4.4 The MOSFET as an Amp and Switch

Reading Assignment: pp. 270-280

Now we know how an enhancement MOSFET works!

Q:

A:

1.

2.

HO: The MOSFET as an Amp and Switch
The MOSFET as an Amp and Switch

Consider this simple MOSFET circuit:

Q: Oh, good—you’re going to waste my time with another of these pointless academic problems. Why can’t you discuss a circuit that actually does something?

A: Actually, this circuit is a fundamental electronic device! To see what this circuit does, we need to determine its transfer function $v_O = f(v_I)$.

Q: Transfer function! How can we determine the transfer function of a MOSFET circuit?!

A: Same as with junction diodes—we determine the output $v_O$ for each device mode, and then determine when (i.e., for what values of $v_I$) the device is in that mode!
First, note that regardless of the MOSFET mode:

\[ V_{GS} = V_I - 0.0 = V_I \]

and:

\[ V_{DS} = V_O - 0.0 = V_O \]

From KVL, we can likewise conclude that:

\[ V_{DS} = V_O = 5.0 - i_D R_D \]

Now let’s ASSUME that the MOSFET is in cutoff, thus ENFORCING \( i_D = 0 \).

Therefore:

\[ V_O = 5.0 - i_D R_D \]

\[ = 5.0 - 0(1) \]

\[ = 5.0 \text{ V} \]

Now, we know that MOSFET is in cutoff when:

\[ V_{GS} = V_I < V_t = 1.0 \]

Thus, we conclude that:

\[ V_O = 5.0 \text{ V} \text{ when } V_I < 1.0 \text{ V} \]
Now, let’s assume that the MOSFET is in **saturation**, thus enforce:

\[
i_d = K(v_{gs} - V_t)^2 = K(v_I - V_t)^2 = 0.75(v_I - 1.0)^2
\]

And thus the output voltage is:

\[
v_o = 5.0 - i_d R_o = 5.0 - 0.75(v_I - 1.0)^2 (1)
\]

Now, we know that MOSFET is in saturation when:

\[
v_{gs} = v_I > V_t = 1.0
\]

and when:

\[
v_{ds} = v_o > v_{gs} - V_t = v_I - 1.0
\]

This second inequality means:

\[
v_o > v_I - 1.0
\]

\[
5.0 - 0.75(v_I - 1.0)^2 > v_I - 1.0
\]

\[
0 > 0.75(v_I - 1.0)^2 + (v_I - 1.0) - 5.0
\]

Solving this quadratic, we find that the **only** consistent solution is:

\[
v_I - 1.0 < 2.0
\]

\[
v_I < 3.0
\]
Meaning that the MOSFET is in saturation when $v_I > 1.0$ and $v_I < 3.0$. Logically, this is same thing as saying the MOSFET is in saturation when $1.0 < v_I < 3.0$.

Thus we conclude:

$$\nu_O = 5.0 - 0.75(v_I - 1.0)^2 \text{ when } 1.0 < v_I < 3.0 \text{ V}$$

Finally, let's ASSUME that the MOSFET is in triode mode, thus we ENFORCE:

$$i_d = k \left[ 2(v_{gs} - v_I)v_{ds} - v_{ds}^2 \right]$$

$$= 0.75 \left[ 2(v_I - 1.0)v_O - v_O^2 \right]$$

And thus the output voltage is:

$$\nu_O = 5.0 - i_d R_o$$

$$= 5.0 - 0.75 \left[ 2(v_I - 1.0)v_O - v_O^2 \right](1)$$

$$= 5.0 - 0.75 \left[ 2(v_I - 1.0)v_O - v_O^2 \right]$$

Rearranging this equation, we get the quadratic form:

$$0.75v_O^2 - (1.5v_I - 0.5)v_O + 5.0 = 0$$

The solutions of which are:

$$v_O = \frac{(1.5v_I - 0.5) \pm \sqrt{(1.5v_I - 0.5)^2 - 15.0}}{1.5}$$
Note because of the ±, there are two possible solutions. However, to be in triode region, the MOSFET must not be in pinchoff, i.e.:

$$V_O = V_{DS} < V_{GS} - V_T = V_I - 1.0$$

This condition is satisfied with the smaller of the two solutions (i.e., the solution with the minus sign):

$$V_O = \frac{(1.5V_I - 0.5) - \sqrt{(1.5V_I - 0.5)^2 - 15.0}}{1.5}$$

So, the above expression provides us with the output voltage if the MOSFET is in triode mode. The question remaining is thus when (i.e., for what values of $V_I$) is the MOSFET in triode mode?

We could do a lot more math to find this answer, but this answer is actually quite obvious!

Recall that we have already determined that:

a) The MOSFET is in cutoff when $V_I < 1.0$ V

b) The MOSFET is in saturation when $1.0 < V_I < 3.0$ V

Since there are only three modes of a MOSFET device, and since the transfer function must—well—be a function, we can conclude (correctly) that the MOSFET will be in triode region when $V_I$ is the value of the only region that is left--$V_I > 3.0$!
Thus we can conclude that:

\[
\nu_O = \frac{(1.5 \nu_I - 0.5) - \sqrt{(1.5 \nu_I - 0.5)^2 - 15.0}}{1.5} \quad \text{when } \nu_I > 3.0 \text{ V}
\]

We now have determined the complete, continuous **transfer function** of this circuit!

\[
\nu_O = \begin{cases} 
0 & \text{when } \nu_I < 1.0 \text{ V} \\
5.0 - 0.75(\nu_I - 1.0)^2 & \text{when } 1.0 < \nu_I < 3.0 \text{ V} \\
\frac{(1.5 \nu_I - 0.5) - \sqrt{(1.5 \nu_I - 0.5)^2 - 15.0}}{1.5} & \text{when } \nu_I > 3.0 \text{ V}
\end{cases}
\]
A: To see how this circuit is **useful**, consider what happens when the **input** voltage $v_I$ is 0 V and 5V.

From the transfer function, we find that if $v_I = 0$, the output voltage will be $v_o = 5.0$. Likewise, if the input voltage is $v_I = 5.0$, the output voltage will be **small**, specifically $v_o = 0.78$ V.

Let's **summarize** these results in a table:

<table>
<thead>
<tr>
<th>$v_I$</th>
<th>$v_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>5.0</td>
</tr>
<tr>
<td>5.0</td>
<td>0.78</td>
</tr>
</tbody>
</table>
This circuit provides a simple example of one of the primary applications of MOSFET devices—digital circuit design. We can use MOSFETs to make digital devices such as logic gates (AND, OR, NOR, etc.), flip-flops, and digital memory.

We typically find that, just like this circuit, when a MOSFET digital circuit is in either of its two binary states (i.e., “0” or “1”), the MOSFETs in the circuit will either be in cutoff \((i_D=0)\) or in triode \((v_{DS} \text{ small})\) modes.

→ Cutoff and Triode are the MOSFET modes associated with digital circuits and applications!
Q: So, just what good is the MOSFET Saturation Mode??

A: Actually, we will find the MOSFET saturation mode to be extremely useful!

To see why, take the derivative of the above circuit's transfer function (i.e., $dV_O/dV_I$):

Sir, it appears to me that the Saturation region is just a useless MOSFET mode between cutoff and triode!
We note that in cutoff and triode:

\[ \left| \frac{dV_O}{dV_I} \right| \approx 0 \]

while in the saturation mode:

\[ \left| \frac{dV_O}{dV_I} \right| >> 1 \]

**Q:** Oh goody. The slope of the transfer function is large when the MOSFET is in saturation. Am I supposed to be impressed by that?! How are these results even remotely important!?

**A:** Since in cutoff and triode \( \frac{dV_O}{dV_I} = 0 \), a small change in input voltage \( V_I \) will result in almost no change in output voltage \( V_O \).

Contrast this with the saturation region, where \( \left| \frac{dV_O}{dV_I} \right| >> 1 \). This means that a small change in input voltage \( V_I \) results in a large change in the output voltage \( V_O \!\).

To see how this is important, consider the case where the input signal has both a DC and a small-signal (AC) component:

\[ V_I(t) = V_I + V_i(t) \]
As a result, the output voltage likewise has both a DC and small-signal component:

\[ v_o(t) = V_o + v_o(t) \]

Now, let's consider only the DC components. We can select the DC input \( V_I \) such that the MOSFET is placed in saturation. The value \( V_I \), along with the resulting DC output \( V_O \), sets a DC bias point for this circuit.

By selecting the right value of \( V_I \) we could set this DC bias point to where the transfer function slope is the greatest:

Now, say we add a small-signal \( v_i \) to this input DC voltage (i.e., \( v_I(t) = V_I + v_i(t) \)). This small signal simply represents a small change in the input voltage from its average (i.e., DC) value. The result is of course as small change in the output voltage—the small-signal output voltage \( v_o(t) \)!
Now for the interesting part (I bet you were wondering when I would get around to it)! The small change in the output voltage will have a much larger magnitude than the small change in the input!

For example, if the input voltage changes by 1 mV (i.e., $v_i = 1$ mV), the output might change by, say, 5 mV (i.e., $v_o = 5$ mV).

**Q:** Goodness! By how much would the output change in our example circuit? How can we determine the small-signal output $v_o$?

Determining how much the output voltage of our circuit will change when we change the input voltage by a small amount is very straightforward—we simply take the derivative of the output voltage $v_O$ with respect to input voltage $v_I$!

By taking the derivative of $v_O$ with respect to $v_I$ (when the MOSFET is in saturation, we find:

$$
\frac{d v_O}{d v_I} = \frac{d \left( 5.0 - 0.75(v_I - 1.0)^2 \right)}{d v_I} = -1.50(v_I - 1.0) \quad \text{for} \quad 1.0 < v_I < 3.0
$$

This expression describes the slope of our circuit's transfer function (for $1.0 < v_I < 3.0$). Note the slope with the largest magnitude occurs when $v_I = 3.0$, providing a slope of -3.0 mV/mV.
Thus, if we DC bias this circuit with $V_I = 3.0 \, \text{V}$ (resulting in $V_O = 2.0 \, \text{V}$), we find that the small signal output will be 3 times the small signal input!

For example, say that the input to our circuit is:

$$v_I = 3.0 + 0.01 \cos \omega t \, \text{V}$$

(i.e., $V_I = 3.0 \, \text{V}$ and $v_I = 0.01 \cos \omega t$). We would find that the output voltage would approximately be:

$$v_O = 2.0 - 0.03 \cos \omega t \, \text{V}$$

(i.e., $V_I = 2.0 \, \text{V}$ and $v_o = -0.03 \cos \omega t$). Note then that:

$$v_o = \frac{d v_O}{d v_I} \bigg|_{v_I = 3.0} v_i$$

$$= -3.0 \, v_i$$

$$= -0.03 \cos \omega t$$

In other words, the magnitude of the small-signal output has a magnitude three times larger than the input magnitude.

We say then that our signal provides small-signal gain—our circuit is also a small-signal amplifier!
I see. A small voltage change results in a big voltage change—it's voltage gain!

The MOSFET saturation mode turns out to be—excellent.

Even the simple circuit of this example is sufficient demonstrates to demonstrate the two primary applications of MOSFET transistors—digital circuits and signal amplification.

Whereas the important MOSFET regions for digital devices are triode and cutoff, MOSFETs in amplifier circuits are typically biased into the saturation mode!