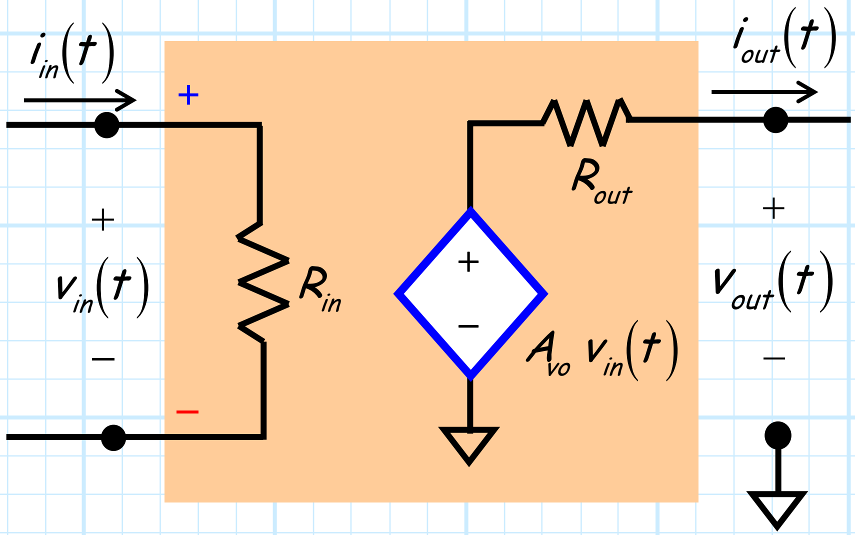
The Ideal Operational Amplifier

We begin by considering the **equivalent circuit** of an **ideal** op-amp:



Note that **output** voltage is defined with respect to **ground** potential, while the input voltage is simply the potential **difference** between the **plus** (+) terminal and the **minus** (-) terminal.

Very large and very small

Of course, we have **three** parameters in this circuit model: input resistance, output resistance, and open circuit voltage gain.

Q: *So what are the **ideal** attributes (R_{in}, R_{out}, and A_{vo}) of an operational amplifier? In other words, what is the **perfect** op-amp?*

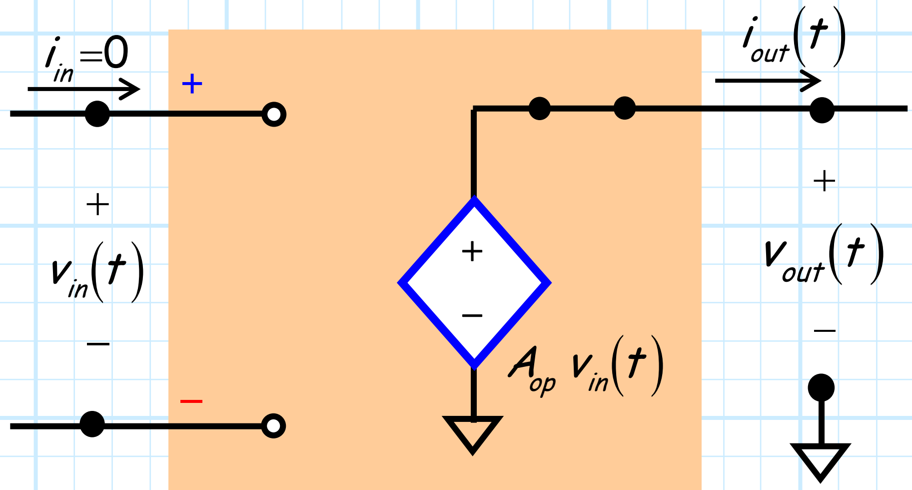
A1: The **input resistance** of a perfect op-amp is infinitely **large** (i.e., $R_{in} = \infty$).

A2: The **output resistance** of a perfect op-amp is **zero** (i.e., $R_{out} = 0$).

A3: The open-circuit **voltage gain** of a perfect op-amp is **very large**, approaching **infinity** ($A_{vo} \cong \infty$)!

The ideal op-amp model

Thus, the equivalent circuit model of an **ideal op-amp** is:



Here we have **changed the notation** of the open-circuit voltage gain.

The value A_{op} is used, where:

$$A_{op} \doteq \lim_{A_{vo}\to\infty} A_{vo}$$

In other words, the gain value A_{op} is **unfathomably large**!

Ideal at all frequencies!

Note then:

1. Since the input resistance is infinite, the **input current is zero**—always!

2. Since the output resistance is zero, the **output** voltage is **equal** to the **open-circuit** output voltage, even when the output load is **not** an open circuit! I.E.,:

$$v_{out}(t) = A_{op}\, v_{in}(t) \quad \leftarrow \text{ regardless of } i_{out}!$$

Q: *What about the **bandwidth** of this "ideal" op-amp; is the model **only** valid for **low-frequencies** ?*

A: Not for an **ideal** op-amp!

The bandwidth of an **ideal** op-amp is likewise **infinite**.

It just seems so perfect

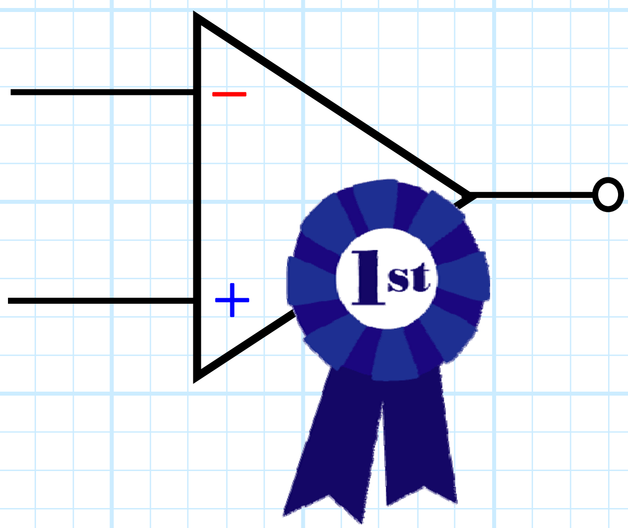
Q: *Wow! Unfathomably **high** voltage gain, **infinite** input resistance (impedance), **zero** output resistance (impedance), and:*

$$v_{out}(t) = A_{op}\, v_{in}(t)$$

***regardless** of the frequency spectrum $V_{in}(\omega)$.*

*This sounds like the **perfect voltage amplifier**!*

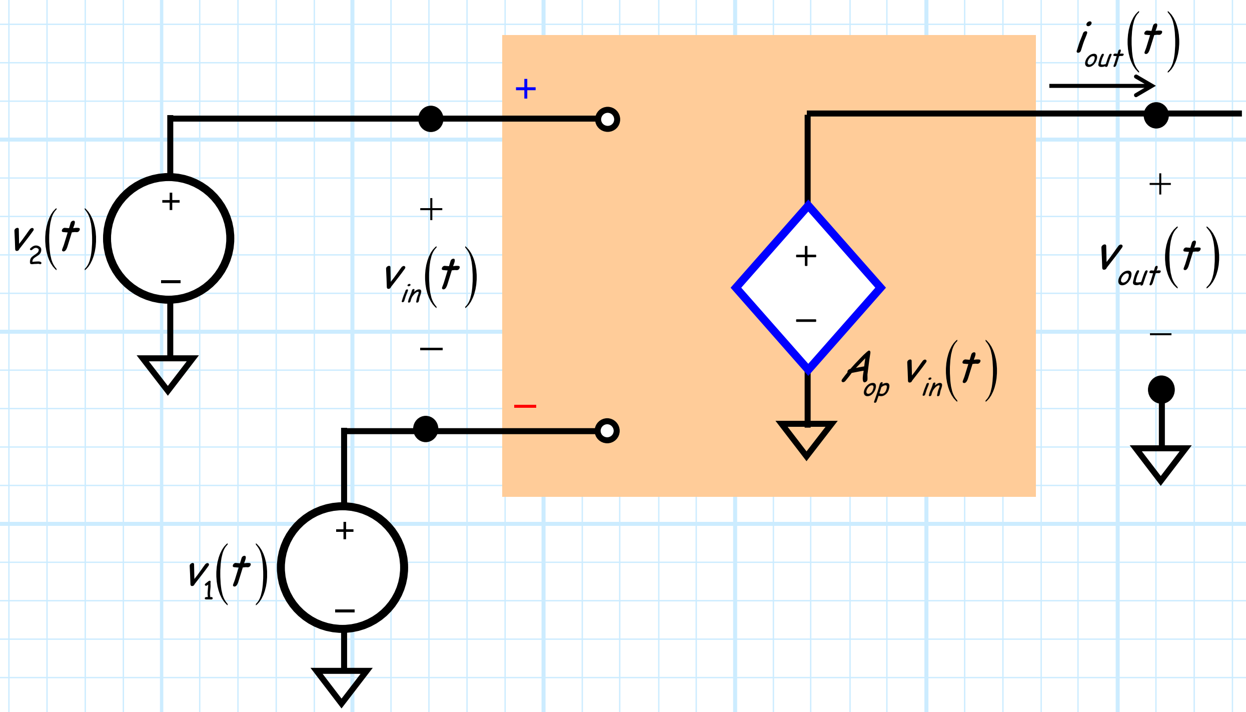
A: It is! That's why we refer to it as the **ideal** op-amp.



Why the output but not the input?

Q: *So why isn't the input voltage with respect to **ground** potential? Why is not the minus (-) input terminal **connected to ground**?*

A: Generally speaking, we find that **two different voltages** will be connected to the two different input terminals:



The input is a differential voltage

From **KVL** it is clear (right?) that the **input voltage** is:

$$v_{in}(t) = v_2(t) - v_1(t)$$

And so the **output** voltage is:

$$v_{out}(t) = A_{op}\left(v_2(t) - v_1(t)\right)$$

Note that the input voltage is simply the **difference** between the **two** input signals v_2 and v_1.

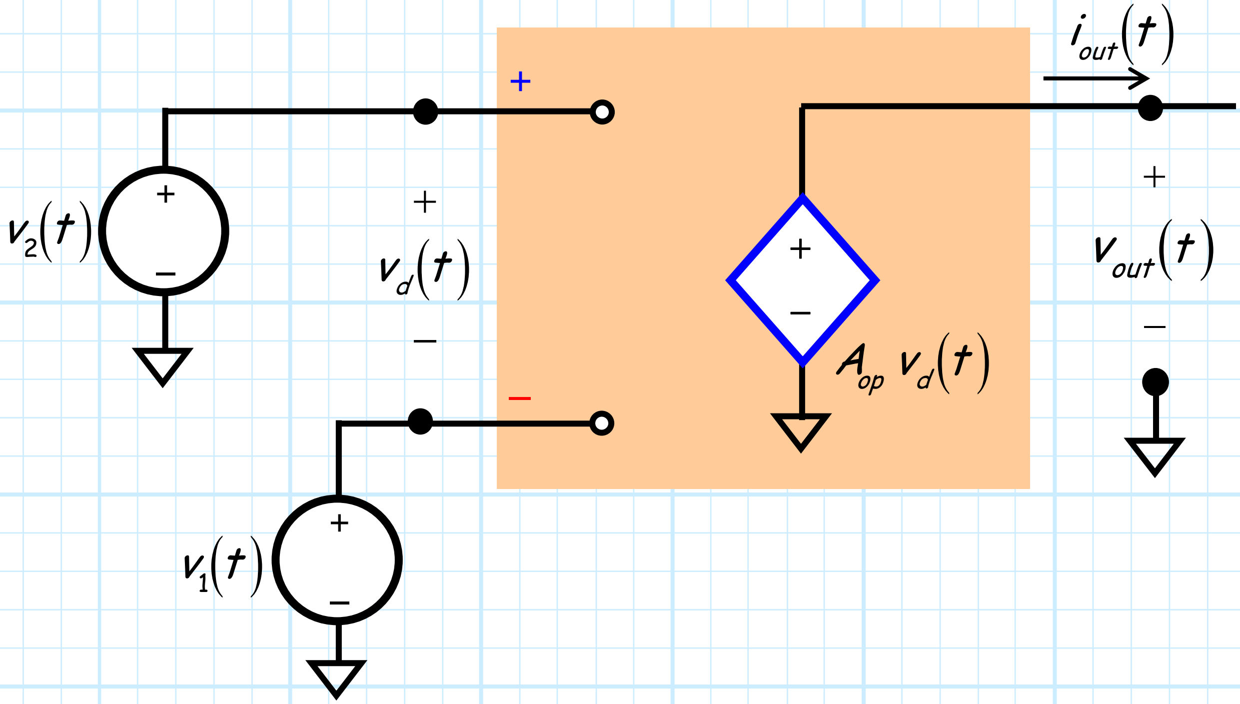
We call this the **differential** input signal:

$$v_d(t) \doteq v_2(t) - v_1(t)$$

It's called a differential amplifier

Thus, we can likewise express the output as:

$$v_{out}(t) = A_{op} v_d(t)$$



Amplifiers of this type—where the input voltage is **not** defined with respect to ground—are referred to as **differential amplifiers**, as they can amplify the **differential mode** of **two** distinct signals (e.g., $v_2(t)$ and $v_1(t)$).

An example

For **example**, say:

$$v_1(t) = 7.0\ \cos(10\pi t) + 2.0\ \cos(5\pi t)$$

and:

$$v_2(t) = 7.0\ \cos(10\pi t) + 5.0\ \cos(5\pi t)$$

the **ideal** op-amp output voltage is therefore:

$$\begin{aligned} v_{out}(t) &= A_{op}\left(v_2(t) - v_1(t)\right) \\ &= A_{op}\ 3.0\ \cos(5\pi t) \end{aligned}$$

➔ What happened to $\cos(10\pi t)$??

The common mode disappears!

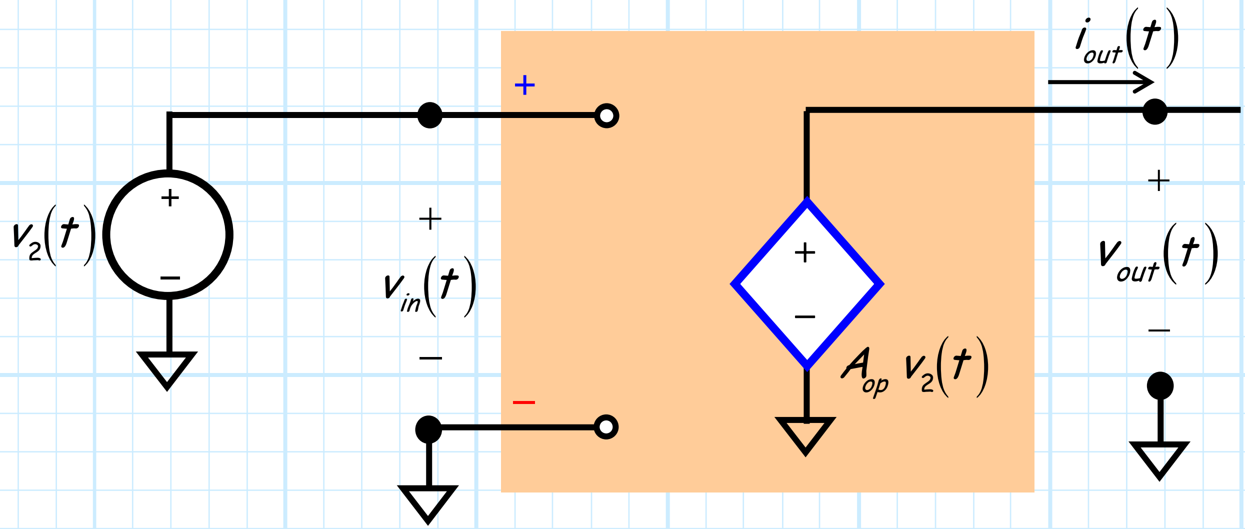
Note:

1. The **difference** between $v_2(t)$ and $v_1(t)$ is amplified.

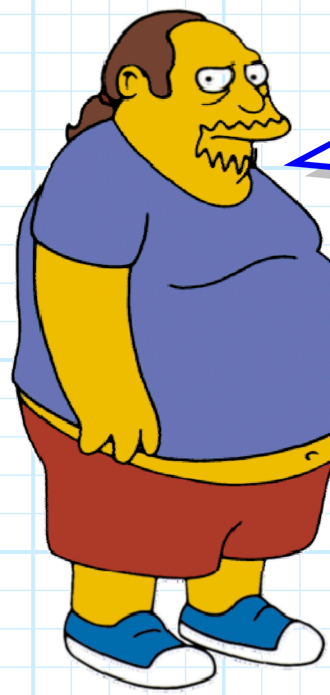
2. The **common** signal ($7\cos 10\pi\ t$) is eliminated by the subtraction .

☞ Difference amplifiers **ideally** have perfect **common-mode rejection.** That is, the common signal between the two inputs has **no effect** on the output signal.

Of course, we can always **connect** a terminal to ground potential (i.e., $v_1(t)=0$), thus making the input voltage a value with respect to ground:



The ideal op-amp; is it bogus?



Q: *I scoff at your so-called **"ideal"** op-amp.*

Although $R_{in} = \infty$ and $R_{out} = 0$ *are obviously correct, I deem your assertion that* $\boldsymbol{A_{op}}$ *should be **unfathomably large** (approaching* ∞*) to be a **silly** notion.*

After all, a gigantic gain A_{op} *would mean that the **output voltage*** $v_{out} = A_{op}(v_2 - v_1)$ *would likewise be **unfathomably** large—the **destructive** implications are **obvious**.*

A: It is true that the output voltage will be very large—**unless** the differential voltage is **unfathomably small**!

Definitely not bogus!

For example, what if the differential voltage is approximately (i.e., almost) **zero**:

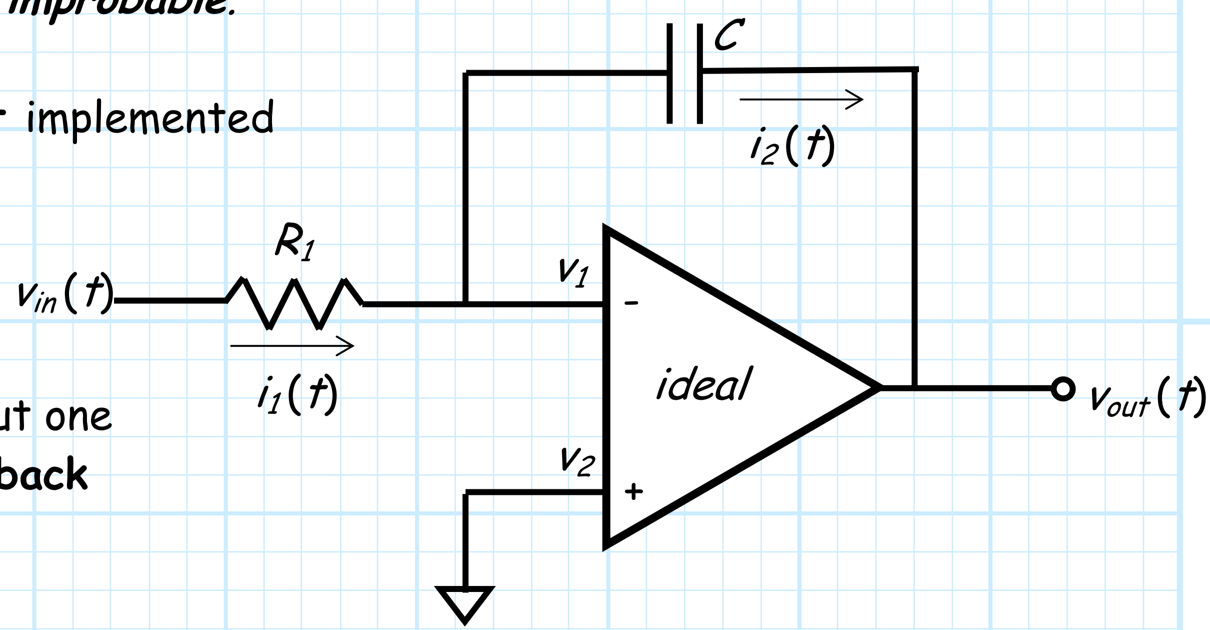
$$v_d(t) \cong 0 \qquad \Rightarrow \qquad v_2(t) \cong v_1(t) \qquad ?$$

In this case, the output voltage may **not** be very large at all!

Q: *Yes, but what is the **likelihood** that the two voltages $v_2(t)$ and $v_1(t)$ are nearly the same? This seems **improbable**.*

A: Op-amps are generally **not** implemented by themselves!

Instead, they typically are but one component of **many** in a **feedback amplifier**.



Get used to the virtual short!

In these applications, we will **indeed** find that $v_2 \cong v_1$—but we will also find that this is a **desirable** condition!

The condition $v_2 \cong v_1$ is know as a **virtual short**.

If this is not true, the output **voltage** of an ideal op-amp will be **unfathomably large**.

As a result, the **virtual short** $v_2 \cong v_1$ is almost **always** the case in useful op-amp circuits.