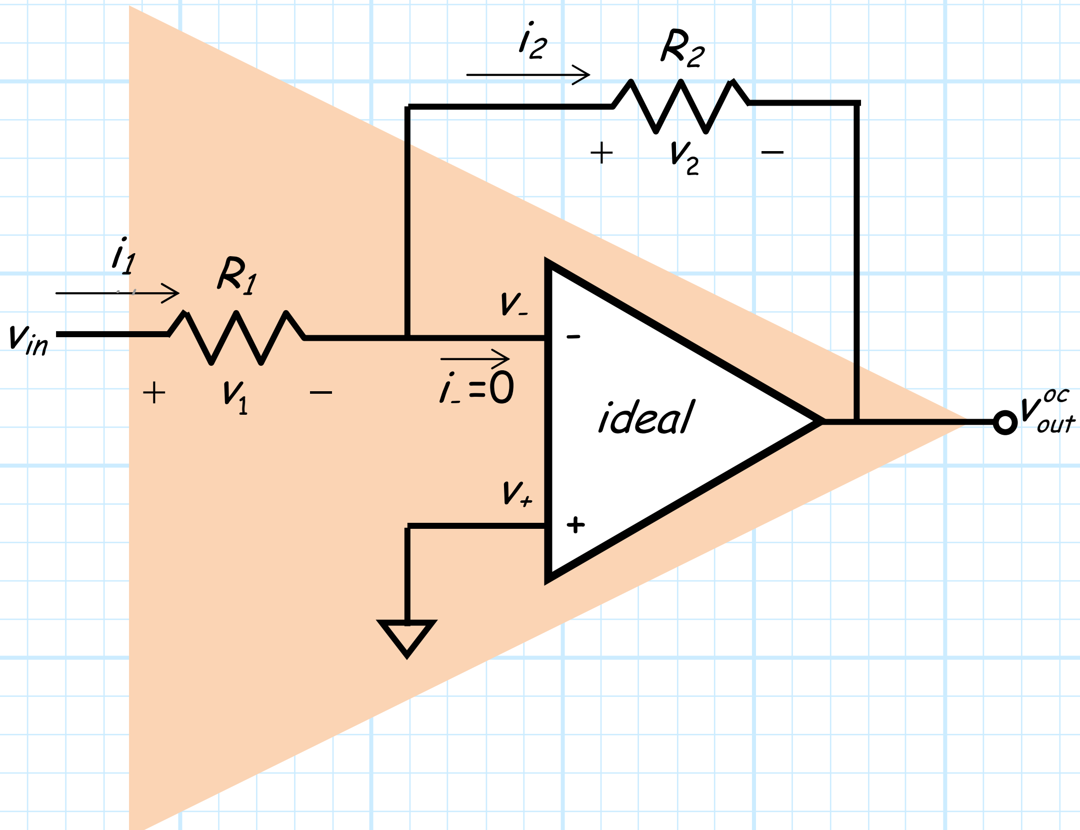
Analysis of the Inverting Amplifier

Consider an **inverting amplifier**:



Note that we use here the **new notation** $v_+ = v_2$ and $v_- = v_1$.

Pay attention to your TA!

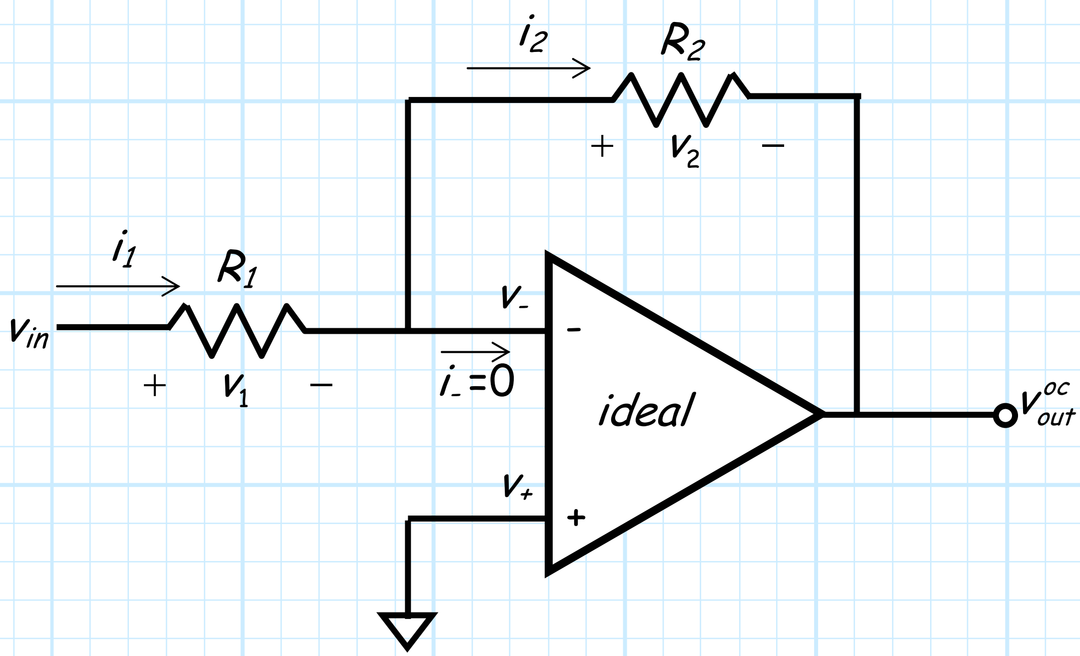
Now what is the **open-circuit voltage gain** of this inverting amplifier?

Let's start the analysis by writing down **all that we know**. First, the **op-amp equation**:

$$v_{out}^{oc} = A_{op}\left(v_{+} - v_{-}\right)$$

Since the non-inverting terminal is **grounded** (i.e., $v_+ = 0$):

$$v_{out}^{oc} = -A_{op}\, v_{-}$$



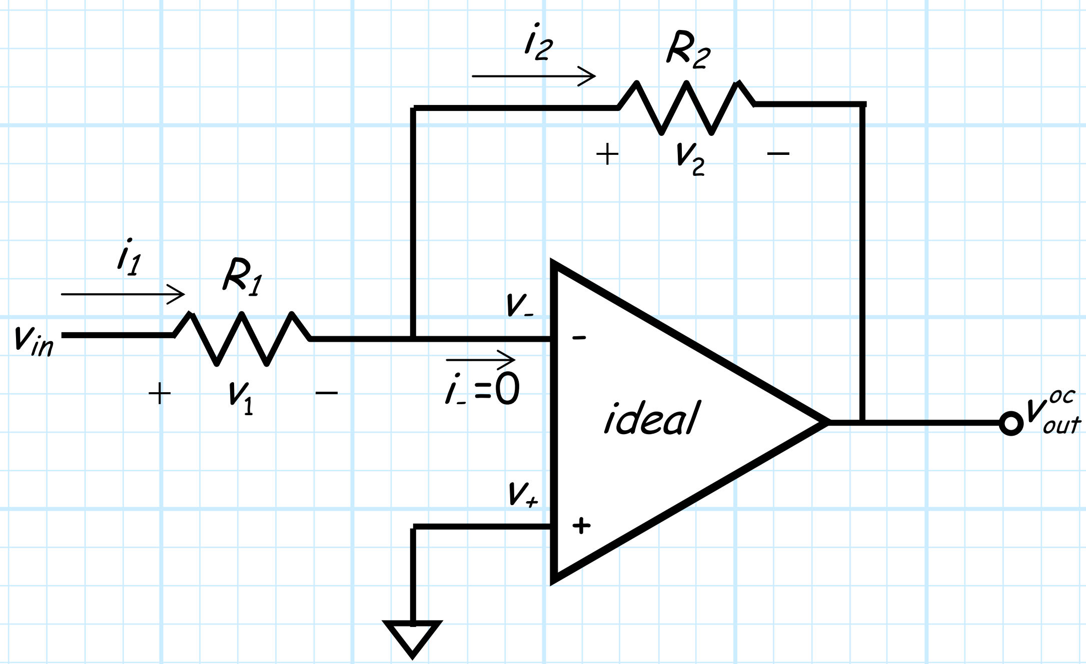
First some KCL…

Now let's apply our **circuit** knowledge to the remainder of the amplifier circuit. For example, we can use KCL to determine that:

$$i_1 = i_- + i_2$$

However, we know that the **input current** i_- of an ideal op-amp is **zero**, as the input resistance is infinitely large.

Thus, we reach the conclusion that: $i_1 = i_2$



And then some Ohm's law…

Likewise, we know from Ohm's Law:

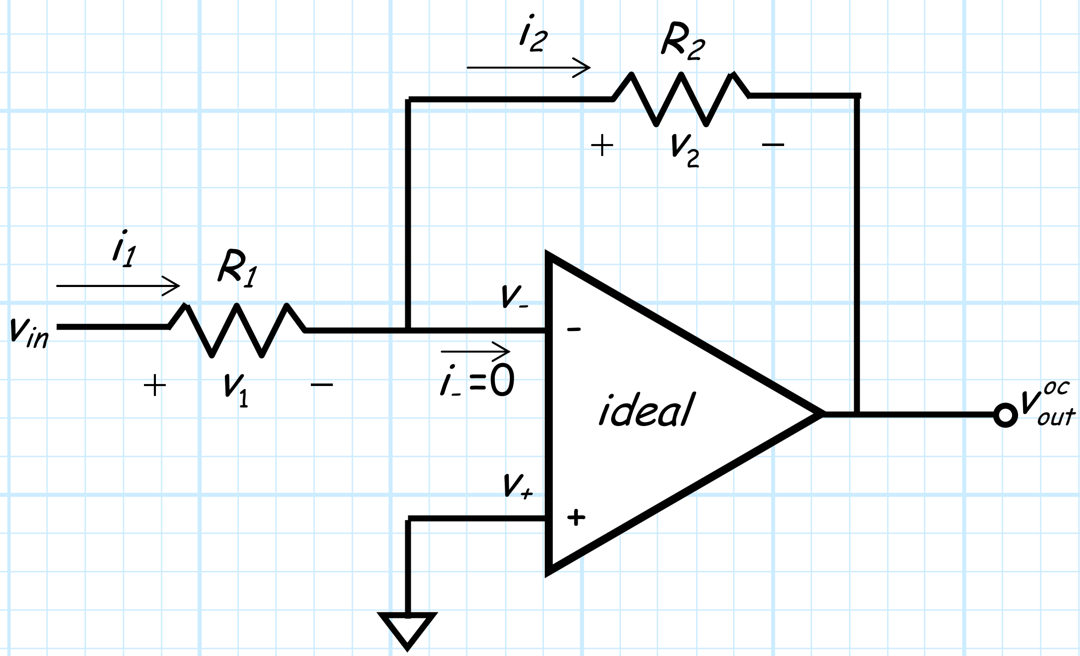
$$i_1 = \frac{v_1}{R_1}$$

and also that:

$$i_2 = \frac{v_2}{R_2}$$

And so combining:

$$\frac{v_1}{R_1} = \frac{v_2}{R_2}$$

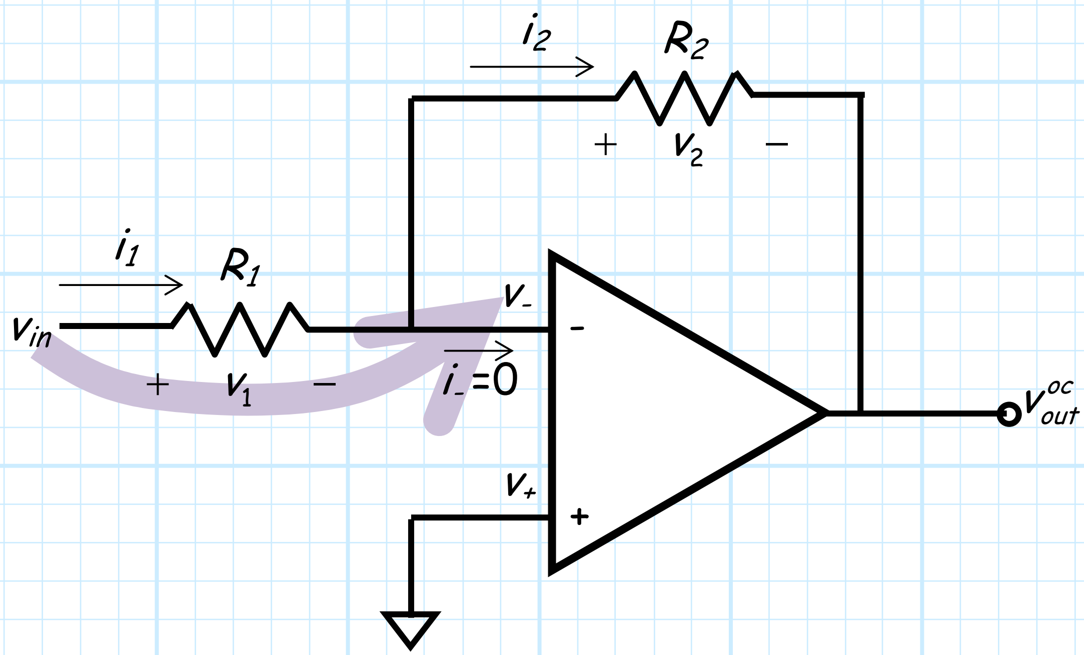


Followed by KVL…

Finally, from KCL we can conclude:

$$v_{in} - v_1 = v_- \qquad \Rightarrow \qquad v_1 = v_{in} - v_-$$

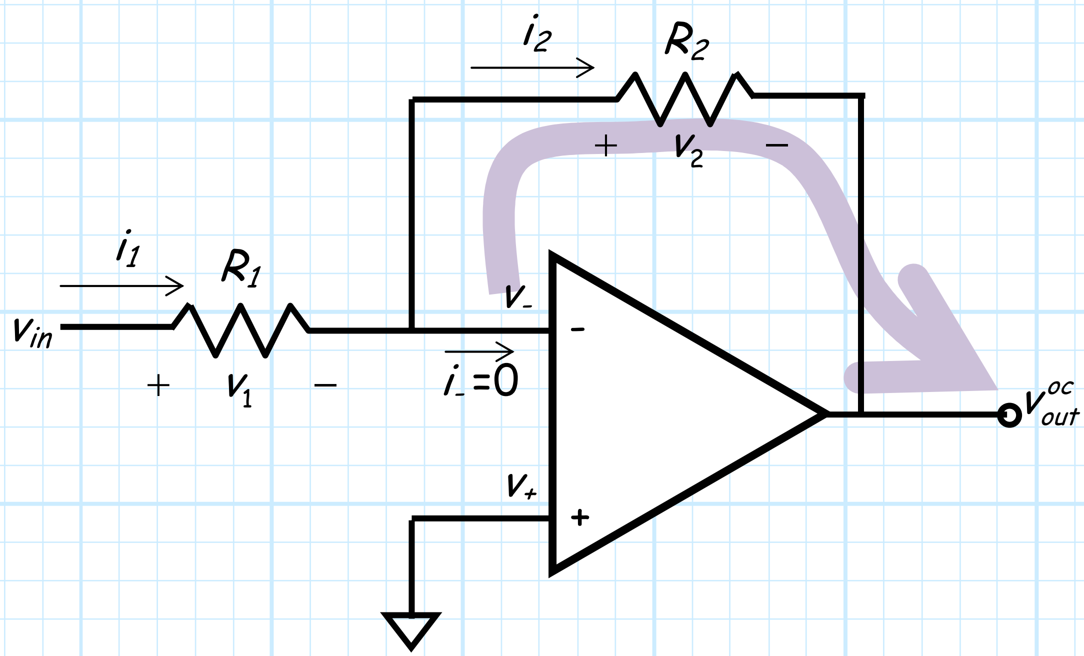
In other "words", we start at a potential of v_{in} volts (with respect to ground), we drop a potential of v_1 volts, and now we are at a potential of v_- volts (with respect to ground).



And yet another KVL…

Likewise, we start at a potential of of v_- volts (with respect to ground), we drop a potential of v_2 volts, and now we are at a potential of v_{out}^{oc} volts (with respect to ground).

$$v_- - v_2 = v_{out}^{oc} \qquad \Rightarrow \qquad v_2 = v_- - v_{out}^{oc}$$



The feed-back equation

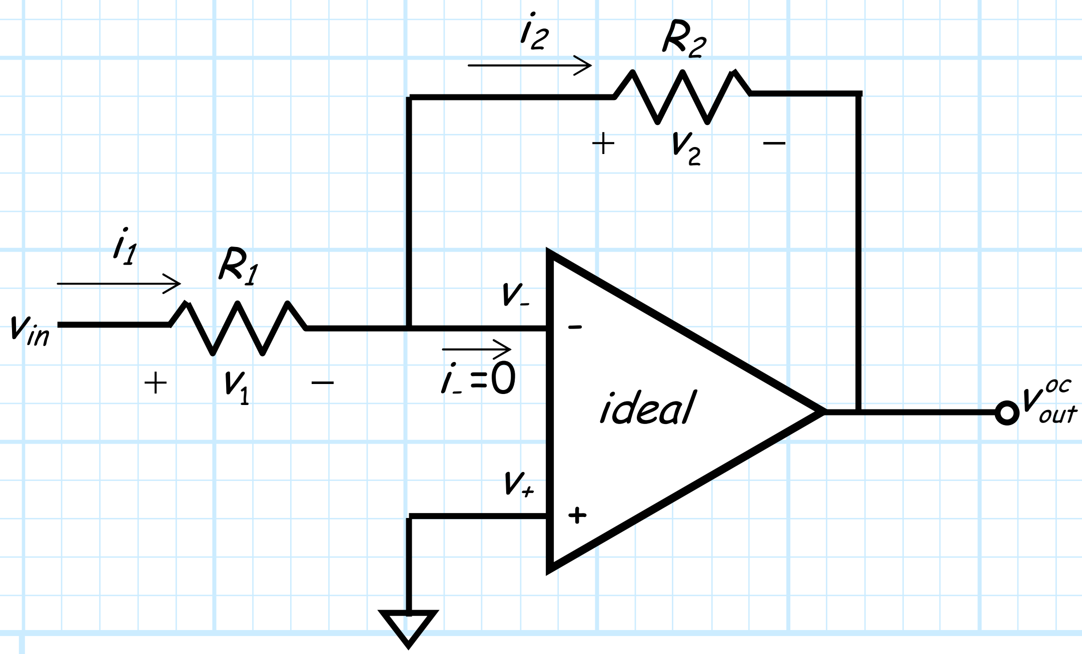
Combining these last three equations, we find:

$$\frac{v_{in} - v_-}{R_1} = \frac{v_- - v_{out}^{oc}}{R_2}$$

Now rearranging, we get what is known as the **feed-back equation**:

$$v_- = \frac{R_2 v_{in} + R_1 v_{out}^{oc}}{R_1 + R_2}$$

Note the feed-back equation relates v_- in terms of output v_{out}^{oc}.



The feed-forward equation

We can combine this feed-back equation with the **op-amp** equation:

$$v_{out}^{oc} = -v_{-} \; A_{op}$$

This op-amp equation is likewise referred to as the **feed-forward** equation.

Note this equation relates the output v_{out}^{oc} in terms of v_{-}.

We can combine the feed-**back** and feed-**forward** equations to determine an expression involving **only** input voltage v_{in} and output voltage v_{out}^{oc}:

$$\frac{R_2 \; v_{in} + R_1 \; v_{out}^{oc}}{R_1 + R_2} = -\frac{v_{out}^{oc}}{A_{op}}$$

...and the open-circuit voltage gain appears!

Rearranging this expression, we can determine the **output** voltage v_{out}^{oc} in terms of **input** voltage v_{in} :

$$v_{out}^{oc} = \left(\frac{-A_{op} R_2}{(R_1 + R_2) + A_{op} R_1} \right) v_{in}$$

and thus the **open-circuit voltage gain** of the inverting amplifier is:

$$A_{vo} = \frac{v_{out}^{oc}}{v_{in}} = \left(\frac{-A_{op} R_2}{(R_1 + R_2) + A_{op} R_1} \right)$$

Recall that the voltage gain A of an **ideal op-amp** is very large—approaching **infinity**.

Thus the open-circuit voltage gain of the **inverting amplifier** is:

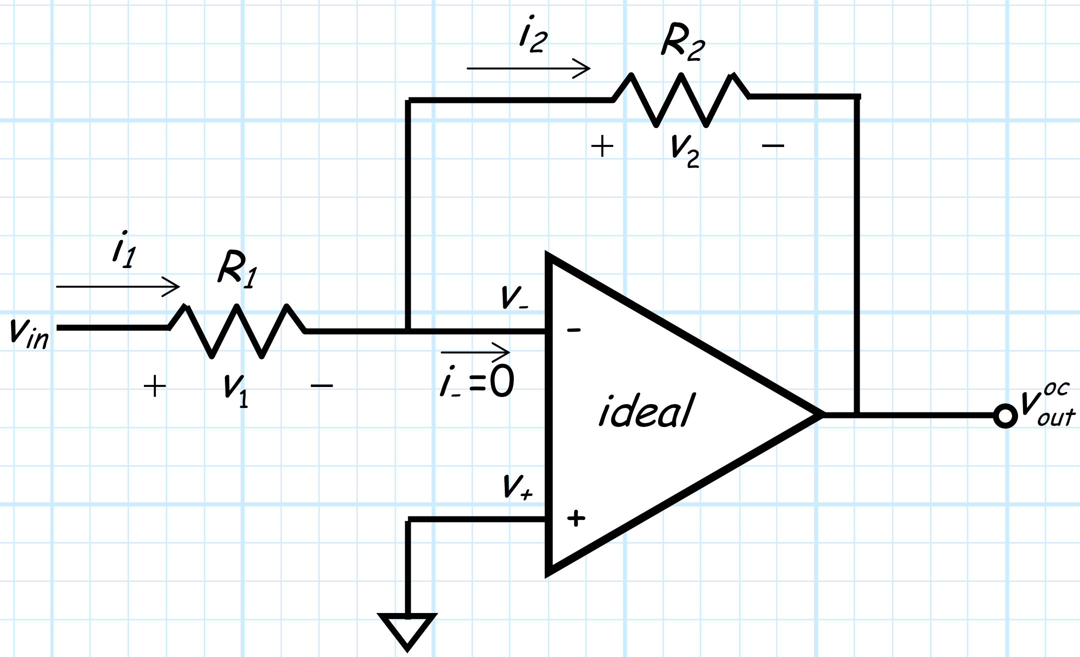
$$\begin{aligned} A_{vo} &= \lim_{A_{op} \to \infty} \left(\frac{-A_{op} R_2}{(R_1 + R_2) + A_{op} R_1} \right) \\ &= \frac{-R_2}{R_1} \end{aligned}$$

Summarizing

Summarizing, we find that for the **inverting amplifier**:

$$A_{vo} = \frac{-R_2}{R_1}$$

$$v_{out}^{oc} = \left(\frac{-R_2}{R_1}\right) v_{in}$$



The non-inverting terminal is at ground potential

One **last thing**. Let's use this final result to determine the value of v_-, the voltage at the **inverting** terminal of the op-amp.

Recall:

$$v_- = \frac{R_2 v_{in} + R_1 v_{out}^{oc}}{R_1 + R_2}$$

Inserting the equation:

$$v_{out}^{oc} = \left(\frac{-R_2}{R_1}\right) v_{in}$$

we find:

$$\begin{aligned} v_- &= \frac{R_2 v_i + R_1 \left({-R_2}/{R_1} \right) v_{in}}{R_1 + R_2} \\ &= \frac{R_2 v_{in} - R_2 v_{in}}{R_1 + R_2} \\ &= 0 \end{aligned}$$

The voltage at the inverting terminal of the op-amp is **zero**!

The logic behind the virtual short

Thus, since the non-inverting terminal is grounded ($v_2 = 0$), we find that:

$$v_- = v_+ \qquad \text{and} \quad \therefore \qquad v_+ - v_- = 0$$

Recall that this should **not** surprise us.

We know that if **op-amp** gain A_{op} is infinitely large, its output v_{out}^{oc} will also be infinitely large (can you say saturation?), **unless** $v_+ - v_-$ is **infinitely small**.

We find that the **actual** value of $v_+ - v_-$ to be:

$$v_+ - v_- = \frac{v_{out}^{oc}}{A_{op}} = \frac{-R_2}{A_{op} R_1} v_{in}$$

a number which approaches **zero** as $A_{op} \rightarrow \infty$!