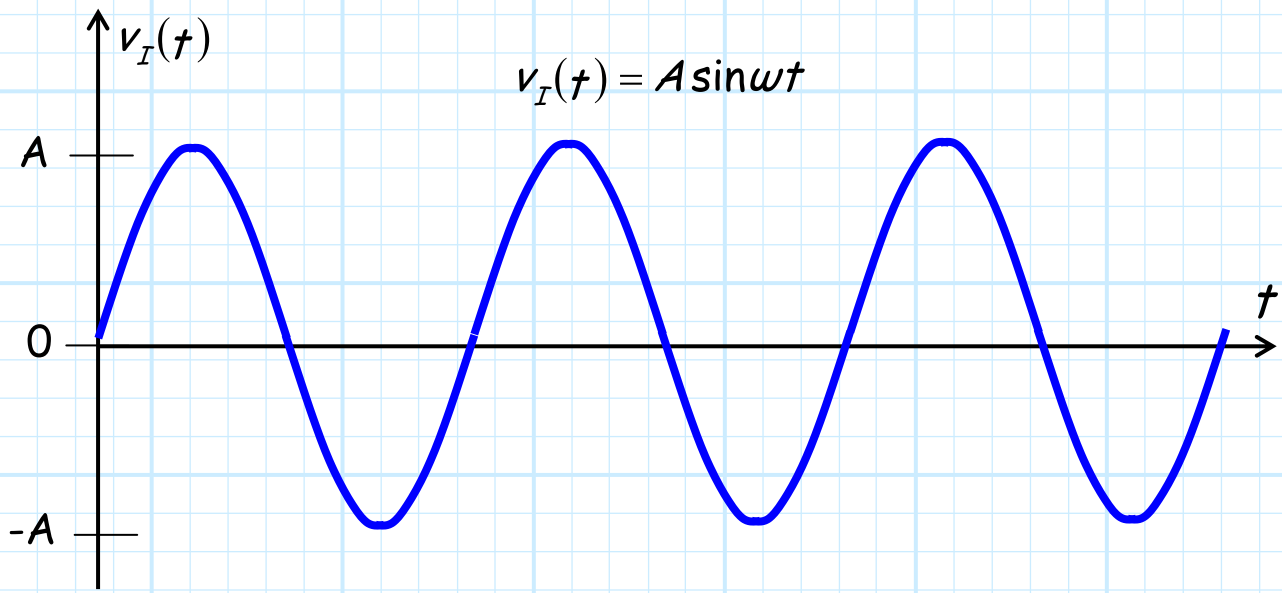


Pretty simple! **When** the input is negative, the output is **zero**, whereas **when** the input is positive, the output is the **same** as the input.

**Q:** *Pretty **pointless** I'd say.*

*This appears to be your most **useless** circuit yet.*

**A:** To see **why** a half-wave rectifier is useful, consider the **typical** case where the input source voltage is a **sinusoidal** signal with **frequency** $\omega$ and peak **magnitude** $A$:

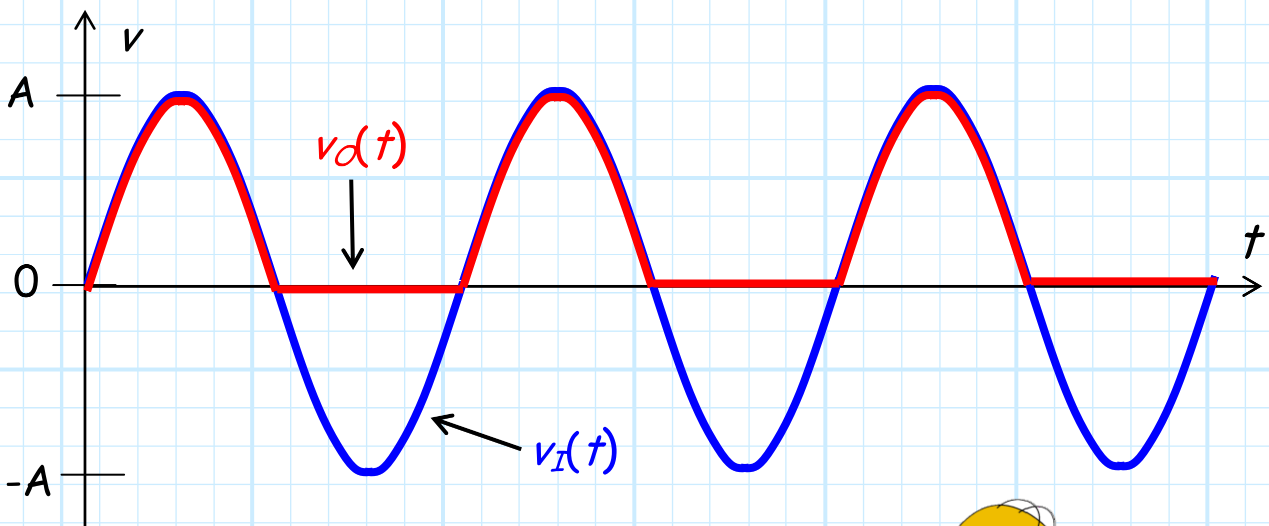


# Half a sine wave

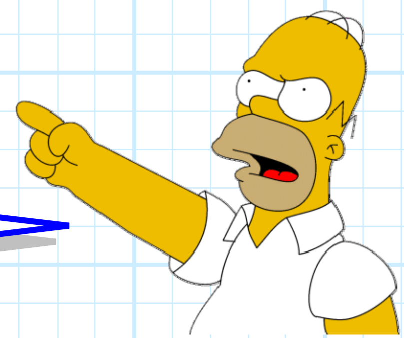
**Think** about what the **output** of the half-wave rectifier would be!

Remember the rule: when $v_I(t)$ is **negative**, the output is **zero**, when $v_I(t)$ is **positive**, the output is **equal** to the input.

The **output** of the half-wave rectifier for **this** example is therefore:




**Q:** *A **half** a sine wave? What kind of lame signal is **that**?*



# The input is a pure AC signal…

**A:** Although it may appear that our rectifier had **little** useful effect on the input signal $v_I(t)$, in fact the difference between input $v_I(t)$ and output $v_O(t)$ is both **important** and **profound**.

To see how, consider first the **DC component** (i.e. the time-averaged value) of the **input** sine wave:

$$\begin{aligned} V_I &= \frac{1}{T}\int_0^T v_I(t)\,dt \\ &= \frac{1}{T}\int_0^T A\sin\omega t\,dt = 0 \end{aligned}$$

Thus, (as you probably already knew) the **DC component** of a sine wave is **zero**—a sine wave is an **AC signal**!

# ...but the output has a DC component as well

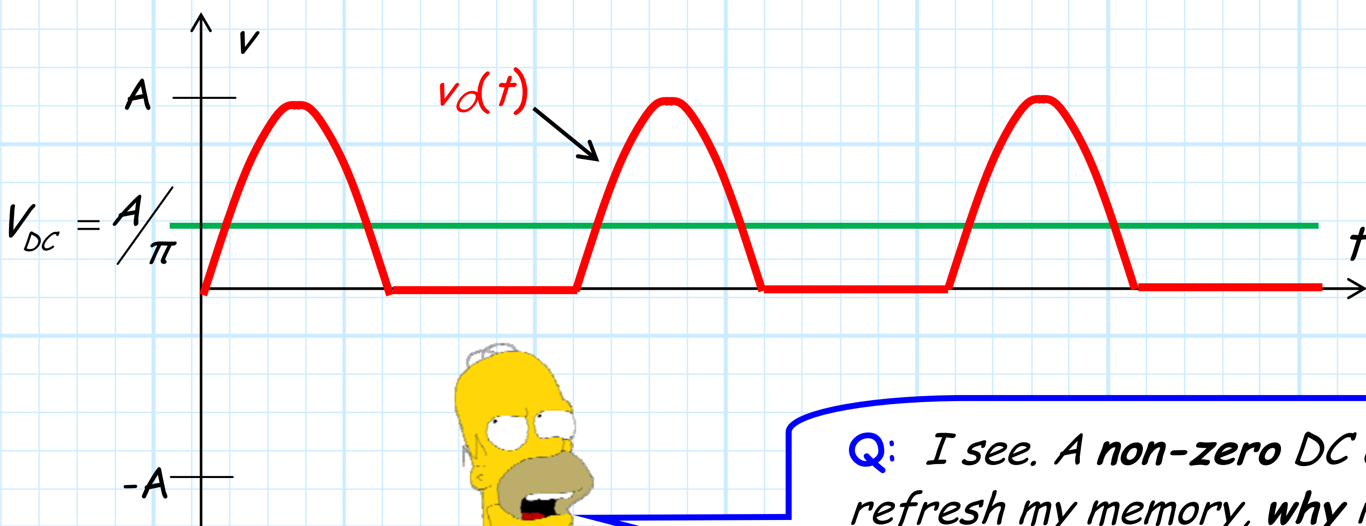
Now, contrast this with the **output** $v_O(t)$ of our half-wave rectifier.

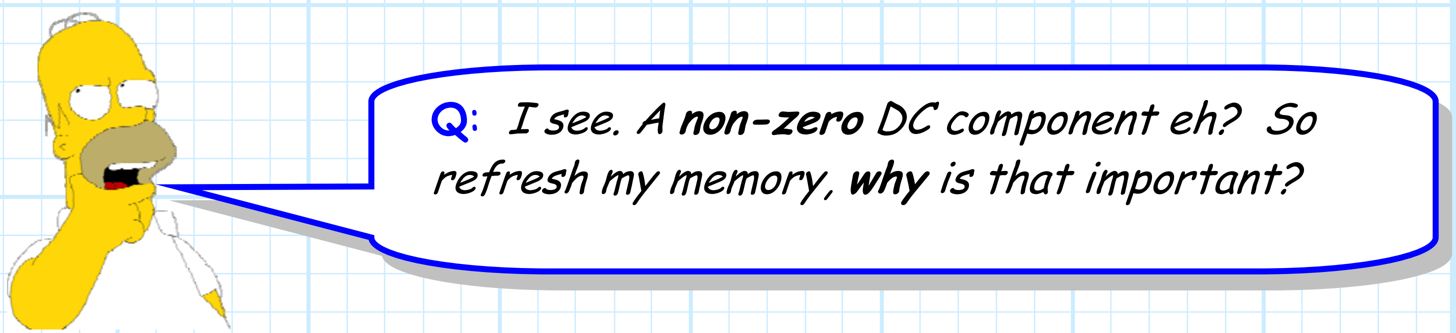
The **DC component** of the **output** is:

$$V_{DC} = \frac{1}{T}\int_0^T v_O(t)\,dt$$

$$= \frac{1}{T}\int_0^{T/2} A\sin\omega t\,dt + \frac{1}{T}\int_{T/2}^{T} 0\,dt = \frac{A}{\pi}$$

Unlike the input, the **output** has a **non-zero** (positive) **DC component** ($V_{DC} = A/\pi$)!



# A DC component is required

**A:** Recall that the **power distribution** system we use is an **AC** system.

The source voltage $v_I(t)$ that we get when we plug our "**power cord**" into the wall socket is a 60 Hz **sinewave**—a source with a **zero DC component**!

The **problem** with this is that most **electronic devices** and systems, such as TVs, stereos, computers, etc., require a **DC voltage(s)** to operate!

**Q:** *But, how can we create a **DC** supply voltage if our power source $v_I(t)$ has **no** DC component??*

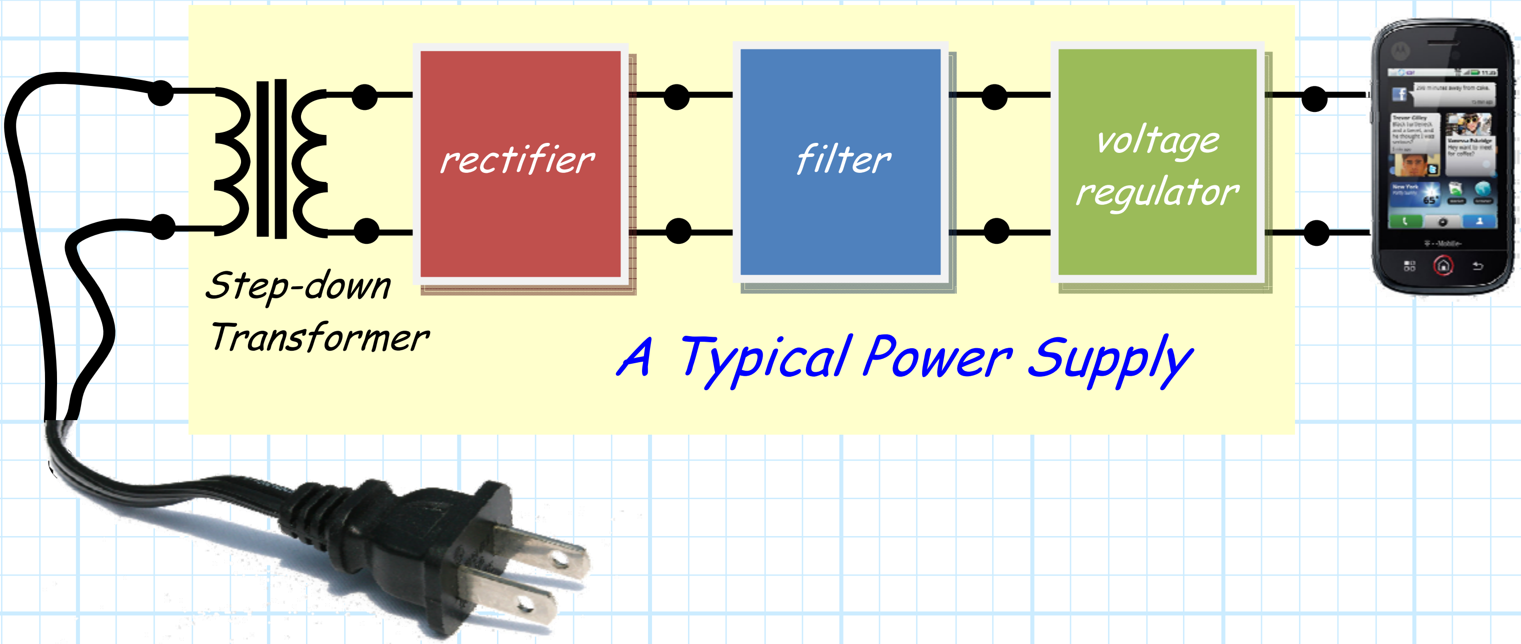
**A:** That's **why** the half-wave rectifier is so **important**!

It takes an AC source with **no** DC component and creates a signal with **both** a AC **and** DC component.

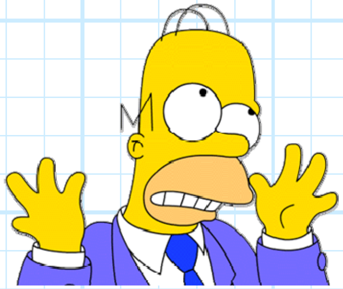
# Remember this?

We can then pass the output of a half-wave rectifier through a **low-pass filter**, which **suppresses** the AC component but lets the **DC** value ($V_O = A/\pi$) pass through.

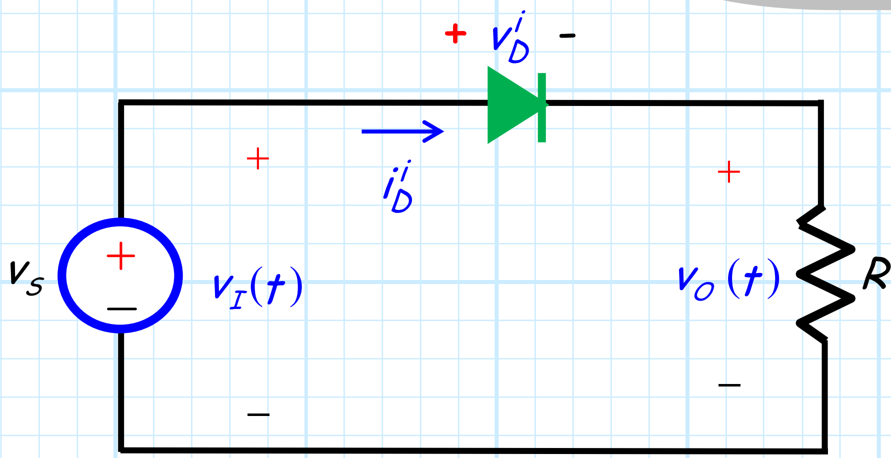
We then **regulate** this output and form a **useful DC voltage source**—one suitable for powering our electronic systems!



# An ideal rectifier requires an ideal diode



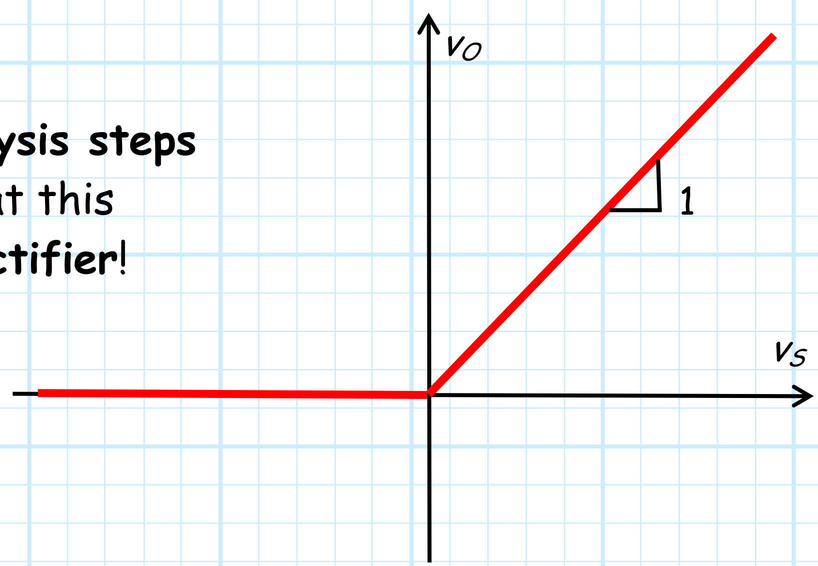
**Q:** *OK, now I see **why** the **ideal** half-wave rectifier might be useful. But, is there any way to actually **build** this magical device?*




**A:** An **ideal** half-wave rectifier can be "built" if we use an **ideal** diode.

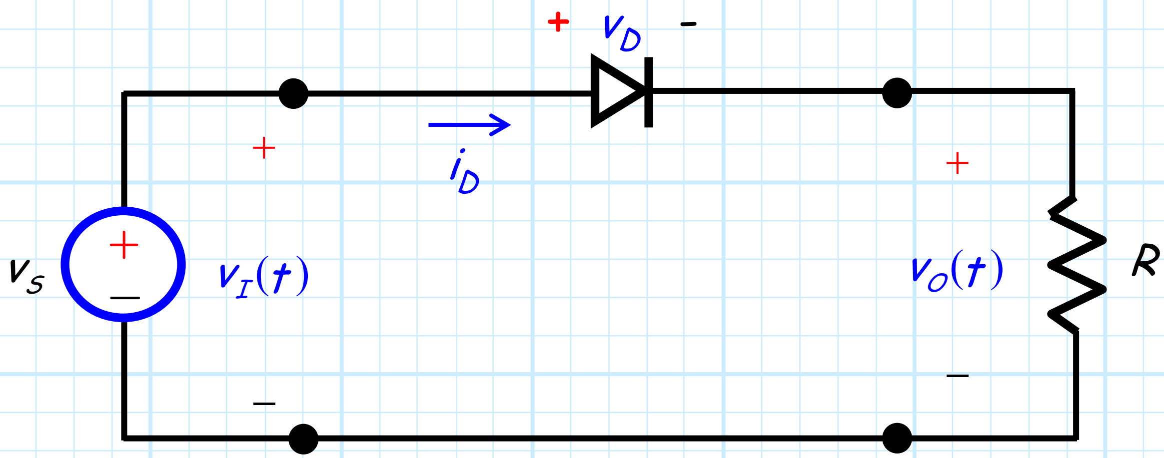
If we follow the transfer function **analysis steps** we studied earlier, then we will find that this circuit is indeed an **ideal half-wave rectifier**!

$$v_O = \begin{cases} 0 & \text{for} \quad v_I < 0 \\ v_s & \text{for} \quad v_I > 0 \end{cases}$$



# But a junction diode works as well

Of course, since **ideal** diodes do **not** exist, we must use a **junction** diode instead:




**Q:** *This circuit looks so **familiar**! Haven't we studied it before?*

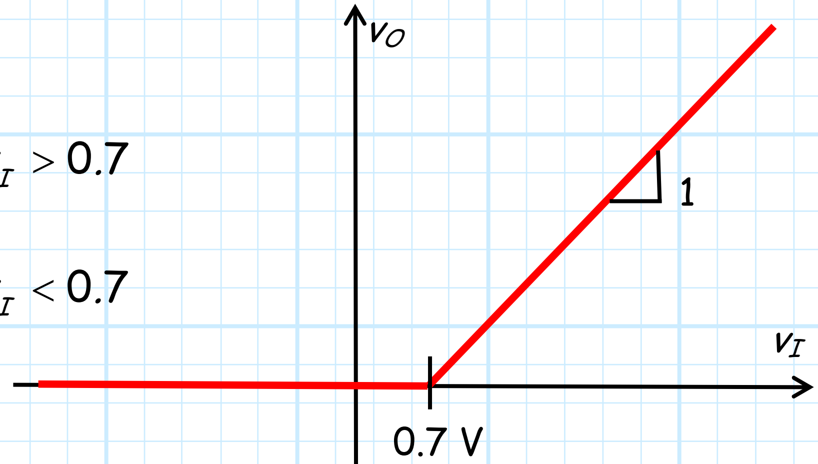
**A:** Yes!

It was an **example** where we determined the junction diode circuit transfer function.

# It's a little different from ideal

Recall that the **result** was:

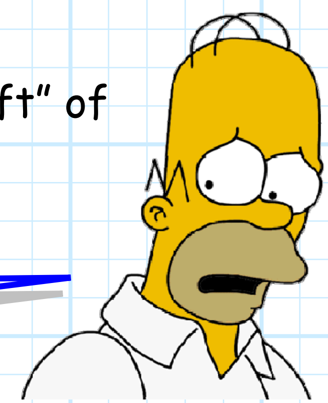
$$v_O = \begin{cases} v_I - 0.7 & for \quad v_I > 0.7 \\ 0 & for \quad v_I < 0.7 \end{cases}$$




Note that this result is **slightly different** from that of the **ideal** half-wave rectifier!

The **0.7 V drop** across the junction diode causes a horizontal "shift" of the transfer function from the ideal case.

**Q:** *So then this junction diode circuit is **worthless**?*



**A:** Hardly! Although the transfer function is **not quite** ideal, it works **well enough** to achieve the goal of signal **rectification**—it takes an input with **no** DC component and creates an output with a **significant** DC component!

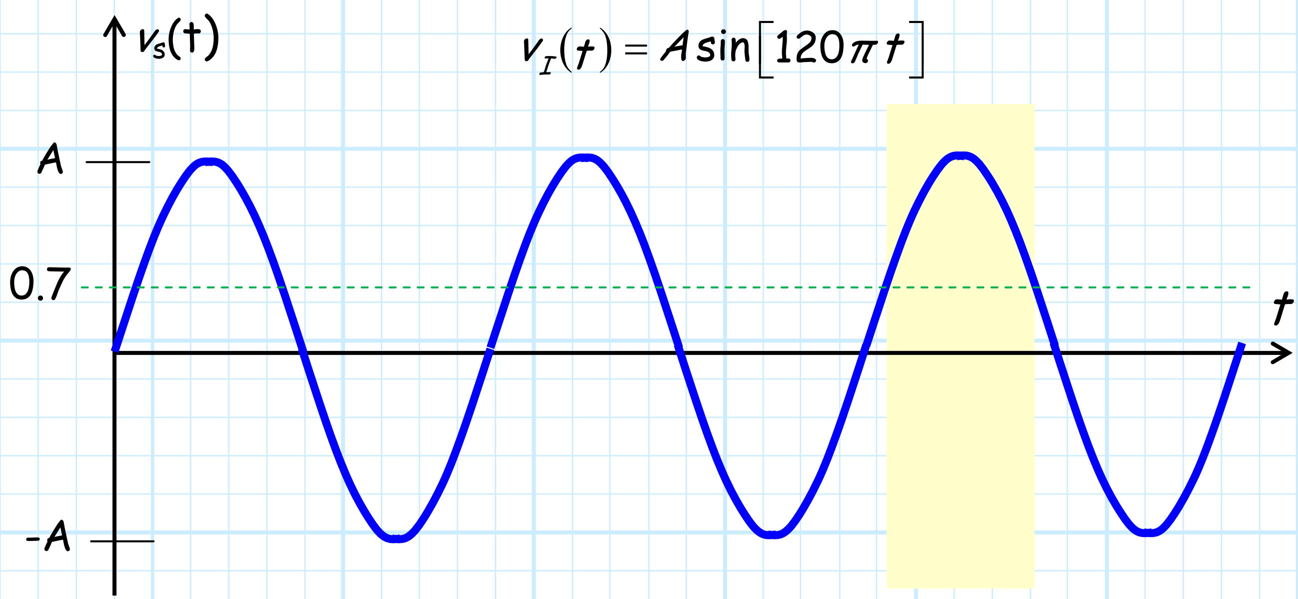
# Let's determine the output

Note what the transfer function "**rule**" is now:

1. When the input is **greater** than 0.7 V, the output voltage is **equal** to the input voltage minus 0.7 V.

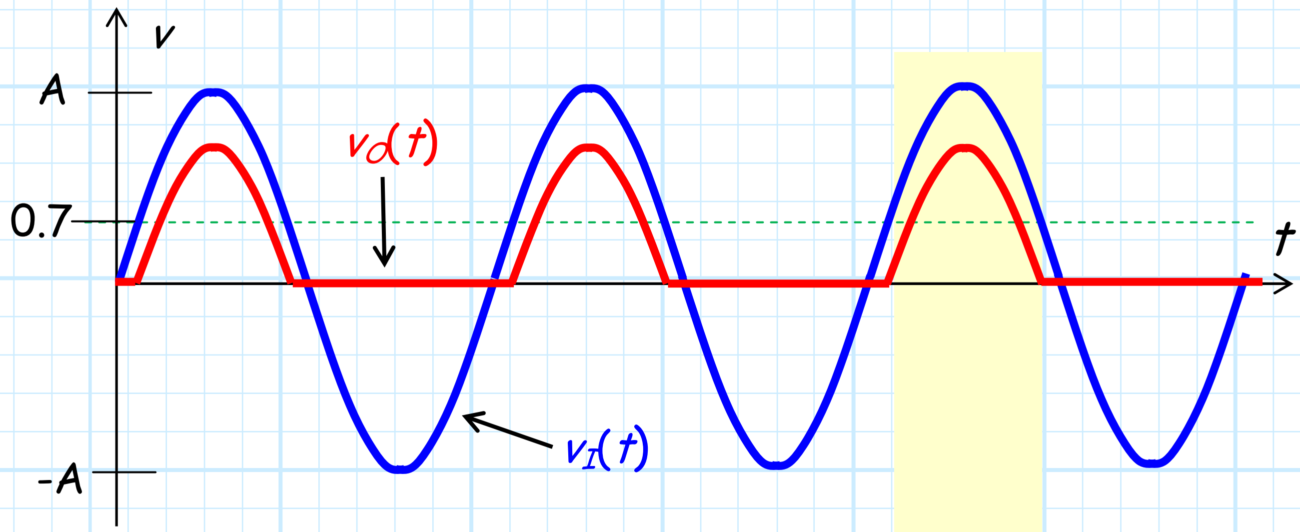
2. When the input is **less** than 0.7 V, the output voltage is **zero**.

So, let's consider **again** the case where the **source** voltage is **sinusoidal** (just like the source from a "wall socket"!):



# Close to ideal—and still plenty of DC

The output of our **junction diode** half-wave rectifier would therefore be:




Although the output is **shifted downward** by 0.7 V (note in the plot above this is **exaggerated**, typically $A >> 0.7\ V$), it should be apparent that the **output** signal $v_O(t)$, unlike the input signal $v_I(t)$, has a **non-zero** (positive) **DC component**.

Because of the 0.7 V shift, this DC component is **slightly smaller** than the **ideal** case. In fact, we find that if $A$>>0.7, this **DC component** is approximately:

$$V_{DC} \approx \frac{A}{\pi} - 0.35\ V$$

In other words, **just** 350 mV less than **ideal**!

# The ideal full-wave rectifier

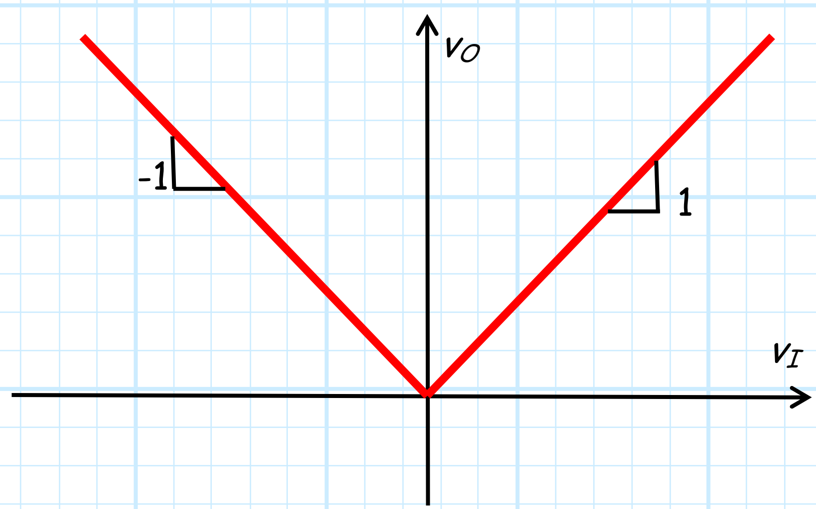
**Q:** *Way back on the first page you said that there were **two** types of rectifiers.*

*I now understand **half-wave** rectification, but what about these so-called **full-wave** rectifiers?*

**A:** Almost forgot!

Let's examine the transfer function of an **ideal full-wave rectifier**:

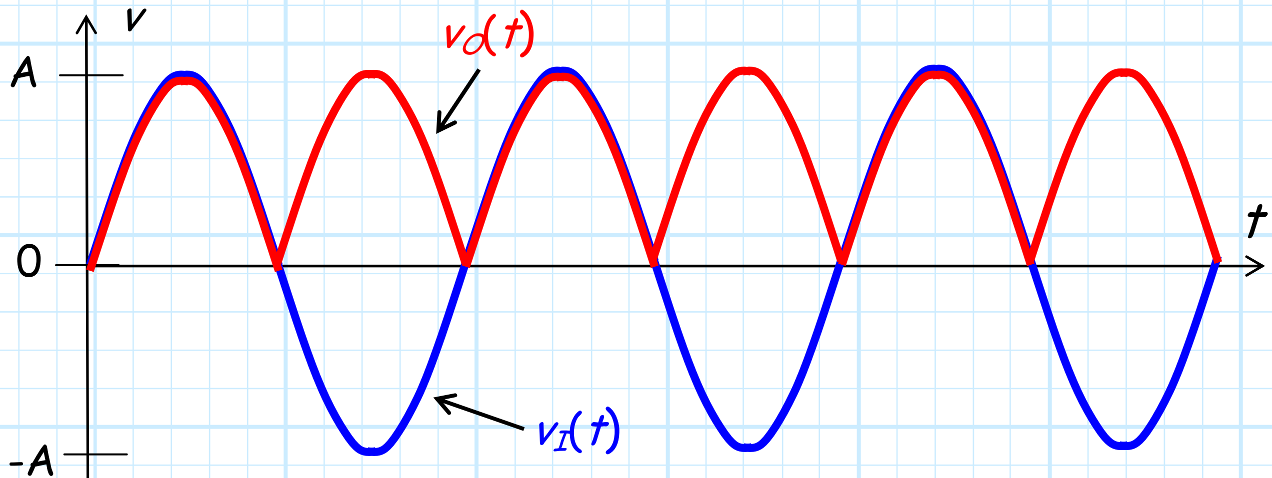
$$v_O = \begin{cases} -v_I & \text{for} \quad v_I < 0 \\ v_I & \text{for} \quad v_I > 0 \end{cases}$$



# The DC component is twice as big

If the ideal half-wave rectifier makes **negative** inputs **zero**, the ideal full-wave rectifier makes **negative** inputs—**positive**!

For **example**, if we again consider our **sinusoidal** input, we find that the output will be:

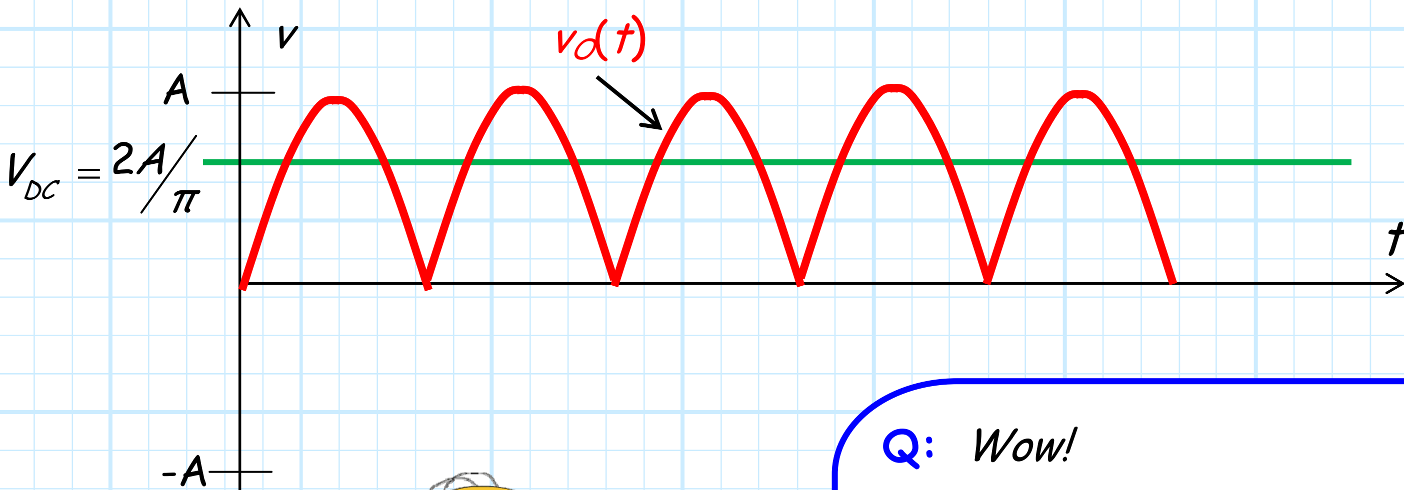



The result is that the output signal will have a DC component **twice** that of the ideal half-wave rectifier!

$$V_{DC} = \frac{1}{T}\int_0^T v_O(t)\,dt$$

$$= \frac{1}{T}\int_0^{T/2} A\sin\omega t\,dt - \frac{1}{T}\int_{T/2}^{T} A\sin\omega t\,dt = \frac{2A}{\pi}$$

# We can build a *nearly* ideal full-wave rectifier with *junction* diodes




**Q:** *Wow!*

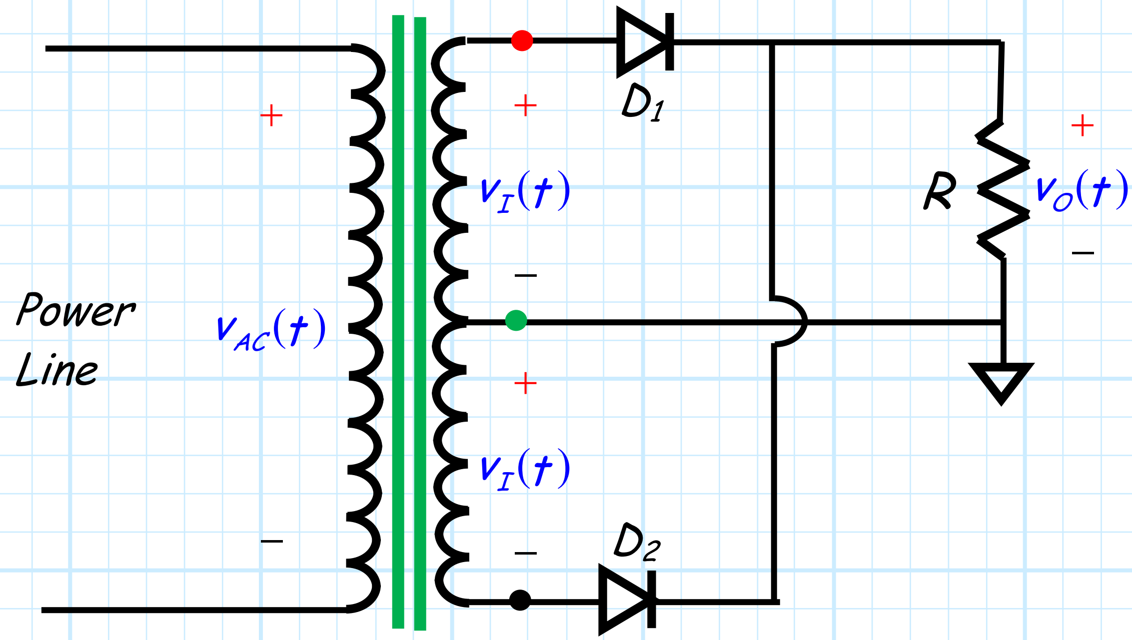
***Full-wave*** *rectification appears to be* ***twice*** *as good as half-wave.*

*Can we* ***build*** *an ideal full-wave rectifier with* ***junction*** *diodes?*

**A:** Although we cannot build an **ideal** full-wave rectifier with **junction** diodes, we can build full-wave rectifiers that are **very close** to ideal with junction diodes!

# The Full-Wave Rectifier

Consider the following **junction diode** circuit:




Note that we are using a **transformer** in this circuit.

The job of this transformer is to **step-down** the large voltage on our power line (120 $V_{rms}$) to some **smaller** magnitude (typically 20-70 $V_{rms}$).

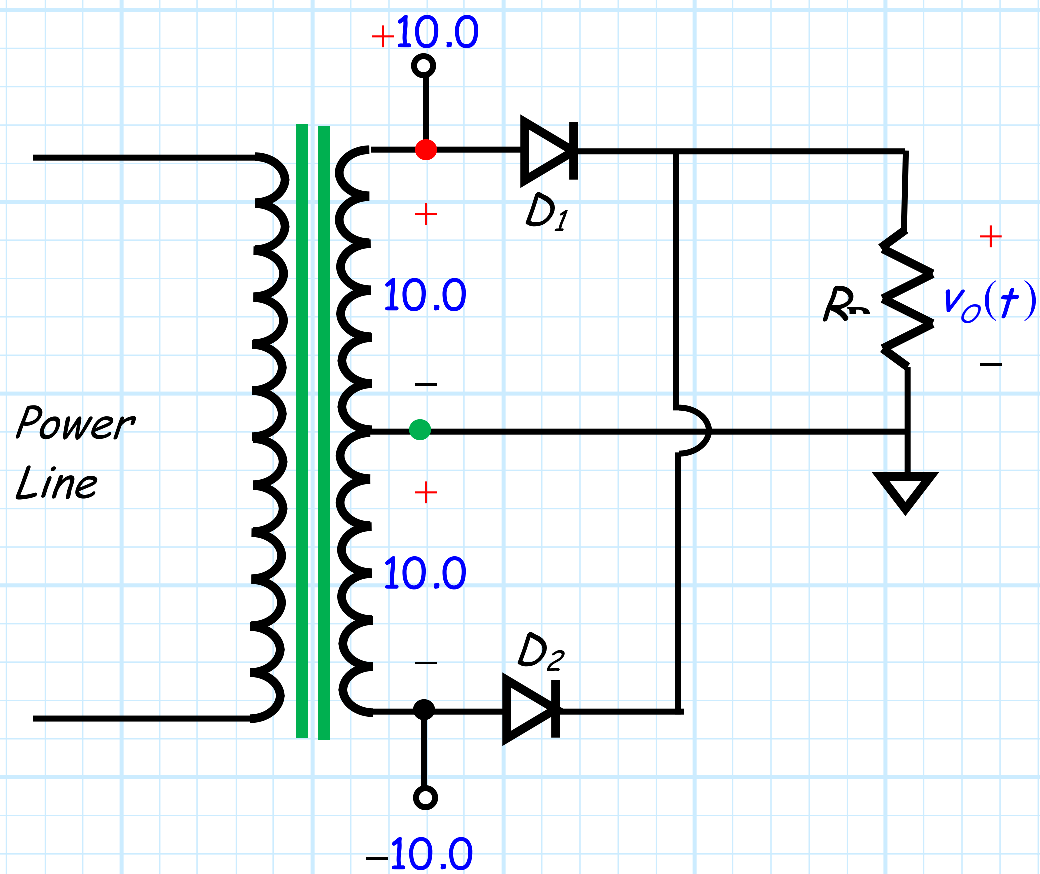
# This point is *very* important!

Note the secondary winding has a **center tap** that is **grounded**.

Thus, the secondary voltage is distributed **symmetrically** on either side of this center tap.

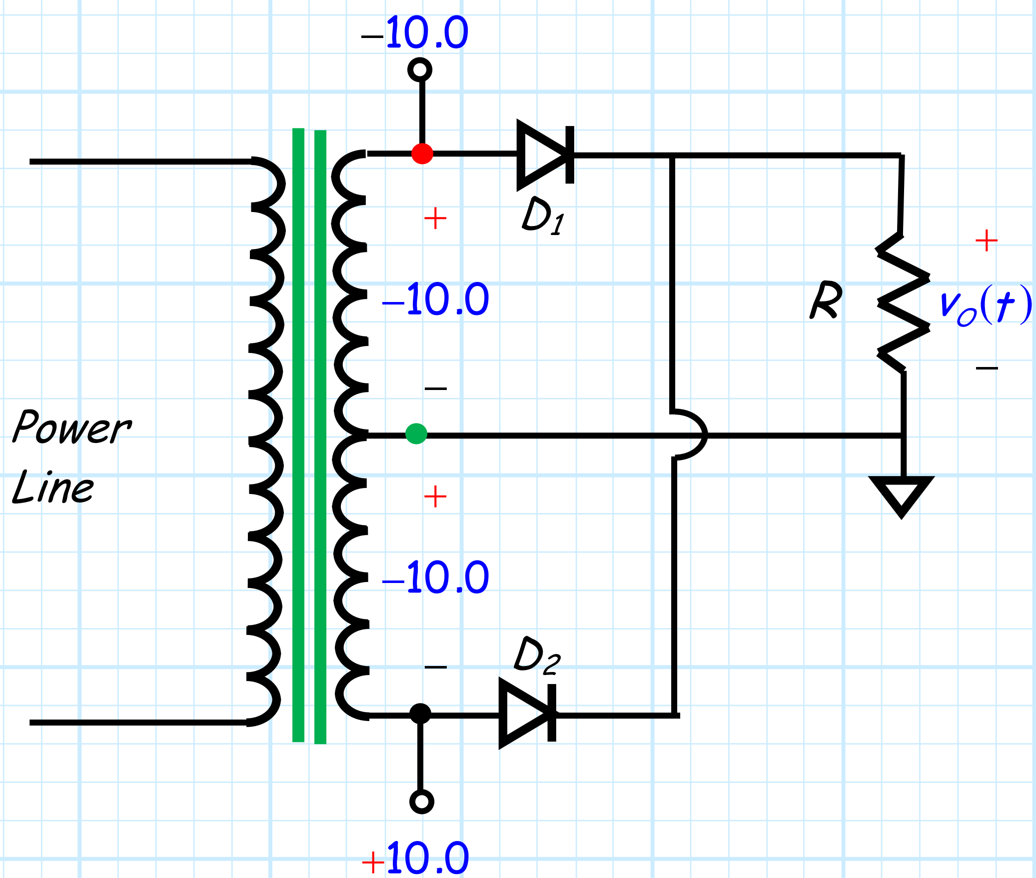
For **example**, if $v_I = 10\ V$, the anode of $D_1$ will be 10 $V$ **above** ground potential.

While, the anode of $D_2$ will be 10 $V$ **below** ground potential (i.e., -10V):



# Make this make sense

Conversely, if $v_I$=-10 V, the anode of $D_1$ will be 10V **below** ground potential (i.e., -10V), while the anode of $D_2$ will be 10V **above** ground potential:

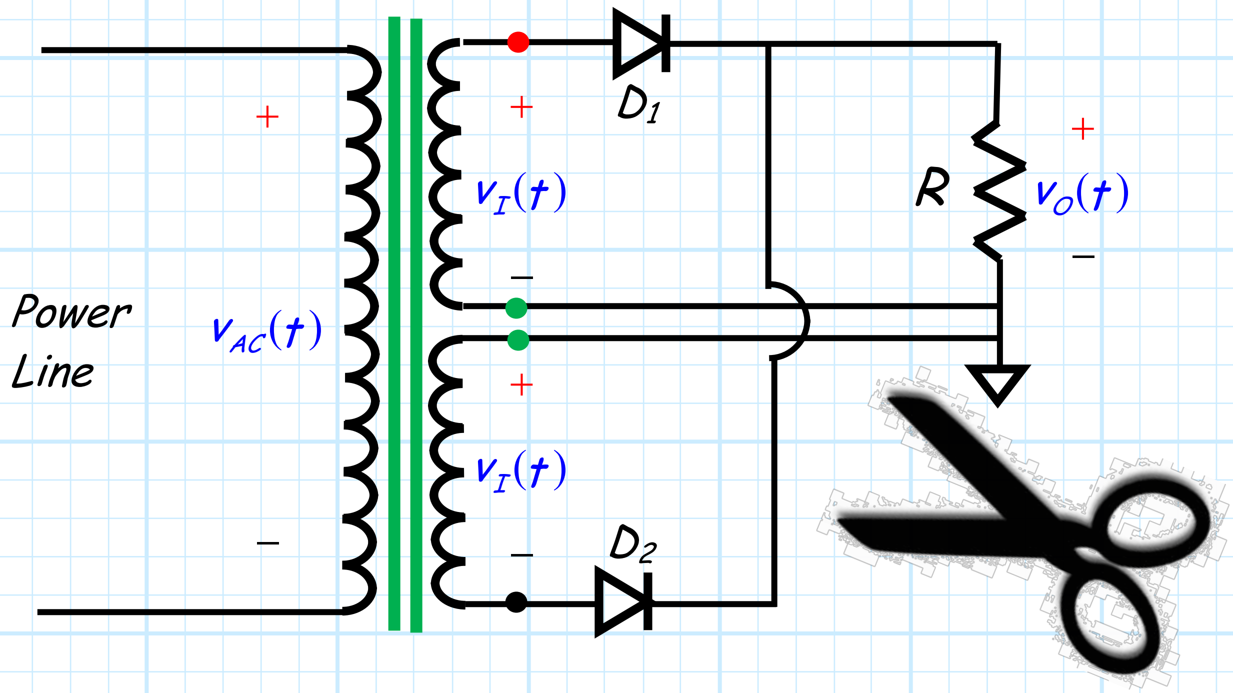


# Let's redraw the circuit…

The more important question is, what is the value of **output** $v_O$?

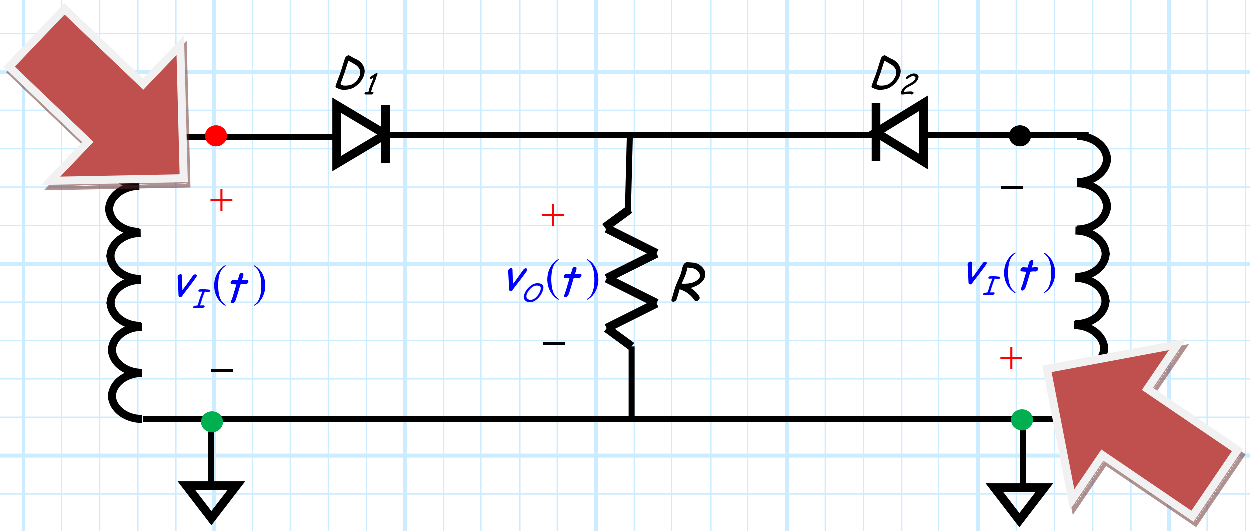
More specifically, how is $v_O$ related to the value of source $v_I$—what is the **transfer fuction** $v_O = f(v_I)$?

To help simplify our analysis, we are going **redraw** this cirucuit in another way. First, we will **split** the secondary winding into two explicit pieces:



# ...nothing has changed—it's the same circuit

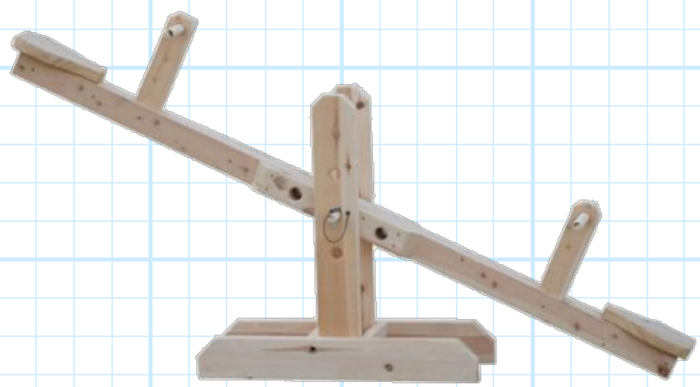
We will now **ignore the primary** winding of the transformer and redraw the remaining circuit as:

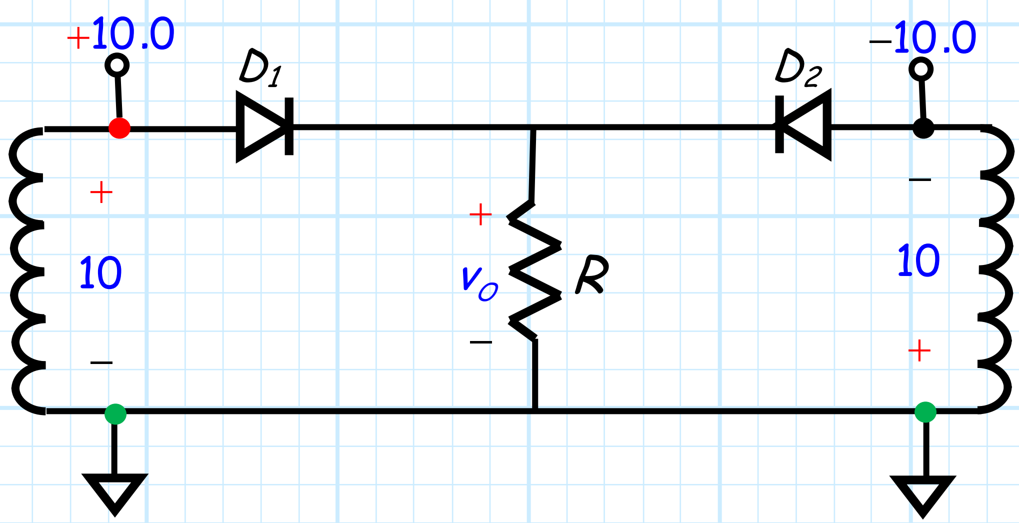



Note that the secondary voltages at either end of this circuit are the **same**, but have **opposite** polarity.

# Just like a teeter-totter; one side goes up, the other side goes down

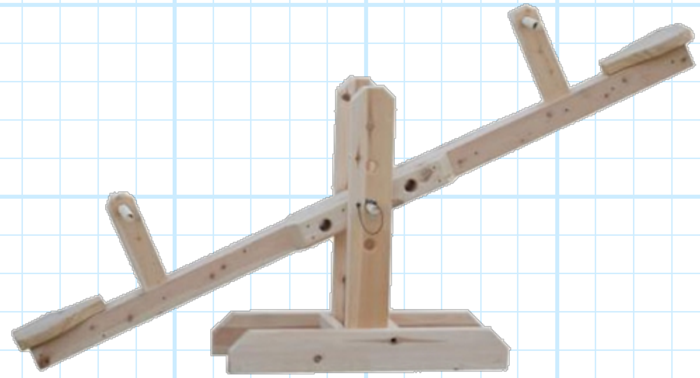
As a result, **if** $v_I$=10, then the anode of diode $D_1$ will be 10 $V$ **above** ground, and the anode at diode $D_2$ will be 10 $V$ **below** ground—just like before!

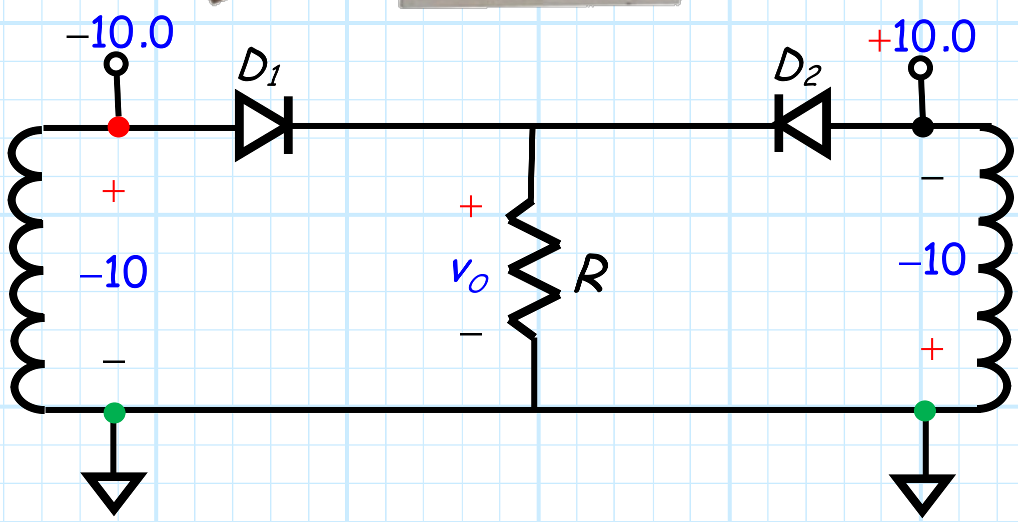




# And vice vesa

And, **if** $v_I$=-10, then the anode of diode $D_1$ will be 10 $V$ **below** ground, and the anode at diode $D_2$ will be 10 $V$ **above** ground—just like before!





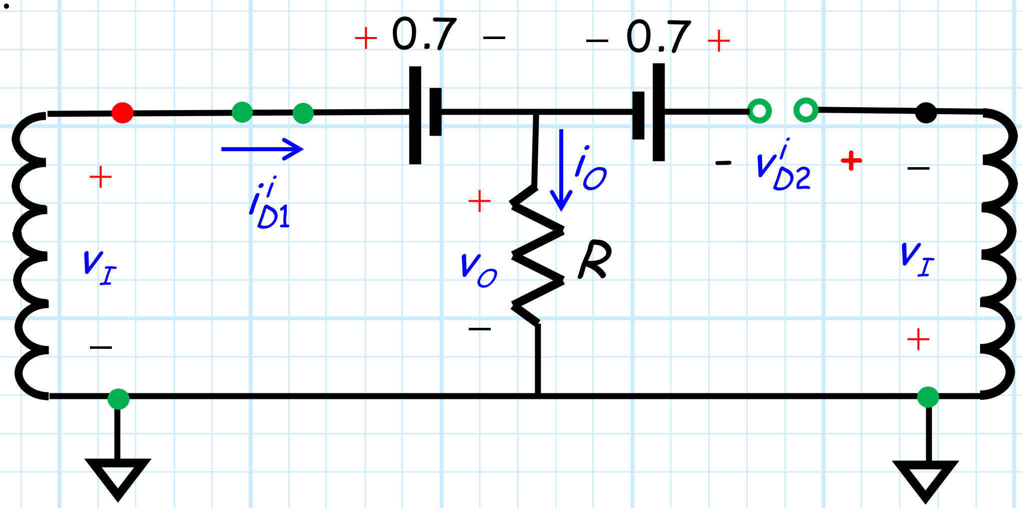

Now, let's attempt to determine the **transfer function** $v_O = f(v_I)$ of this circuit!

# Yikes! Three things to find!

First, we will replace the junction diodes with **CVD models**.

Then let's ASSUME $D_1$ is **forward** biased and $D_2$ is **reverse** biased, thus ENFORCE $v_{D1}^i = 0$ and $i_{D2}^i = 0$.

Thus ANALYZE:




Note that we need to determine **3** things:

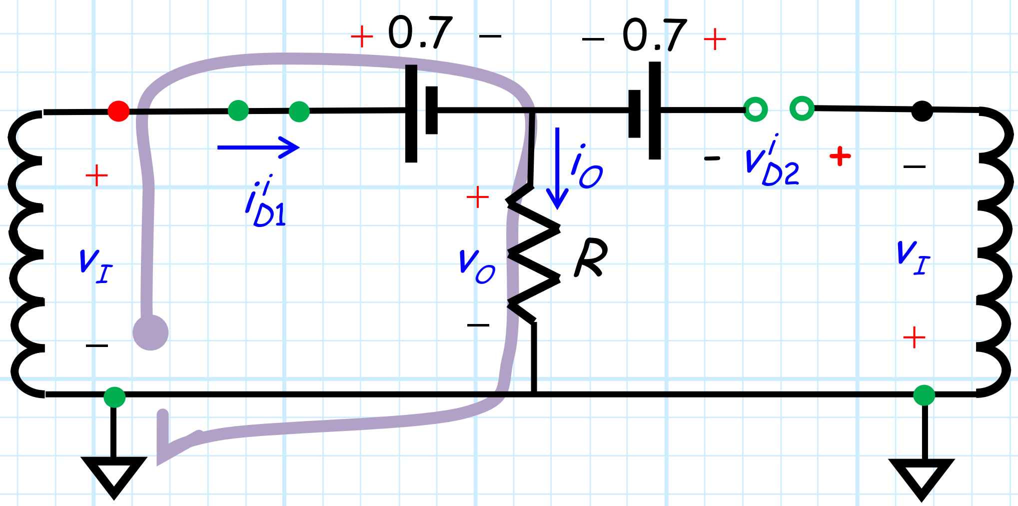
* the **ideal diode current** $i_{D1}^i$,

* the **ideal diode voltage** $v_{D2}^i$,

* and the **output voltage** $v_O$.

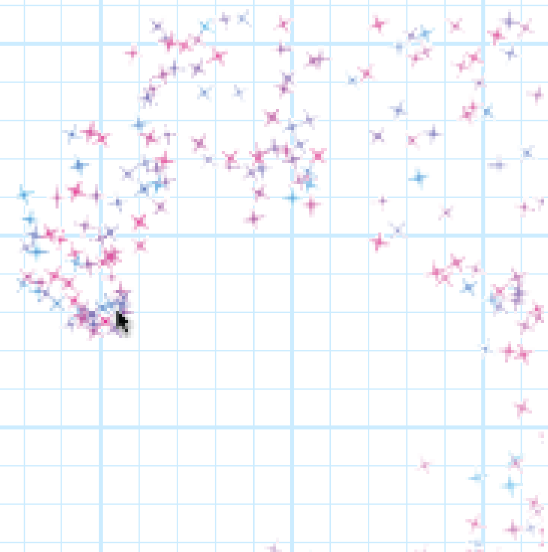
# Sprinkle on some KVL pixie dust

However, **instead** of finding numerical values for these 3 quantities, we must express them in terms of **input voltage** $v_I$!




From **KCL**:

$$i_O = i_{D1}^i + i_{D2}^i = i_{D1}^i + 0 = i_{D1}^i$$



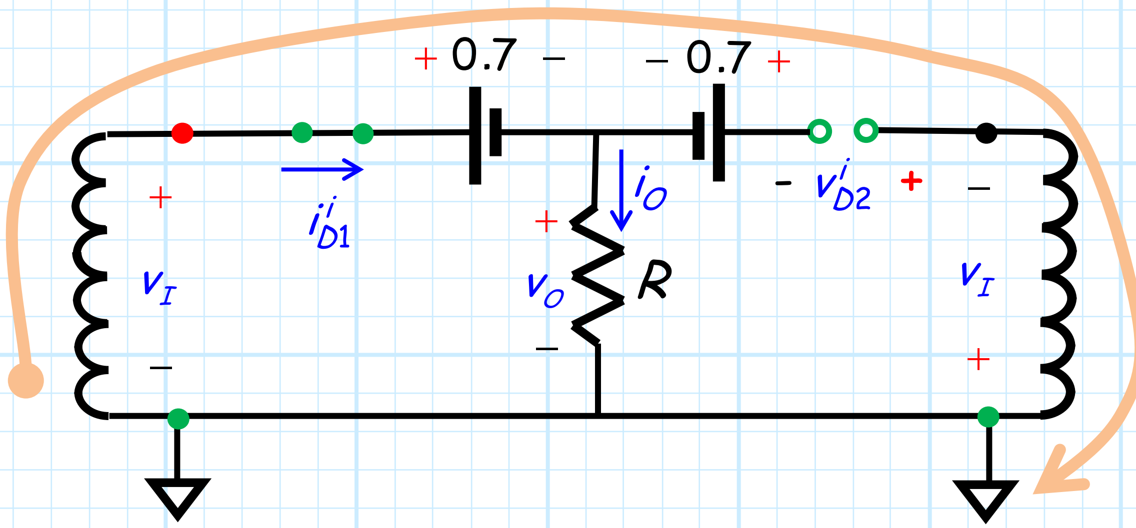
From **KVL**:

$$v_I - 0 - 0.7 - R\, i_{D1}^i = 0$$

Thus the **ideal diode current** is:

$$i_{D1}^i = \frac{v_I - 0.7}{R}$$

# It's getting dusty in here




Likewise, from **KVL**:

$$v_I - 0 - 0.7 + 0.7 + v'_{D2} + v_I = 0$$

Thus, the **ideal diode voltage** is:

$$v'_{D2} = -v_I - v_I + 0.7 - 0.7 = -2v_I$$

And finally, from Ohm's Law:

$$v_O = i'_{D1} R = \left( \frac{v_I - 0.7}{R} \right) R = v_I - 0.7$$

# Does *not* mean that $v_I$ is bigger than 0.7

Thus, the **output voltage** is:

$$v_O = v_I - 0.7$$

Now, we must determine **when** both $i'_{D1} > 0$ and $v'_{D2} < 0$.

When **both** these conditions are true, the output voltage will be $v_O = v_I - 0.7$.

When one **or** both conditions $i'_{D1} > 0$ and $v'_{D2} < 0$ are **false**, then our assuptions are **invalid**, and $v_O \neq v_I - 0.7$.

Using the results we just determined, we know that $i'_{D1} > 0$ **when**:

$$\frac{v_I - 0.7}{R} > 0$$

Solving for $v_I$:

$$\begin{aligned} \frac{v_I - 0.7}{R} &> 0 \\ v_I - 0.7 &> 0 \\ v_I &> 0.7 \text{ V} \end{aligned}$$

# Make this make sense

Likewise, we find that $v_{D2}^{i} < 0$ **when**:

$$-2v_I < 0$$

Solving for $v_I$:

$$-2v_I < 0$$
$$2v_I > 0$$
$$v_I > 0$$

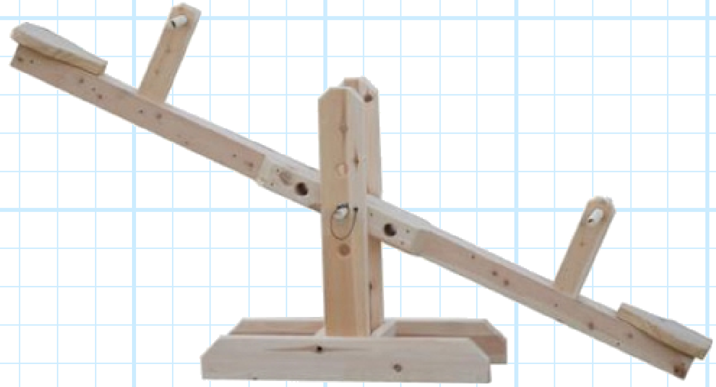
Thus, our assumptions are correct **when** $v_I > 0.0$ **AND** $v_I > 0.7$.

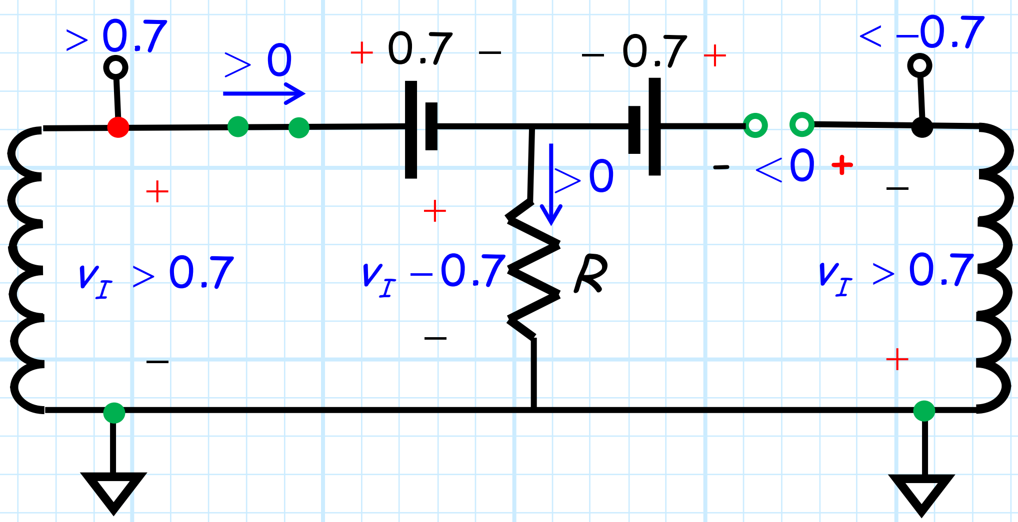
→THINK about this. This is the **same** thing as saying our assumptions are valid when $v_I > 0.7$!

Thus, we have found that the following statement is true about **this** (but only this!) circuit:

$$\rightarrow \; v_O = v_I - 0.7 \;V \quad \text{when} \quad v_I > 0.7 \;V$$

# We're not done yet!






$$\rightarrow \quad v_O = v_I - 0.7 \; V \quad \text{when} \quad v_I > 0.7 \; V$$

Note that this statement does **not** constitute a **function** (what about $v_I < 0.7$?), so we must **continue** with our analysis!

# A good engineer is fretful, paranoid and fatalistic


**Q:** *Wait! I'm **concerned** about something.*

*We found that the voltage across the second ideal diode is:*

$$v_{D2}^{i} = -2v_I$$

*From the CVD model, that means the voltage across **junction** diode $D_2$ is approximately:*

$$v_{D2} = 0.7 + v_{D2}^{i} = 0.7 - 2v_I$$

*Since we know this is true only **when:***

$$v_I > 0.7 \ \ V$$

*the diode voltage $v_{D2} = 0.7 - 2v_I$ must be **negative.***

# Avoid breakdown!

*Moreover, this **negative** voltage is proportional to **twice** the **input** voltage!*

*Thus, if the **input** voltage is **large**, the voltage across this **junction** diode might be **very, very negative**.*

*Shouldn't I be **concerned** about this junction diode going into **breakdown**?*

**A:** You sure should!

**If** the junction diode goes into **breakdown**, the transfer function will **not** be what we expected.

You'd beter use junction diodes with sufficiently large **Zener breakdown voltages**!

# Peak Inverse Voltage: more on this later

**Q:** *But **how** large is **sufficiently** large?*

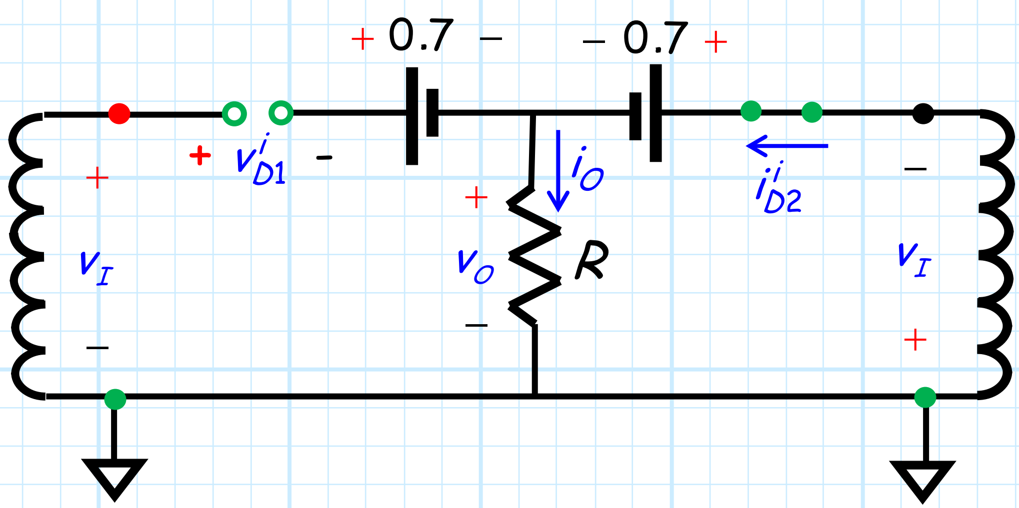
**A:** If we know precisely the **input** voltage function $v_I(t)$, we can find the **"worst case" scenario**—the **most** negative voltage value that occurs across this junction diode.


We call the magnitude of this value the **Peak Inverse Voltage** (more on this later)—the $V_{ZK}$ of our Zener diodes had better be larger that this value!

# Back to the analysis; I'll skip some stuff

OK, back to the anaysis, say we **now** ASSUME that $D_1$ is **reverse** biased and $D_2$ is **forward** biased, so we ENFORCE $i_{D1}^{i} = 0$ and $v_{D2}^{i} = 0$.

Thus, we ANALYZE **this** circuit:

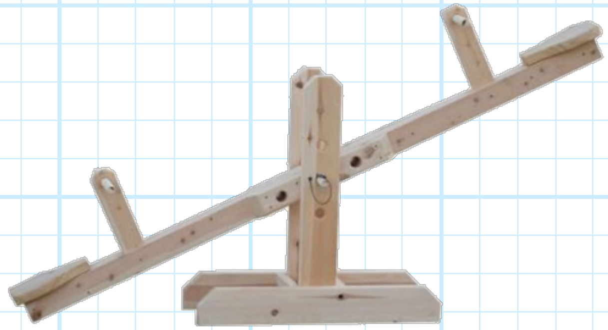


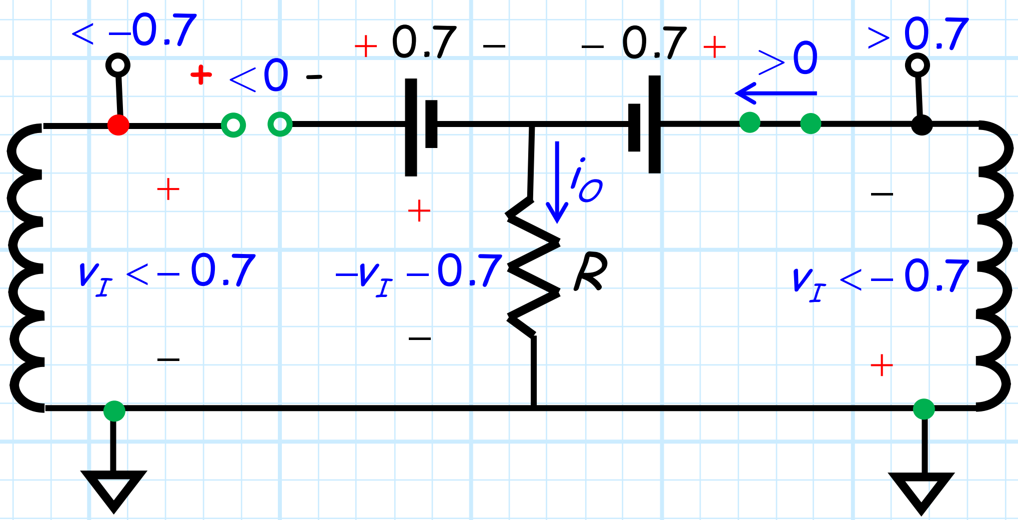

Using the **same proceedure** as before, we find that $v_O = -v_I - 0.7$, and both our assumptions are true **when** $v_I < -0.7$ V.

# And we're still not finished!

In other "words":

$$\rightarrow v_O = -v_I - 0.7 \ \ V \quad \text{when} \quad v_I < -0.7 \ \ V$$





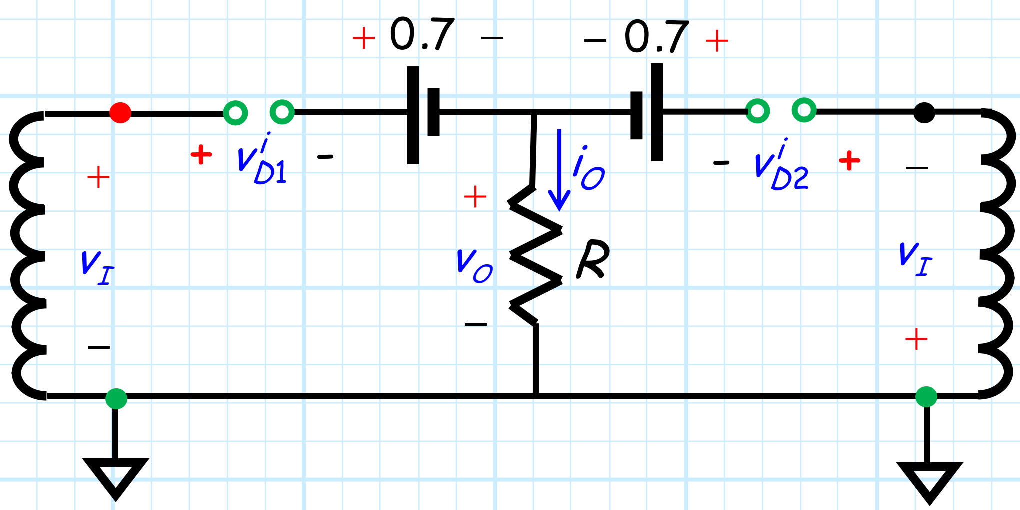

Note we are still **not** done!

We do not have a complete transfer **function** (what happens when $-0.7 \ \ V < v_I < 0.7 \ \ V$ ?).

# Now we're done!

Finally then, we ASSUME that **both** ideal diodes are **reverse** biased, so we ENFORCE $i_{D1}^{i} = 0$ and $i_{D2}^{i} = 0$.

Thus ANALYZE:

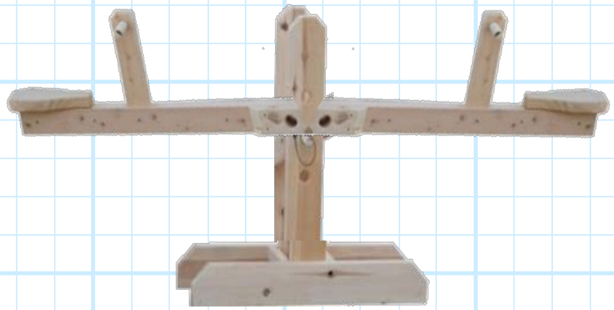


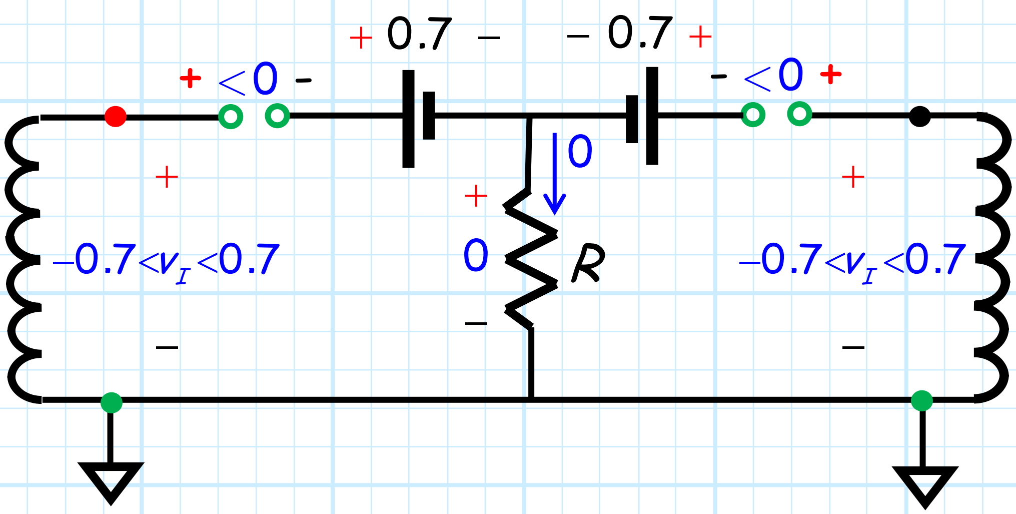

Following the **same proceedures** as before, we find that $v_I = 0$, and both assumptions are true **when** $-0.7 < v_I < 0.7$.

In other words:

$$\rightarrow v_I = 0 \quad \text{when} \quad -0.7 < v_I < 0.7$$

# A simple analysis

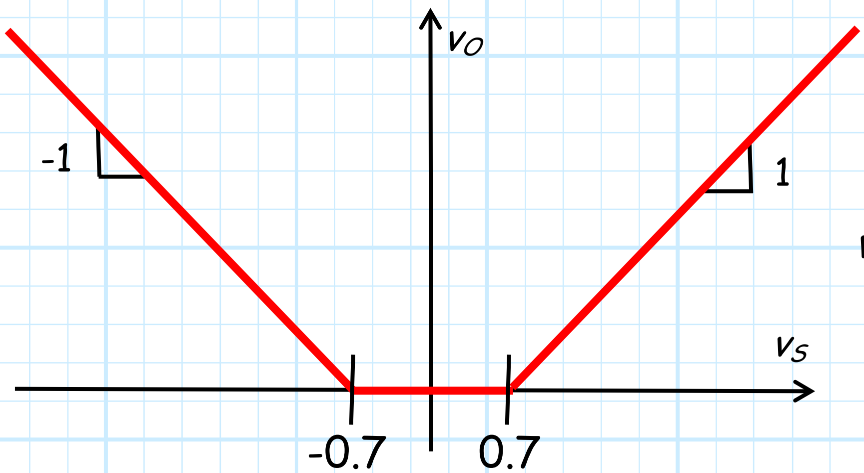




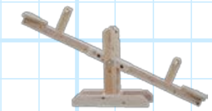
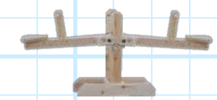
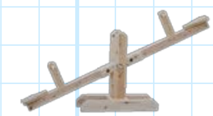

**Now** we have a **function**!

# Smells like an ideal full-wave rectifier!

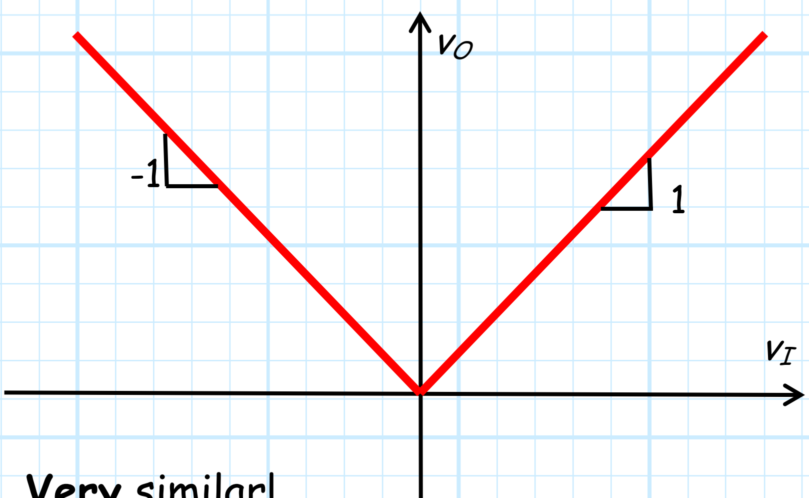
The **transfer function** of this circuit is:




$$v_O = \begin{cases} v_I - 0.7V & \text{for} \quad v_I > 0.7V \\ 0V & \text{for} \quad -0.7 > v_I > 0.7V \\ -v_I - 0.7V & \text{for} \quad v_I < -0.7V \end{cases}$$

Note how this **compares** to the transfer function of the **ideal** full-wave rectifier:

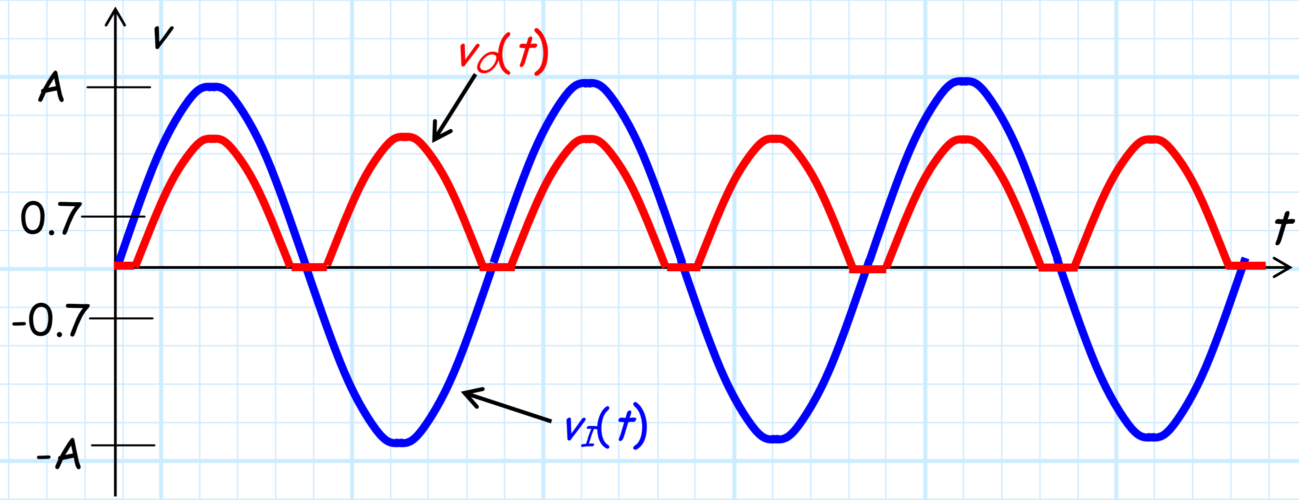



$$v_O = \begin{cases} -v_I & \text{for} \quad v_I < 0 \\ v_I & \text{for} \quad v_I > 0 \end{cases}$$

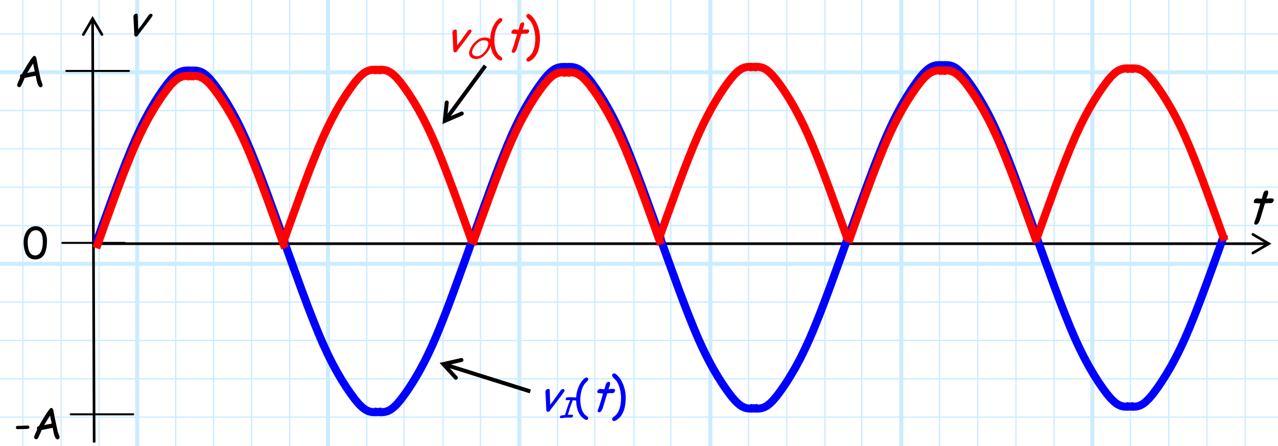
**Very** similar!

# See?

Likewise, compare the output of this **junction diode** full-wave rectifier:




to the output of an **ideal** full-wave rectifier:



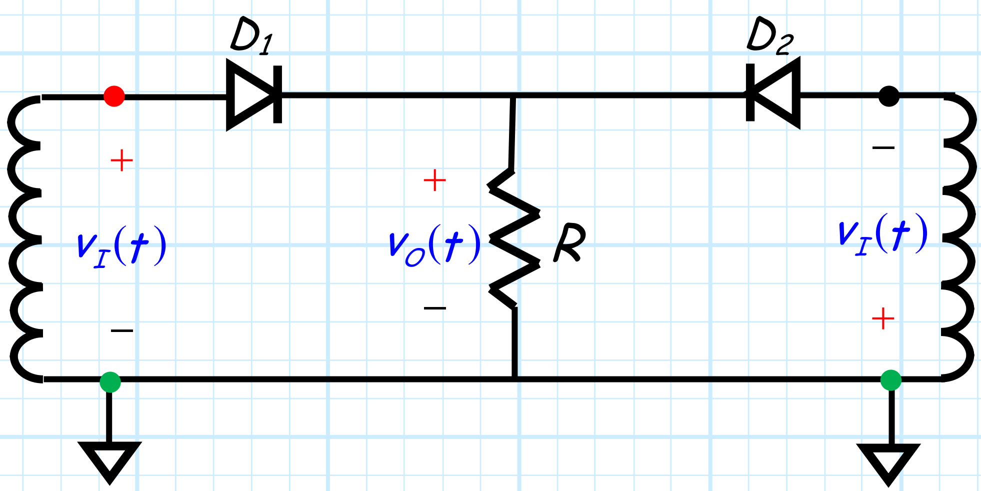

Again we see that the junction diode full-wave rectifier output is **very close** to ideal.

# The DC component of the output is *nearly* ideal!

In fact, if $A \gg 0.7$ V, the **DC component** of this junction diode full wave rectifier is approximately:

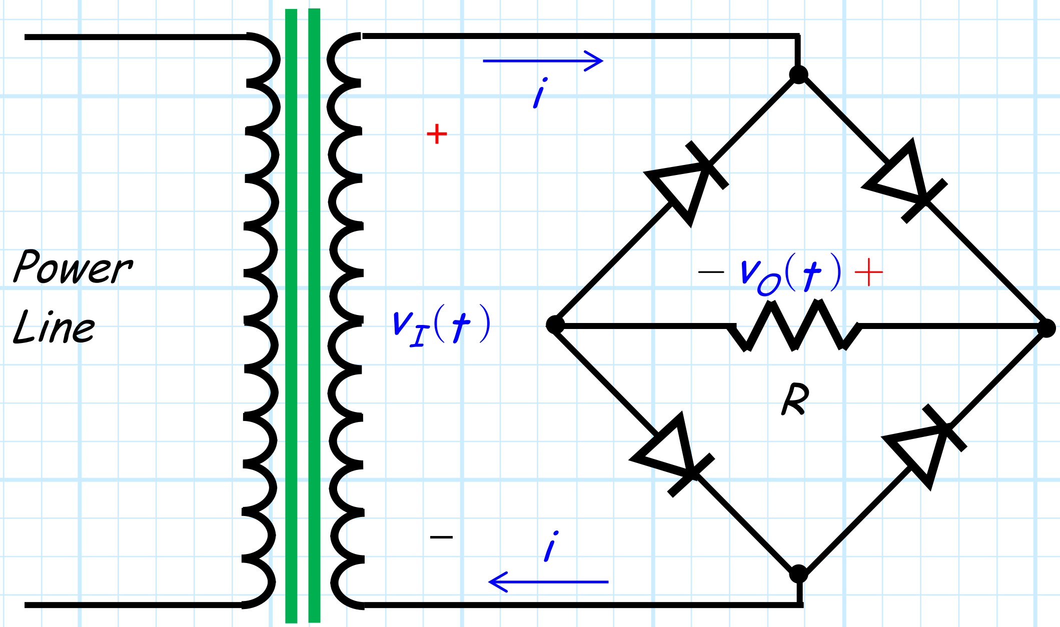
$$V_{DC} \approx \frac{2A}{\pi} - 0.7 \text{ V}$$

**Just** 700 mV less than the **ideal** full-wave rectifier DC component!



# The Bridge Rectifier

Now consider this **junction diode** rectifier circuit:

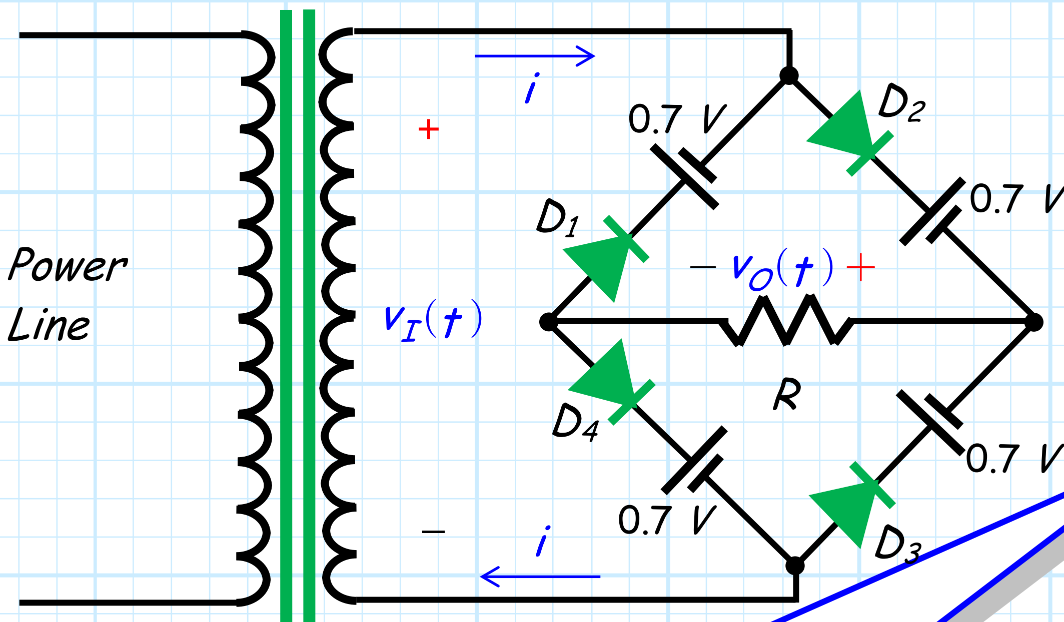



We call this circuit the **bridge rectifier**.

→ Let's **analyze** it and see what it does!

# 16 possible assumptions!

First, we **replace** the junction diodes with the **CVD model**:



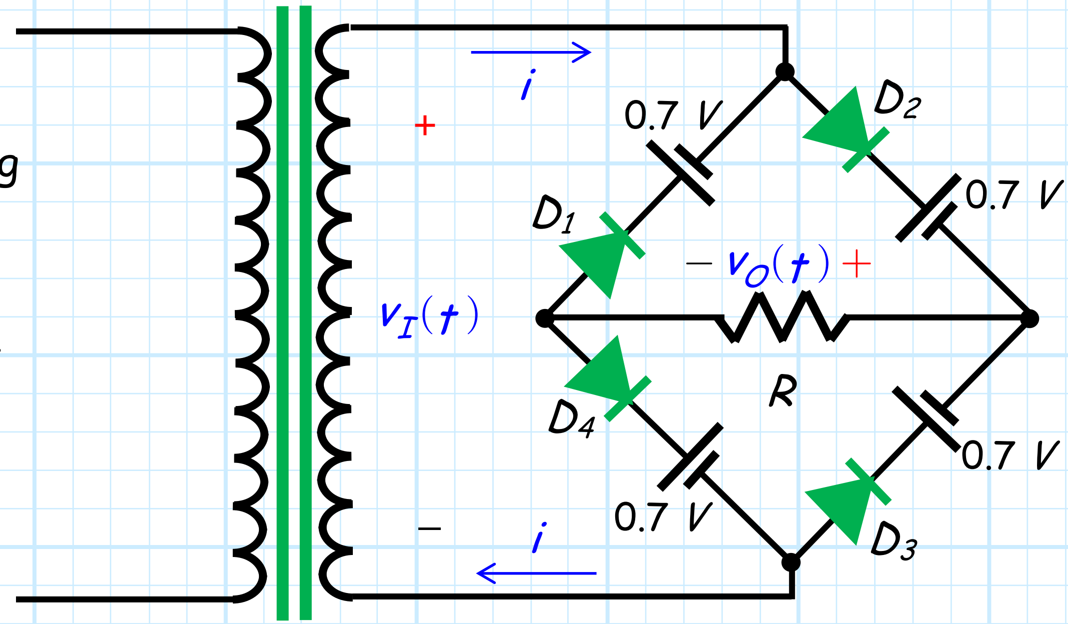

**Q:** *__Four__ gul-durn __ideal__ diodes! That means __16__ sets of dad-gum assumptions!*

**A:** True! However, there are only **three** of these sets of assumptions are actually **possible**!

# But only 3 sets are possible

Consider the **current** $i$ flowing through the rectifier.

This current of course can be positive, negative, or zero.




It turns out that there is only **one** set of diode assumptions that would result in positive current $i$, **one** set of diode assumptions that would lead to negative current $i$, and **one** set that would lead to zero current $i$.

**Q:** *But what about the remaining 13 sets of dog gone diode assumptions?*

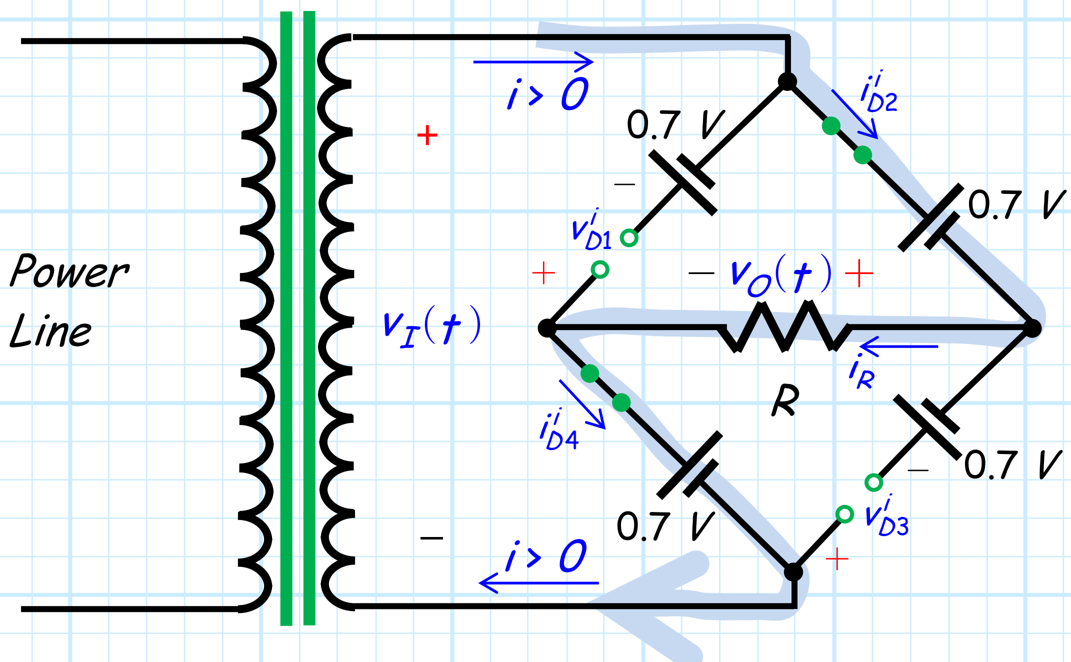
**A: Regardless** of the value of source $v_S$, the remaining 13 sets of diode assumptions simply **cannot occur** for this particular circuit design!

# $i > 0$

Let's look at the **three** possible sets of assumptions, first starting with $i > 0$.

The rectifier current $i$ can be **positive** only **if** these assumptions are true:

* Ideal diodes $D_1$ and $D_3$ are **reverse** biased.
* Ideal diodes $D_2$ and $D_4$ are **forward** biased.



# Breakdown: it appears to be less likely

Analyzing this circuit, we find from KVL that the **output voltage** is:

$$v_O = v_I - 1.4 \text{ V}$$

and the forward biased **ideal diode currents** are from KCL and Ohm's Law:

$$i = i_{D2}^{i} = i_{D4}^{i} = i_R = \frac{v_I - 1.4}{R}$$

and, finally the reverse biased **ideal diode voltages** are from KVL:

$$v_D^{i} = -v_I$$

**Q:** *Hey! I notice that the reverse bias voltage for this bridge rectifier is* ***much less negative*** *than that of the full-wave rectifier (i.e., $v_D^{i} = -2v_I$ for the full-wave rectifier).*

*Does that mean* ***breakdown is less likely****?*

**A:** Absolutely! This is an **important feature** of the **bridge** rectifier (more on this **later**!).

# Way to be paranoid!
# Now, back to the analysis…

Thus, $i_D^i > 0$ when:

$$\frac{v_I - 1.4}{R} > 0$$
$$v_I - 1.4 > 0$$
$$v_I > 1.4 \text{ V}$$

and $v_D^i < 0$ when:

$$-v_I < 0$$
$$v_I > 0$$

Therefore, we **find** that for this circuit that:

$$v_O = v_I - 1.4\,V \quad \text{when} \quad v_I > 0 \quad \textbf{and} \quad v_I > 1.4\,V$$

Applying some logic, we see that this simply means

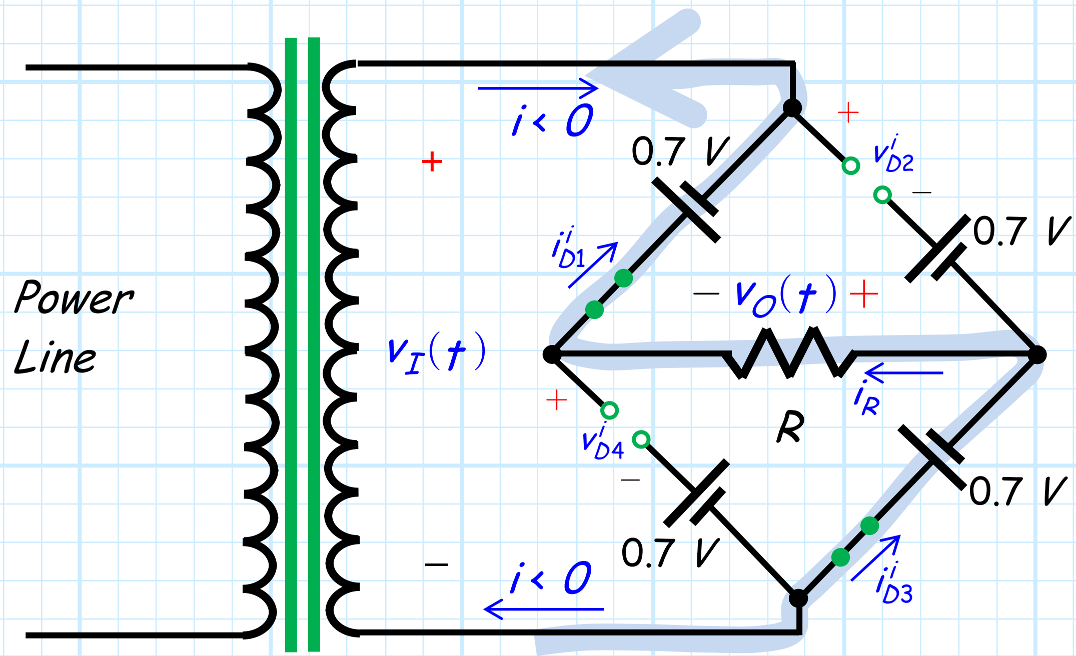
$$\rightarrow \quad v_O = v_I - 1.4\,V \quad \text{when} \quad v_I > 1.4\,V$$

# $i < 0$

The rectifier current $i$ can be **negative** only **if** these assumptions are true:

* Ideal diodes $D_1$ and $D_3$ are **forward** biased.

* Ideal diodes $D_2$ and $D_4$ are **reverse** biased.



# Verify this yourself

Analyzing this circuit, we find from KVL that the **output voltage** is:

$$v_O = -v_I - 1.4 \text{ V}$$

while the forward biased **ideal diode currents** are both determined from KCL and Ohm's Law:

$$-i = i_{D1}^{i} = i_{D3}^{i} = i_R = \frac{-v_S - 1.4}{R}$$

and the reverse biased **ideal diode voltages** are found from KVL to be:

$$v_{D1}^{i} = v_{D3}^{i} = v_I \qquad \text{(remember this for later!)}$$

# Applying logic: it will get you far in life

Thus, $i_D^{i} > 0$ when:

$$\frac{-v_I - 1.4}{R} > 0$$

$$-v_I - 1.4 > 0$$

$$-v_I > 1.4 \text{ V}$$

$$v_I < -1.4 \text{ V}$$

and, $v_D^{i} < 0$ when:

$$v_I < 0$$

Therefore, we **find** that for this circuit that:

$$v_O = v_I - 1.4\,V \quad \text{when} \quad v_I < 0 \quad \textbf{and} \quad v_I < -1.4\,V$$

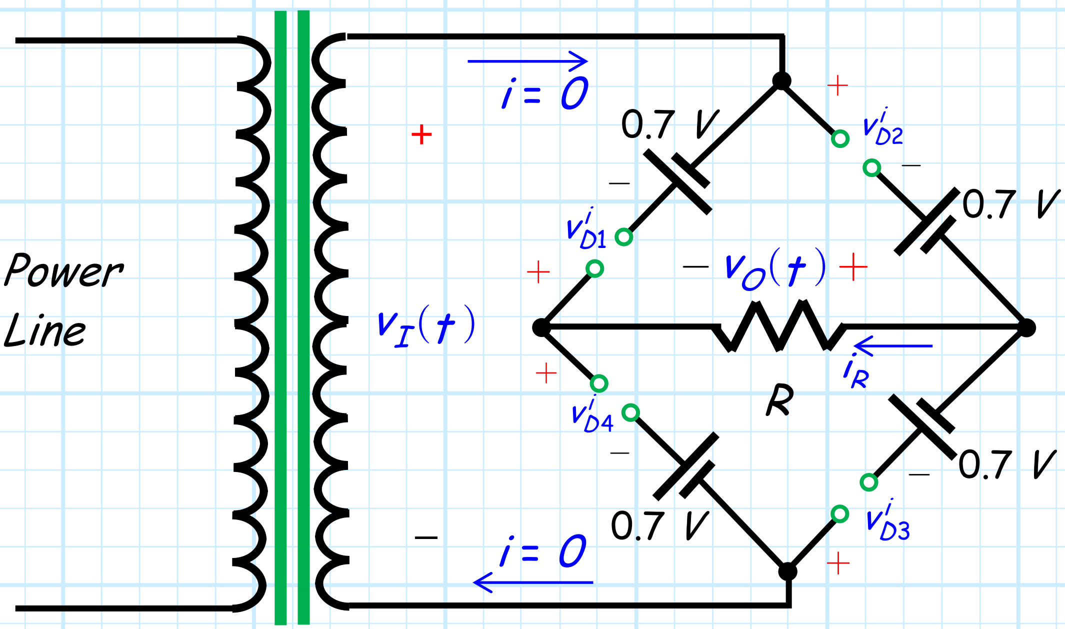
Applying some **logic**, we see that this simply means:

$$\rightarrow \quad v_O = -v_I - 1.4 \text{ V} \quad \text{when} \quad v_I < -1.4 \text{ V}$$

# $i = 0$

The rectifier current $i$ can be **zero** only **if** these assumptions are true:

**All** ideal diodes are **reverse** biased!



# Verify this yourself

Analyzing this circuit, we find that the **output voltage** is:

$$v_O = R\, i_R = R\, i = 0$$

while the **ideal diode voltages** of $D_2$ and $D_4$ are each:

$$v_{D2}^{i} = \frac{v_I - 1.4}{2} = v_{D4}^{i}$$

and the **ideal diode voltages** of $D_1$ and $D_3$ are each:

$$v_{D1}^{i} = \frac{-v_I - 1.4}{2} = v_{D3}^{i}$$

# This logic is a bit simpler

Thus, $v'_{D2} < 0$ when:

$$\frac{v_I - 1.4}{2} < 0$$
$$v_I - 1.4 < 0$$
$$v_I < 1.4$$

and, $v'_{D1} < 0$ when:

$$\frac{-v_I - 1.4}{2} < 0$$
$$-v_I - 1.4 < 0$$
$$-v_I < 1.4$$
$$v_I > -1.4$$

Therefore, we also **find** for this circuit that:

$$v_O = 0 \quad \text{when both} \quad v_s < 1.4V \text{ and } v_s > -1.4V$$

Or, in other "words":

$$\rightarrow v_O = 0 \quad \text{when} \quad -1.4V < v_I < 1.4V$$

# It's true: class of 1983!

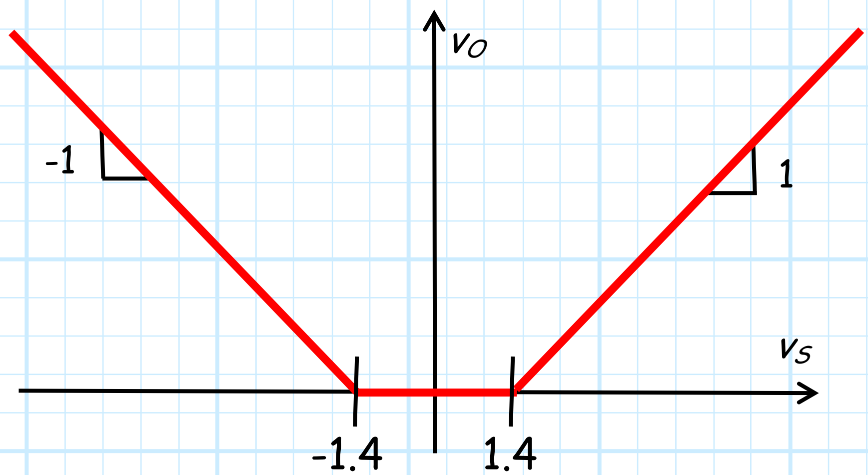
**Q:** *You know, that dang Mizzou grad said we only needed to consider these **three** sets of diode assumptions, yet I am **still** concerned about the other 13.*

*How can we be **sure** that we have analyzed every **possible** set of **valid** diode assumptions?*

**A:** We know that we have considered **every** possible case, because when we combine the three results we find that we have a piece-wise linear **function**!

# Smells like a full-wave rectifier!

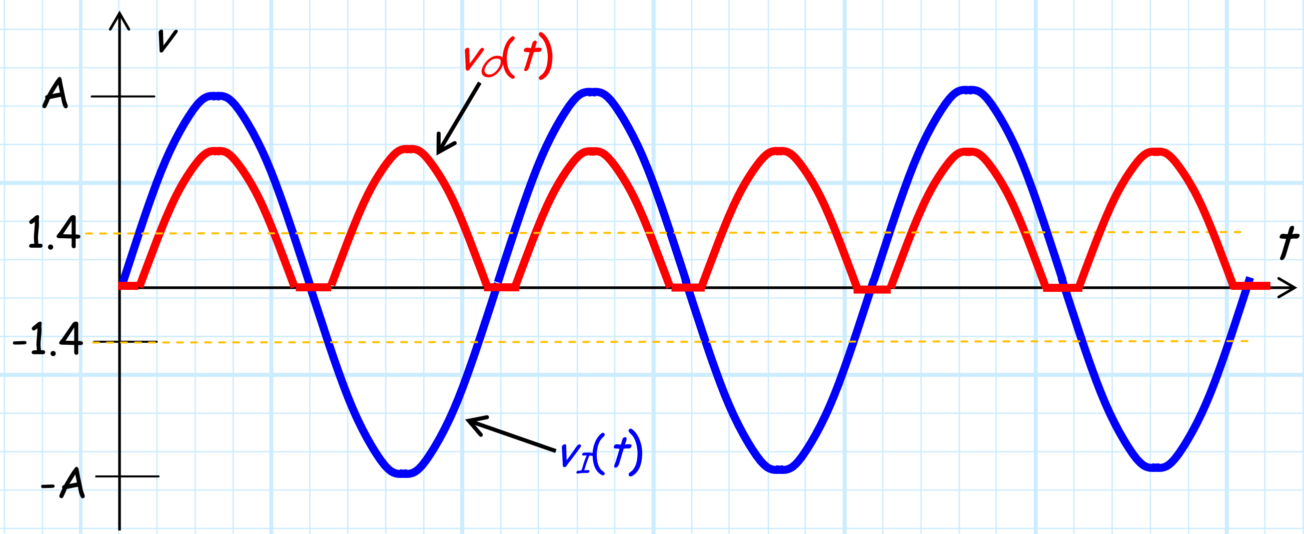
$$v_O = \begin{cases} -v_S - 1.4\text{ V} & \text{if} \quad v_S < -1.4\text{ V} \\ 0 & \text{if} \quad -1.4 < v_S < 1.4\text{ V} \\ v_S - 1.4\text{ V} & \text{if} \quad v_S > 1.4\text{ V} \end{cases}$$




Note that the **bridge** rectifier is a **full-wave** rectifier!

# Close to the ideal result

**If** the input to this rectifier is a **sine wave**, we find that the **output** is approximately that of an ideal **full-wave rectifier**:




We see that the junction diode bridge rectifier output is **very close** to ideal.

In fact, if $A \gg 1.4$ V, the **DC component** of this junction diode bridge rectifier is approximately:

$$V_{DC} \approx \frac{2A}{\pi} - 1.4 \text{ V}$$

**Just** 1.4 V less than the **ideal** full-wave rectifier DC component!

# Peak Inverse Voltage

**Q:** *I'm so **confused!** The bridge rectifier and the full-wave rectifier **both** provide full-wave rectification.*

*Yet, the bridge rectifier use **4** junction diodes, whereas the full-wave rectifier only uses **2**.*

***Why** would we **ever** want to use the **bridge rectifier**?*

**A:** First, a slight **confession**—the results we derived for the **bridge** and **full-wave** rectifiers are **not precisely** correct!

Recall that we used the junction diode **CVD model** to determine the transfer function of each rectifier circuit.

→ The problem is that the CVD model does **not** predict junction diode **breakdown**!

# Doc, it hurts when I do this

If the **input** voltage $v_I$ becomes too **large**, the junction diodes can in fact **breakdown**—but the transfer functions we derived do **not** reflect this fact!

**Q:** *You mean that we must **rework** our analysis and find **new** transfer functions!?*

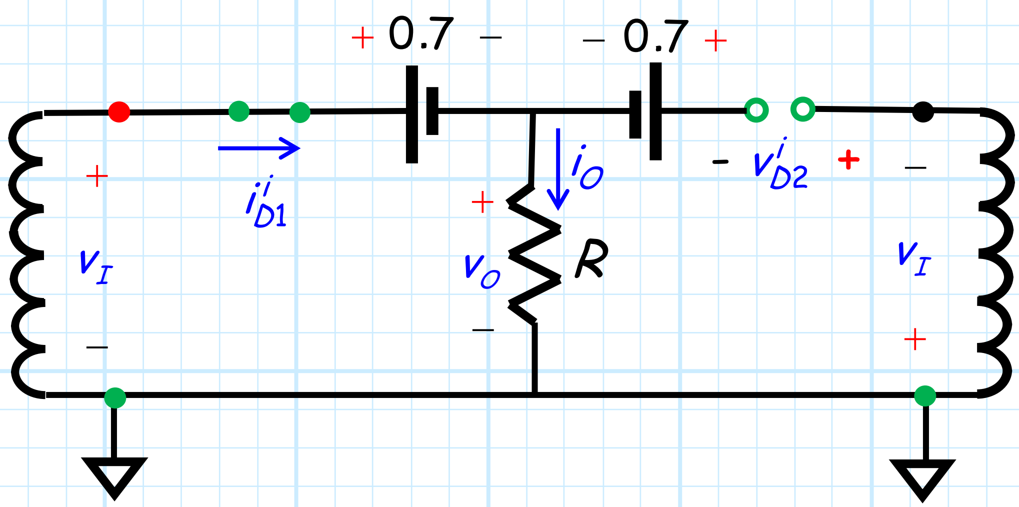
**A:** Fortunately **no**.

Breakdown is an **undesirable** mode for circuit rectification.

Our job as engineers is to design a rectifier that **prevents** it from occuring—that why the **bridge** rectifier is helpful!

# Whew! That's a big negative number

To see **why**, consider the voltage across a **reversed biased** junction diode in **each** of our rectifier circuit designs.




Recall that the voltage across a **reverse biased ideal diode** in the **full-wave rectifier** design was:

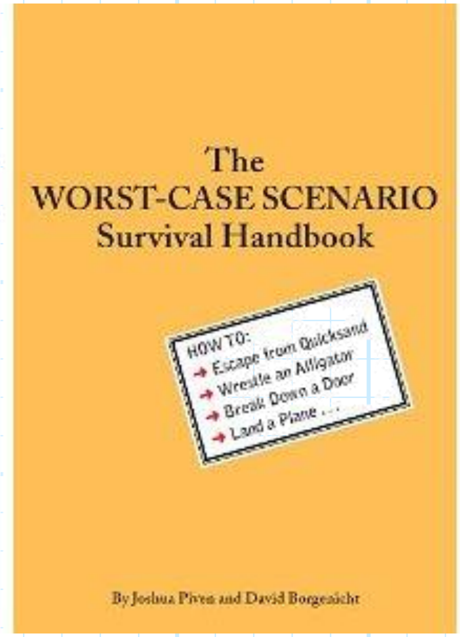
$$v'_{D2} = -2v_I$$

so that the voltage across the **junction** diode is approximately (according to the CVD model):

$$v_{D2} = v'_{D2} + 0.7 = -2v_I + 0.7$$

# Like getting me for all your classes

Now, we wish to determine the **worst case** scenario, with respect to **negative** diode voltage.




We seek $v_D^{min}$, the **minimun** (i.e., most **negative**) value that the diode voltage will **ever** be—at least, for a **given** input $v_I \; t$ .

I.E.:

$$v_D^{min} \leq v_D \; t \quad \text{for all time } t$$

→The value $v_D^{min}$ is a **negative** number!

Recall that for the junction diode to avoid **breakdown**, the diode voltage must be **greater** than $-V_{ZK}$ for all time $t$ (i.e., $v_D \; t > -V_{ZK}$).

# He's not trying to fly; he's indicating "safe"

The **worst case** scenario occurs at the **time** when the diode voltage is at its **most negative** (i.e., when $v_D(t) = v_D^{min}$).

Thus, we know we are **safe** (no breakdown—**ever**!) **IF**:

$$v_D^{min} > -V_{ZK}$$

Now, assume that the **source** voltage is a **sine wave** $v_I(t) = A \sin \omega t$.

We find that diode voltage is at it **most negative** (i.e., breakdown danger!) when the **source** voltage is at its **maximum** value $v_I^{max} = A$.

Therefore:

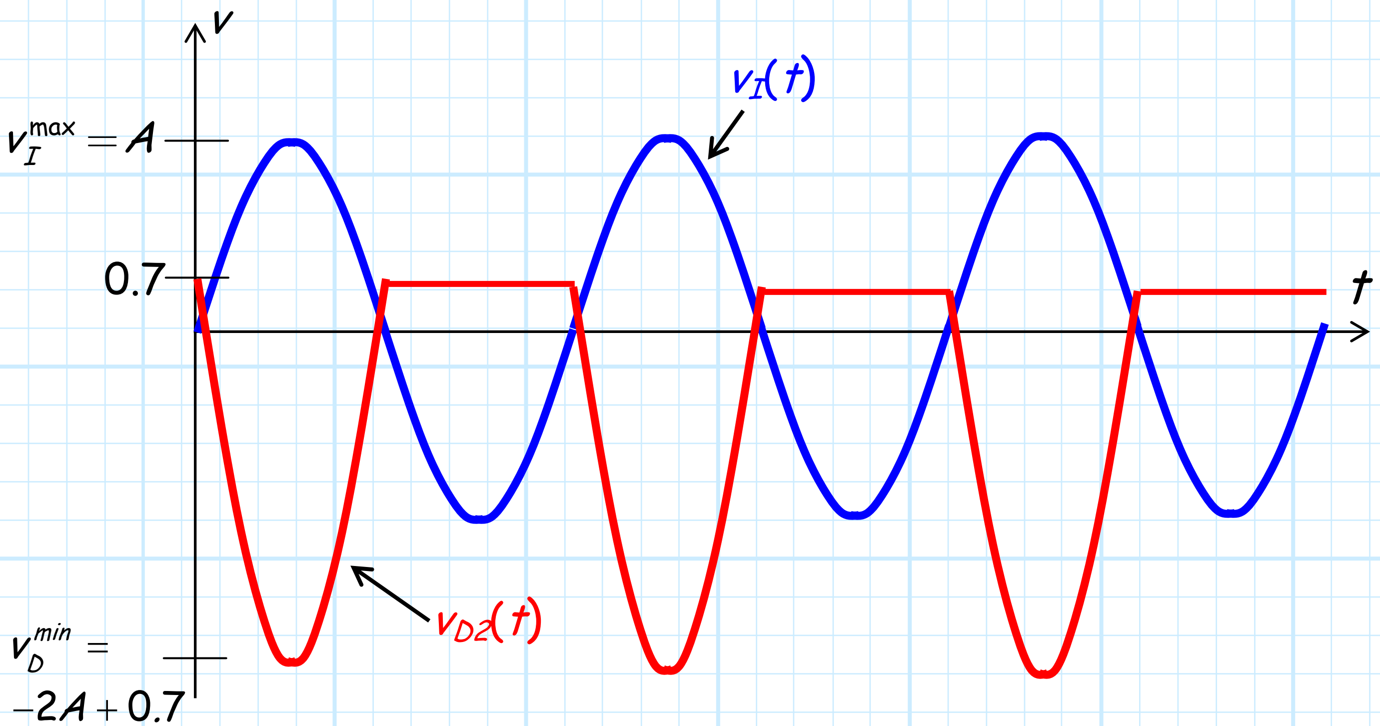
$$v_D^{min} = -2v_I^{max} + 0.7 = -2A + 0.7$$

Of course, the **largest** junction diode voltage occurs when in **forward** bias:

$$v_D^{max} \cong 0.7\ V$$

# Wow: that diode voltage goes way negative!

**Plotting** both **input** $v_I(t) = A \sin \omega t$ and **diode voltage** $v_{D2}(t)$ for the **full-wave** rectifier:




Note that this **minimum** diode voltage $v_D^{min}$ is **very negative**, with an absolute value ($\left|v_D^{min}\right| = 2A - 0.7$) nearly **twice** as large as the source magnitude $A$.

# Peak Inverse Voltage: it's a positive value

Since $v_D^{min}$ is negative, we take its magnitude, "converting " it into a positive value we call the **Peak Inverse Voltage** ($PIV$):

$$\boxed{PIV = \left| v_D^{min} \right|}$$


→ The PIV is a **positive number**!

For example, the PIV of this **full-wave rectifier**, with a **sinusoidal input** is:

$$PIV = \left| v_D^{min} \right| = 2A - 0.7$$

# The input *and* the circuit—PIV depends on *both* !

It is **crucial** that you understand that the Peak Inverse Voltage (PIV) is dependent on **two** things:

1. the rectifier **circuit design**, and

2. the **input** voltage $v_I \; t$ .


**Q:** *So, **why** do we need to determine PIV?*

*I'm not sure I see what **difference** this value makes.*

**A:** The Peak Inverse Voltage specifies the **worst case scenario** with respect to **negative** diode voltage.

It allows us to **answer** one important question—**will** the junction diodes in our rectifier **breakdown**?

# You're safe if PIV is less than $V_{ZK}$

To **avoid breakdown**, we earlier found that $v_D^{min}$ **must be greater** than $-V_{ZK}$:

$$v_D^{min} > -V_{ZK} \quad \text{to avoid breakdown}$$

Multiplying by -1, we **equivalently** state:

$$-v_D^{min} < V_{ZK} \quad \text{to avoid breakdown}$$

But since $v_D^{min}$ is negative, the value $-v_D^{min}$ is **positive**, and so

$$-v_D^{min} = \left| v_D^{min} \right| = PIV$$

Inserting the result in the earlier inequality, we find that breakdown is avoided **if**:

$$\boxed{PIV < V_{ZK}}$$

# In summary:

→ **If** the PIV is **less** than the Zener breakdown voltage of your rectifier diodes. I.E.,

$$\textbf{if} \quad PIV < V_{ZK} \, ,$$

then we know that your junction diodes will **remain** in either **forward** or **reverse** bias for **all time** $t$.

*Your rectifier will operate "properly"!*

→ However, **if** the PIV is **greater** than the Zener breakdown voltage of your rectifier diodes. I.E.:

$$\textbf{if} \quad PIV > V_{ZK} \, ,$$

then you know that our junction diodes will **breakdown** for at least **some** small amount of time $t$.

*Then the rectifier will NOT operate properly!*

# But what if PIV is too big?

**Q:** *So what do we do if PIV **is** greater than $V_{ZK}$?*

*How do we **fix** this problem?*

**A:** We have **three** possible **solutions**:

1. Use junction diodes with **larger** values of $V_{ZK}$ (**but** they exist!).

2. Reduce the input voltage (e.g., magnitude $A$), **but** this will decrease you DC component $V_{DC}$

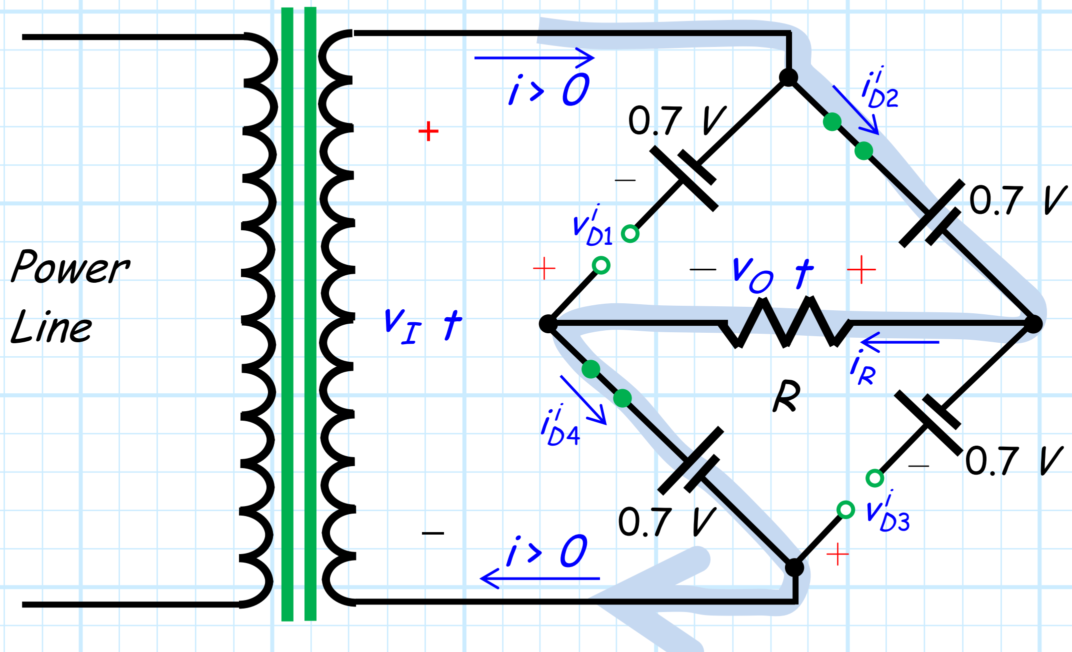
3. Use the **bridge** rectifier design—**no buts** about it!

# The bridge rectifier to the rescue

**Q:** *The **bridge** rectifier!*

*How does that solve our **breakdown** problem?*

**A:** To see how a **bridge** rectifier can be **useful**, let's determine its Peak Inverse Voltage **PIV**.



# Danger, danger!

First, we recall that the voltage across a **reverse biased ideal diode** was:

$$v_{D1}^{i} = -v_S$$

so that the voltage across the **junction** diode is approximately:

$$\begin{aligned} v_{D1} &= v_{D1}^{i} + 0.7 \\ &= -v_I + 0.7 \end{aligned}$$

Now, assume that the **source** voltage is a **sine wave** $v_I(t) = A \sin \omega t$.

We found that diode voltage is at it **most negative** (i.e., breakdown danger!) when the **source** voltage is at its **maximum** value $v_I^{max} = A$.

I.E.,:

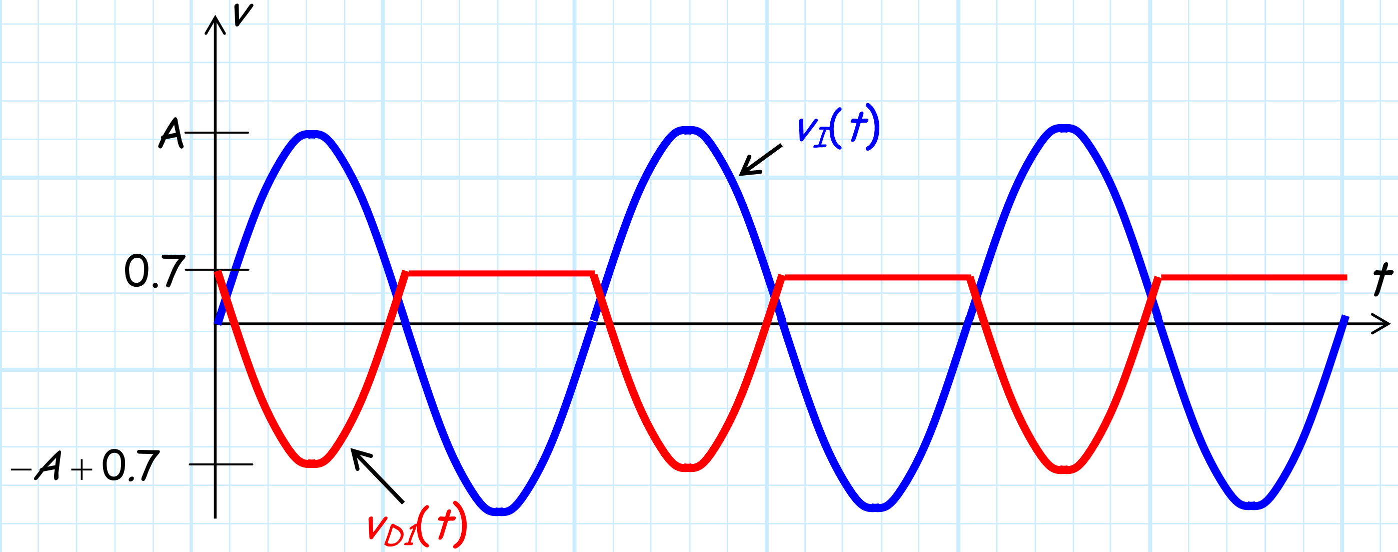
$$v_D^{min} = -v_I^{max} + 0.7 = -A + 0.7$$

# Hmm… It's negative, but not extremely so

Of course, the **largest** junction diode voltage occurs when in forward bias:

$$v_D^{max} = 0.7 \text{ V}$$

Plotting both **input** $v_I(t) = A \sin \omega t$ and **diode voltage** $v_{D1}(t)$ for the **bridge** rectifier:




Note that this minimum diode voltage is **negative**, with an absolute value ($\left|v_D^{min}\right| = A - 0.7$), approximately **equal** to the value of the **source magnitude** $A$.

# The bridge rectifier PIV is about half the full-wave PIV

Thus, the **PIV** for a **bridge** rectifier with a **sinusoidal source** voltage is:

$$PIV_{brg} = \left| v_D^{min} \right| = A - 0.7$$

Note that this bridge rectifier value is approximately **half** the PIV we determined for the **full-wave** rectifier design:

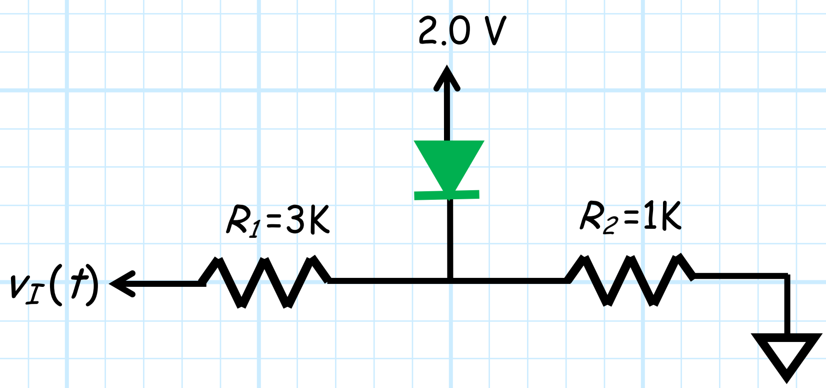
$$PIV_{fw} = \left| v_D^{min} \right| = 2A - 0.7$$

Thus, the source voltage (and the output DC component) of a **bridge** rectifier can be **twice** that of the full-wave rectifier design.

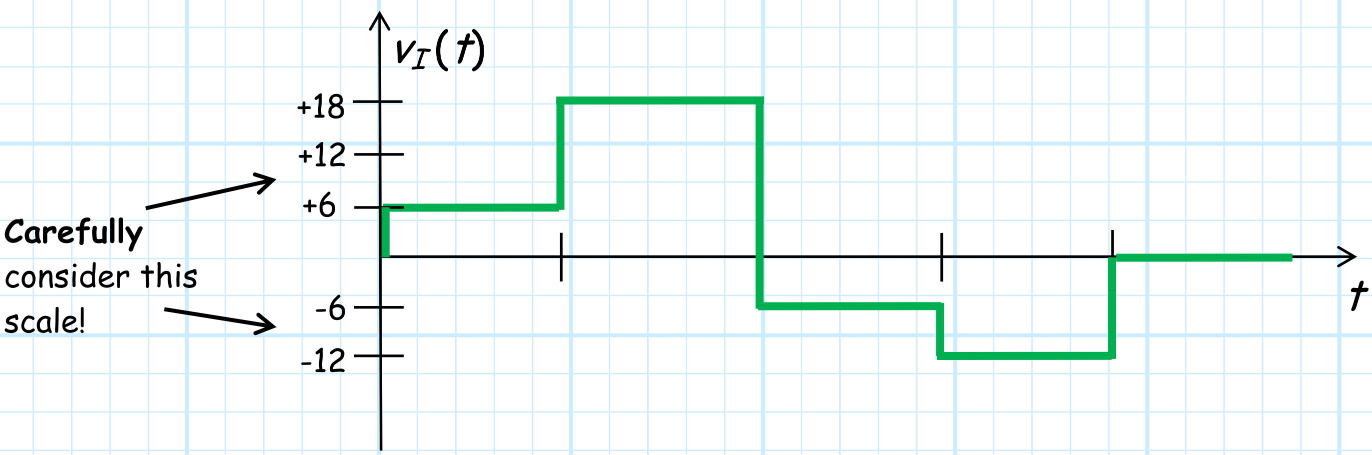
→ This is why the **bridge** rectifier is a very **useful** rectifier design!

# Example: Peak Inverse Voltage

The diode in the circuit below is **IDEAL**.




The input to this circuit ( $v_I(t)$ ) is plotted below.




Let's determine the **Peak Inverse Voltage (PIV)** of the **ideal** diode in this circuit.

**Q:** *This is **so** confusing! Is **this** the right equation:*

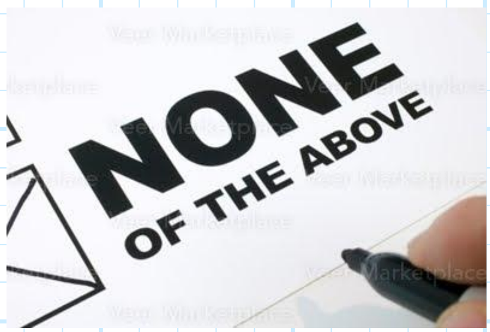
$$PIV = A - 0.7$$

*Or, do I use **this** equation:*

$$PIV = 2A - 0.7 \quad ??$$

*Likewise, is A=18, or is A=12, or is it some **other** number ?*

**A: None** of the above!




The result:

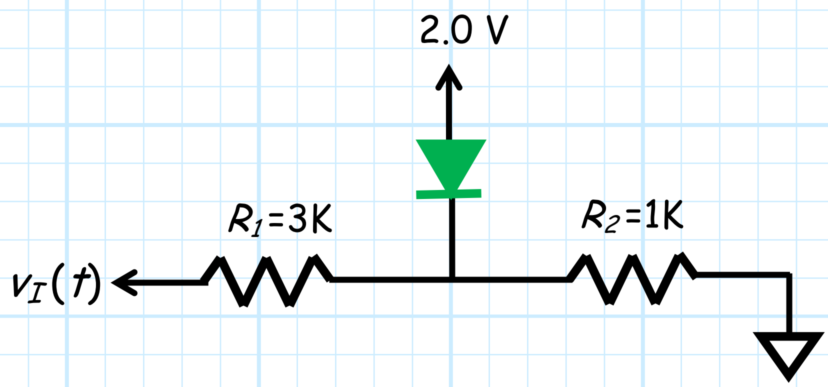
$$PIV = A - 0.7$$

is the PIV for the following **specific** situation—and this situation **only**:

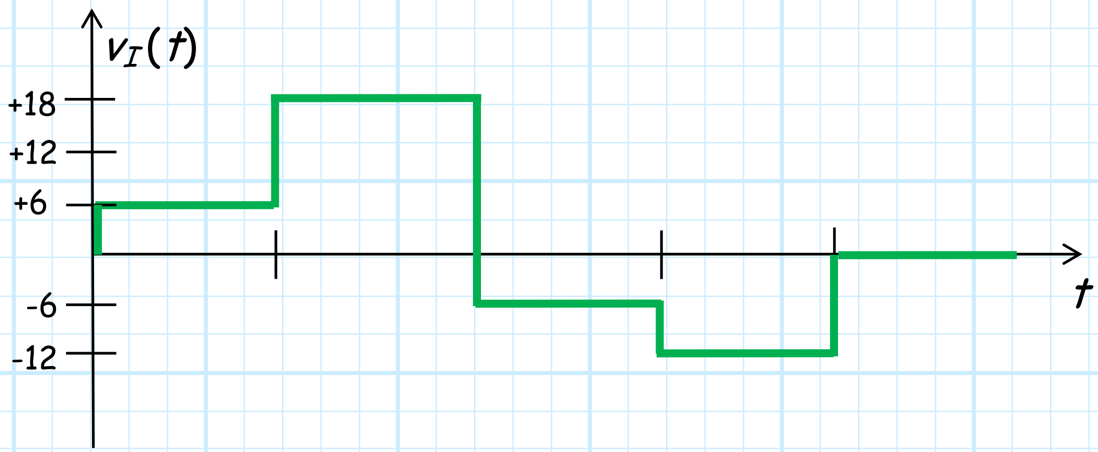
1. the diode circuit is a **bridge rectifier**, with

2. an **input** signal $v_I(t) = A \sin \omega t$

Of course, the circuit for **this** problem is **most definately not** a bridge rectifier (it doesn't even have an output!):




And the input signal is **most definitely not** a sinusiod.




Thus, $PIV = A - 0.7$ is **most definitely not** "the right equation"!

**Q:** *Oh, so then I should use the **other** one:*

$$PIV = 2A - 0.7 \quad ?$$

**A:** Ahem; the result:

$$PIV = 2A - 0.7$$

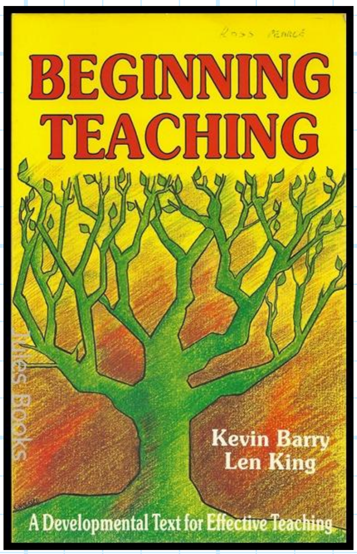
is the PIV for this **specific** situation—and this situation **only**:

1. the diode circuit is a **full-wave rectifier**, with

2. an **input** signal $v_I(t) = A \sin \omega t$

Of course, the circuit in this problem is **not** a full-wave rectifier (it **doesn't** even have an **output**!), and the input signal is **not** a sinusiod.

Thus, you most definitely **do not** "use" the equation $PIV = 2A - 0.7$!

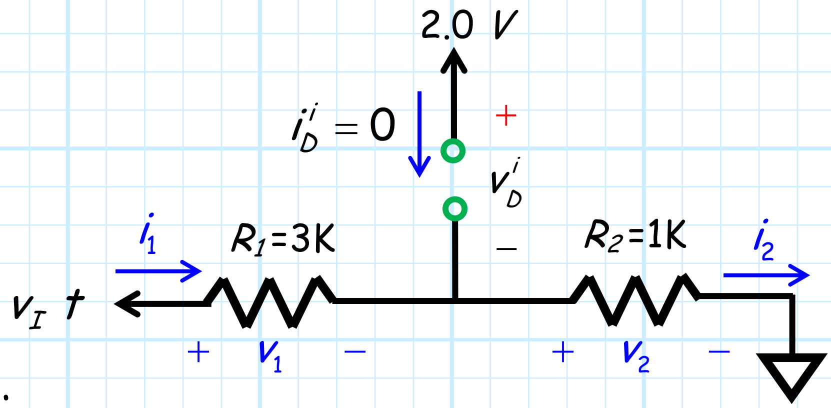



**Q:** *Apparently,* ***you*** *could* ***"use"*** *a book on* ***teaching****, because these are the* ***only two*** *PIV equations that* ***you gave us!***

**A:** **I** did not **"give"** you these equations—they appeared as a result of a careful and detailed analysis of the **specific situations** we encountered.

You **now** have encountered (with this problem) a completely **different** situation—**you** can find the "right equation to use" only after **you** carefully, patiently, completely analyze this **new, specific** situation!

To begin, you first ASSUME that the ideal diode is **reverse** biased, and thus ENFORCE the condition that $i_D^i = 0$:

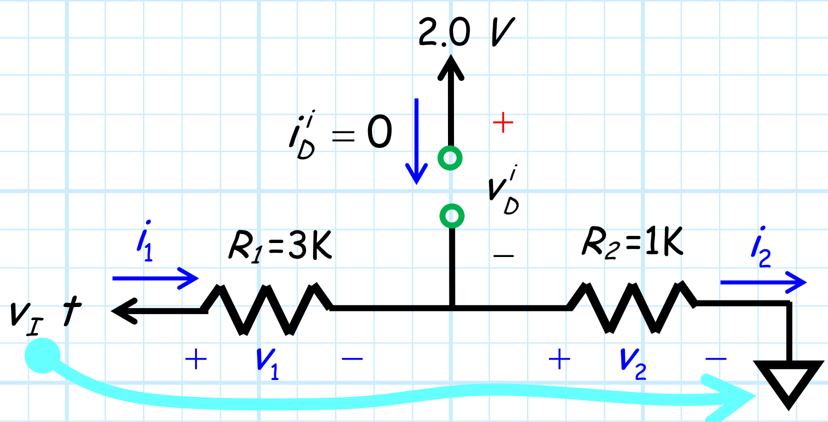



From **KCL**:

$$\begin{aligned} i_2 &= i_1 + i_D^i \\ &= i_1 + 0 \\ &= i_1 \end{aligned}$$

From **Ohm's Law** and the results **above**:

$$v_1 = i_1 R_1 = 3\, i_1 \qquad\qquad v_2 = i_2 R_2 = 1\ i_2 \ = 1\ i_1$$




Now from **KVL**:

$$v_I - v_1 - v_2 = 0$$

Therefore,

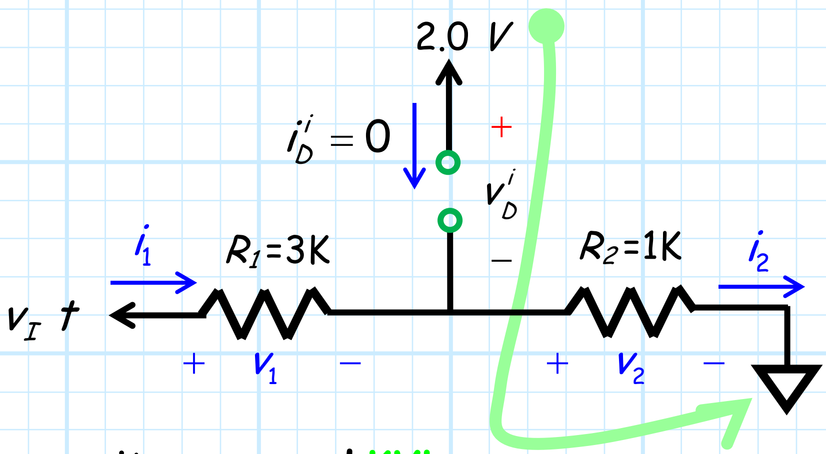
$$\begin{aligned} v_I &= v_1 + v_2 \\ &= 3 i_1 + 1\ i_1 \\ &= 4 i_1 \end{aligned}$$

And so:

$$i_1 = 0.25\, v_I$$

meaning:

$$v_2 = 1\ i_1 \ = 0.25 v_I$$




Now you can write a **second KVL**:

$$2 - v_D^i - v_2 = 0 \qquad \Rightarrow \qquad v_D^i = 2 - v_2 = 2 - 0.25 v_I$$

So you have concluded that the (reverse biased) **ideal diode voltage** is:

$$v_D^i = 2 - 0.25\, v_I$$

This result is true, provided that(!) your reverse bias **assumption** is correct ($v_D^i < 0$)! You can quickly solve the inequality to determine **WHEN** this is true:

$$\begin{aligned} 2 - 0.25\, v_I &< 0 \\ -0.25\, v_I &< -2 \\ -v_I &< -8 \\ v_I &> 8 \end{aligned}$$

Thus:

$$\rightarrow v_D^i = 2 - 0.25\, v_I \quad \text{when} \quad v_I > 8$$

The important (**very** important!) thing to note here is that the ideal **diode voltage** is negative **only** when the **input** voltage is significantly **positive** (i.e., $v_I > 8$).

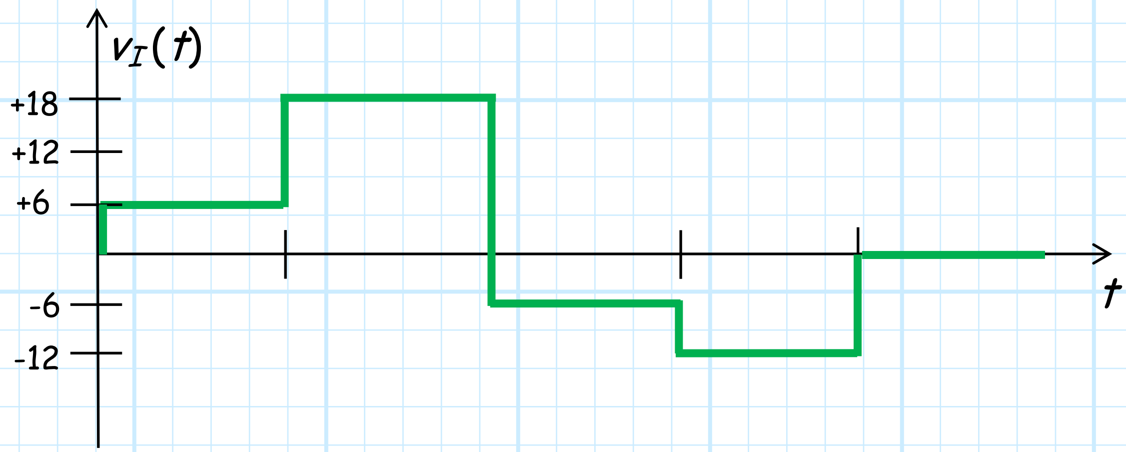
Moreover, the **larger** (i.e., more positive) **the input** voltage becomes, the **more negative** the ideal **diode voltage** becomes!

Thus, you come to the conclusion for **this** specific circuit (it may **not** be true for other circuits!), that the **minimum** ideal diode voltage ($v_D^{i\,min}$) occurs when the input voltage is at its **maximum** (most positive). I.E.:

$$v_D^{i\,min} = 2 - 0.25\, v_I^{max}$$

**Q:** *But **how** do I know what $v_I^{max}$ is? You said that $v_I^{max} \neq A$; so then what is the **"right equation"** to **"use"**?*

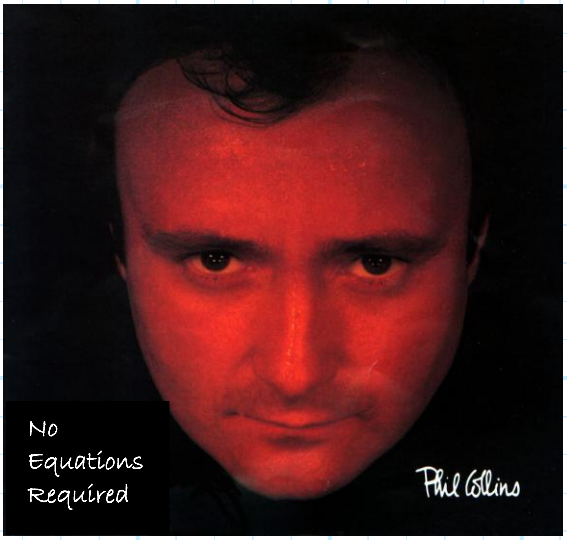
**A:** **Sigh**; there **is** no equation. You know **what** the input voltage is—it is given in the **problem statement**:



Note that the **input** voltage **changes** with respect to **time**.

**At first**, the input voltage is **6.0** $V$, later on it is at **-6.0 V** and then after that **-12.0 V**. The **important** question is: when is the input voltage at its **maximum** (i.e., most positive)?

Hopefully, by simply **looking** at the input voltage signal (**no equations** required!) it is evident that **18.0** $V$ is the most positive the input voltage **ever is**!




Thus,

$$v_I^{max} = 18.0$$

Note that $v_I^{max} = 18.0 > 8.0$; the ideal diode is **definitely** reverse biased!

And so, during the **time period** when the input voltage is at this **maximum** value, the ideal **diode voltage** will be at its **minimum** (i.e., most negative):

$$\begin{aligned} v_D^{i\,min} &= 2 - 0.25\, v_I^{max} \\ &= 2 - 0.25\ \ 18.0 \\ &= -2.5\, V \end{aligned}$$

Thus, the **Peak Inverse Voltage** (PIV) is:

$$PIV = \left| v_D^{i\,min} \right| = \left| -2.5 \right| = \underline{\underline{2.5\, V}}$$

To confirm this, let's **plot** the ideal **diode voltage** as a function of **time**.

**Q:** *Wait! We **only** know the ideal diode voltage when* $v_I > 8.0\,V$.

***What** is the ideal diode voltage when* $v_I < 8.0\,V$ *?*

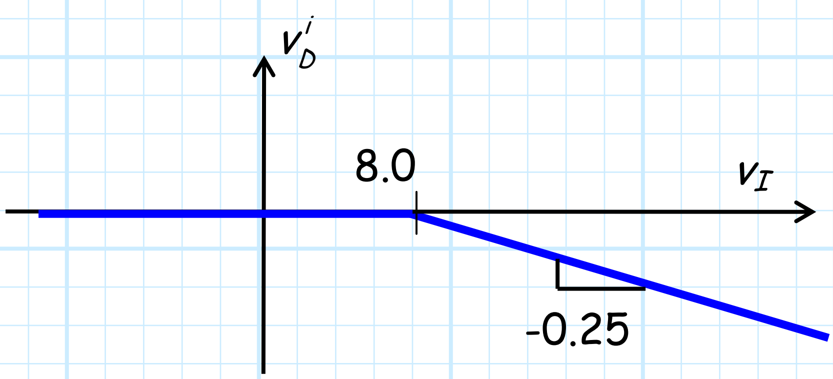
**A:** Remember, the ideal diode is **reverse** biased when $v_I > 8.0\,V$. Thus, when $v_I < 8.0\,V$ the ideal diode is **forward** biased.

→ And you **know** the voltage of a **forward biased ideal diode**!

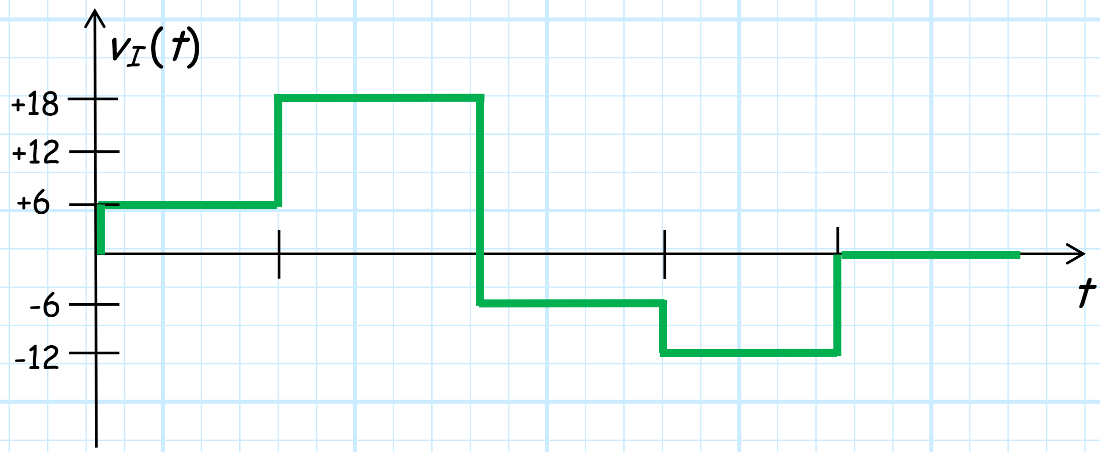
$$v_D^i = 0 \quad \text{if forward biased}$$

Thus, the **continuous** function relating the ideal diode voltage to the input voltage is:

$$v_D^i = f(v_I) = \begin{cases} 2 - 0.25\,v_I & \text{when } v_I > 8.0\,V \\ 0 & \text{when } v_I < 8.0\,V \end{cases}$$

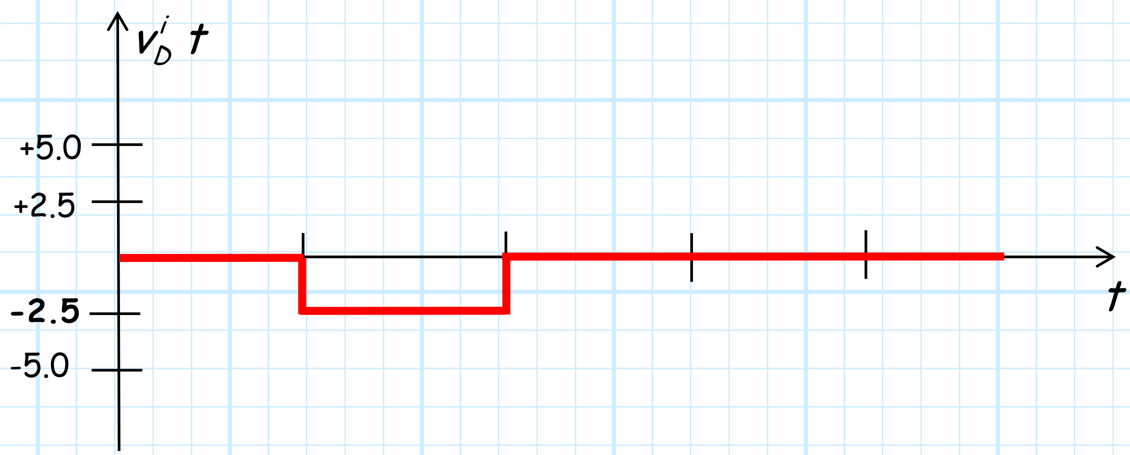


Evaluating this with the **input** signal:




you see that the input is greater than 8.0 $V$ **only** during the period when it is equal to **18.0 $V$**! Thus, **only** during this time period is the ideal diode voltage **non-zero**.

Specifically, you have found that **when** $v_I = v_I^{max} = 18.0$, the ideal **diode voltage** is $v_D^i = -2.5\,V$.




Thus, you have **confirmed** that $v_D^{i\,min} = -2.5\,V$, and so:

$$PIV = \left|v_D^{i\,min}\right| = |-2.5| = \underline{\underline{2.5\,V}}$$

**Q:** *Wasn't this a bit of an* ***academic exercise****; I mean after all, an* ***ideal*** *diode* ***doesn't*** *have a* ***breakdown*** *region!*

**A:** True enough, this **was** an academic exercise. It did this to **simplify** the analysis a bit.

However, **if** you are analyzing a circuit with a **junction** diode, you still need to first find the minimum **ideal** diode voltage $v_D^{i\,min}$ of the **ideal** diode in the **CVD model**.

Then, the minimum voltage of the **junction** diode is simply the minimum voltage across the **CVD model**, which (of course!) is:

$$v_D^{min} = v_D^{i\,min} + 0.7$$

Thus, the PIV is:

$$PIV = \left|v_D^{min}\right| = \left|v_D^{i\,min} + 0.7\right|$$

Now this PIV value had **better** be **smaller** than the Zener breakdown voltage of the junction diode, or else **breakdown will** occur!